

## Lecture 4

### Resultant

The properties of force, moment, and couple were developed in the previous four lecture. Now we are ready to describe the resultant action of a group or system of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduced the system to its simplest form to describe its action. The .resultant of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied. Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics

The most common type of force system occurs when the forces all act in a single plane, say, the x-y plane, as illustrated by the system of three forces  $F_1$ ,  $F_2$ , and  $F_3$  in Fig. 1. We obtain the magnitude and direction of the resultant force  $R$  by forming the force polygon shown in part b of the figure, where the forces are added head to-tail in any sequence. Thus, for any system of coplanar forces we may write

$$\begin{aligned} \mathbf{R} &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F} \\ R_x &= \Sigma F_x \quad R_y = \Sigma F_y \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\ \theta &= \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x} \end{aligned}$$

Graphically, the correct line of action of  $R$  may be obtained by preserving the correct lines of action of the forces and adding them by the parallelogram law. We see this in part a of the figure for the case of three forces where the sum  $R_1$  of  $F_2$  and  $F_3$  is added to  $F_1$  to obtain  $R$ . The principle of transmissibility has been used in this process.

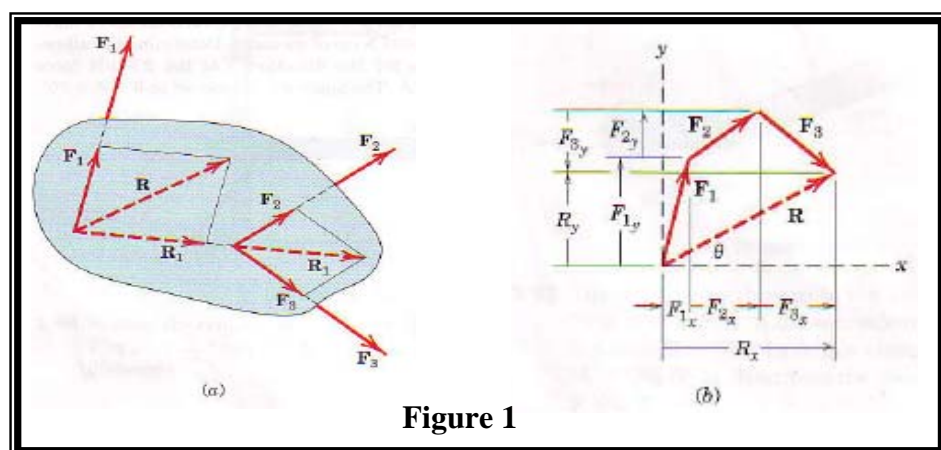


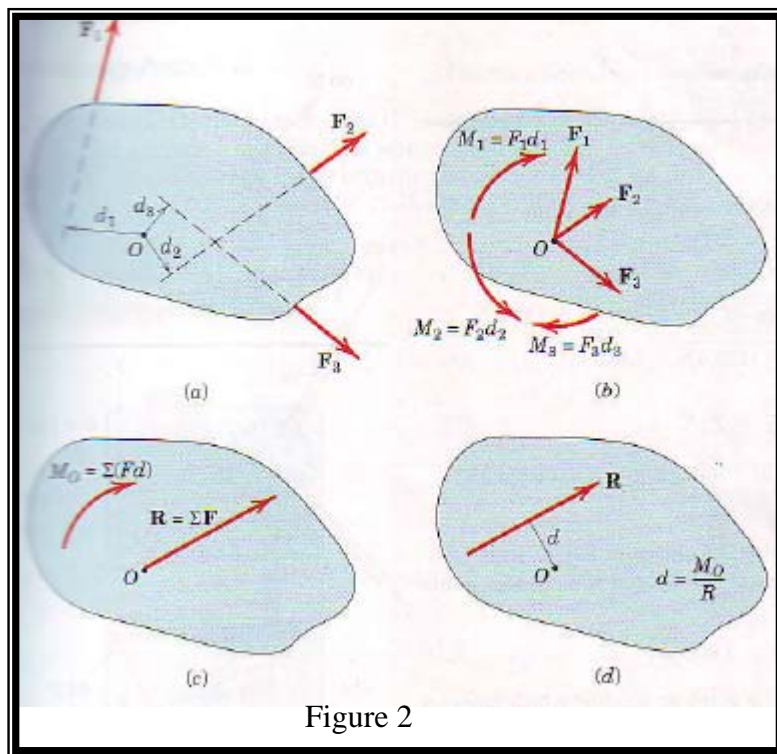
Figure 1

### Algebraic. Method

We can use algebra to obtain the resultant force and its line of action

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs.2a and b, where  $M_1$ ,  $M_2$ , and  $M_3$  are the couples resulting from the transfer of forces  $F_1$ ,  $F_2$ , and  $F_3$  from their respective original lines of action to lines of action through point  $O$ .

2. Add all forces at O to form the resultant force R, and add all couples to form the resultant couple Mo. We now have the single force-couple system, as shown in Fig. 2c.
3. In Fig. 2d, find the line of action of R by requiring R to have a moment of Mo about point O. Note that the force systems of Figs. 2a and 2d are equivalent, and that  $\Sigma(Fd)$  in Fig. 2a is equal to  $Rd$  in Fig. 2d



### principle of Moments

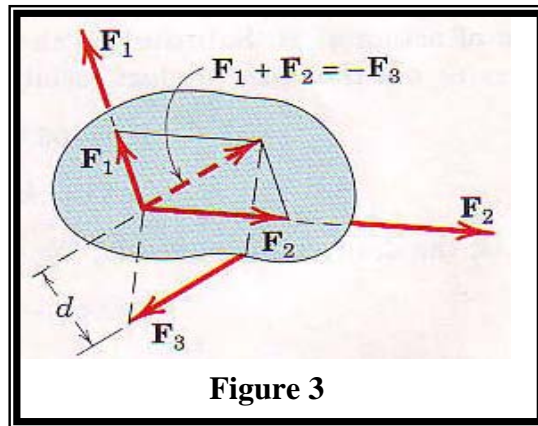
This process is summarized in equation form by

$$\begin{aligned} \mathbf{R} &= \Sigma \mathbf{F} \\ M_O &= \Sigma M = \Sigma (Fd) \\ Rd &= M_O \end{aligned}$$

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The first two of Eqs. 2 reduce a given system of forces to a force-couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of R, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of nonconcurrent force system; we call this extension the principle of moments. For a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum  $\Sigma M_O$  about that point is zero. Thus, the line of action of the resultant  $R = \Sigma F$ , determined by the first of Eqs. 2, passes through point O. For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force R for a given force system is zero, the resultant of the system need not be zero because the

resultant may be a couple. The three forces in Fig. 3, for instance, have a zero resultant force but have a resultant clockwise couple  $M=F_3d$



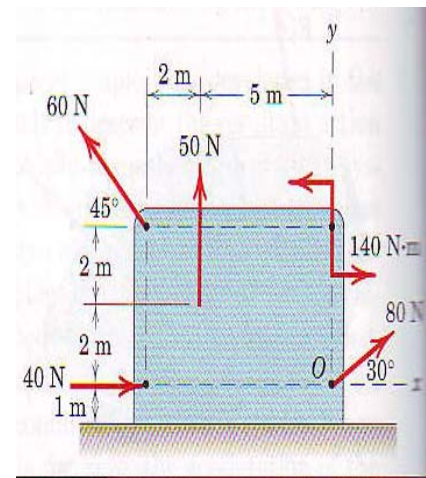
## Examples

### Example1

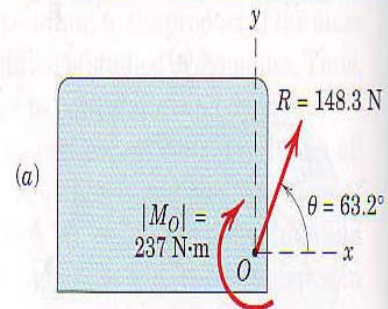
Determine the resultant of the four forces and one couple which act on the plate shown.

**Solution**

Point O is selected as a convenient reference point for the force-couple system that is to represent the given system



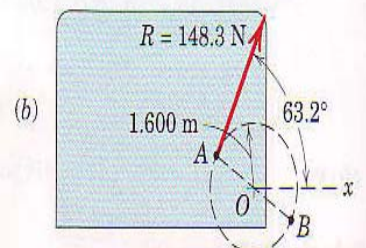
$[R_x = \Sigma F_x]$	$R_x = 40 + 80 \cos 30^\circ - 60 \cos 45^\circ = 66.9 \text{ N}$	
$[R_y = \Sigma F_y]$	$R_y = 50 + 80 \sin 30^\circ + 60 \cos 45^\circ = 132.4 \text{ N}$	
$[R = \sqrt{R_x^2 + R_y^2}]$	$R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$	<i>Ans.</i>
$\left[ \theta = \tan^{-1} \frac{R_y}{R_x} \right]$	$\theta = \tan^{-1} \frac{132.4}{66.9} = 63.2^\circ$	<i>Ans.</i>
$[M_O = \Sigma (Fd)]$	$M_O = 140 - 50(5) + 60 \cos 45^\circ(4) - 60 \sin 45^\circ(7)$	
	$= -237 \text{ N}\cdot\text{m}$	



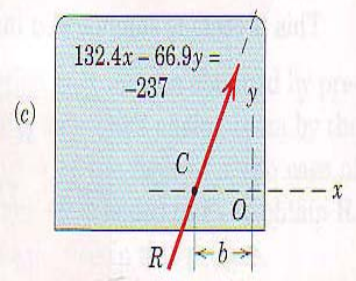
The force-couple system consisting of R and Mo is shown in Fig.a

We now determine the final line of action of R such that R alone represents the original system

$[Rd =  M_O ]$	$148.3d = 237$	$d = 1.600 \text{ m}$	<i>Ans.</i>
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Hence, the resultant R may be applied at point on the line which makes a  $63.2^\circ$  angle with the x-axis and is tangent at point A to a circle of 1.6m radius with center O, as shown in part b of the figure. We apply the equation





$Rd = M_O$  in an absolute-value sense (ignoring any sign of  $M_O$ ) and let the physics of the situation, as depicted in Fig.a, dictate the final placement of R. had  $M_O$  been counterclockwise, the correct line of action of R would have been the tangent at point B.

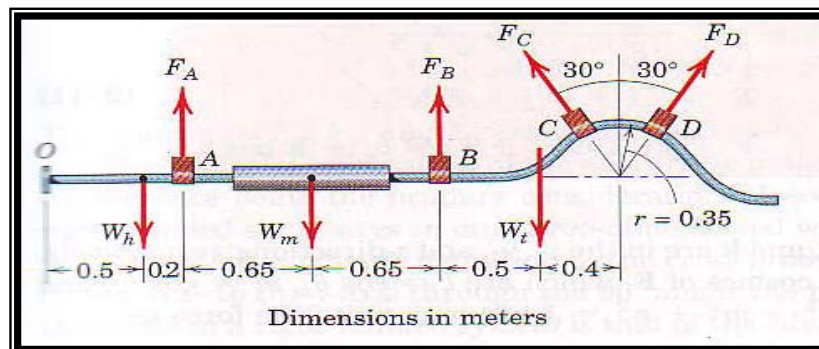
The resultant R may also be located by determining its intercept distance b to point C on the x-axis, Fig.c. with  $R_x$  and  $R_y$  acting through point C, only  $R_y$  exerts a moment about O so that

$$R_y b = |M_O| \quad \text{and} \quad b = \frac{237}{132.4} = 1.792 \text{ m}$$

Alternatively, the y-intercept could have been obtained by noting that the moment about O would be due to  $R_x$  only.

### Example 2

An exhaust system for pickup truck is shown in the Figure. The weights  $W_h$ ,  $W_m$ , and  $W_t$  of the headpipe, muffler, and tailpipe are 10, 100, and 50 N, respectively, and act at the indicated points. If the exhaust pipe hanger at point A is adjusted so that its tension  $F_A$  is 50 N, determine the required forces in the hangers at points B, C, and D so that the force couple system at point O is zero. Why is a zero force couple system at O desirable?



### Solution

For a Zero Force - couple system at point O :

$$R_x = \sum F_x = -F_C \sin 30^\circ + F_D \sin 30^\circ = 0$$

$$F_D \sin 30^\circ = F_C \sin 30^\circ$$

$$F_D = F_C = F$$

$$R_y = \sum F_y = 50 - 10 - 100 - 50 + F_B + F_C \cos 30^\circ + F_D \cos 30^\circ = 0$$

$$0 = -110 + F_B + 2F \cos 30^\circ$$

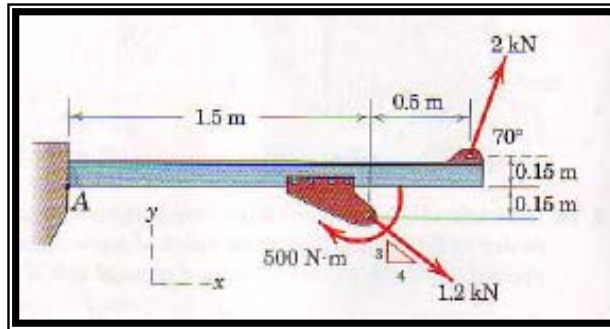
$$F_B = 110 - 2F \cos 30^\circ$$

$$\sum M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30^\circ(2.9) = 0$$

$$F = F_C = F_D = 6.42 \text{ N} \quad , \quad F_B = 98.9 \text{ N}$$

### Example 3

The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown, and their effect on the design of the attachment at A must be determined. Replace the two forces and couple by an equivalent couple M and resultant force R at A



### Solution

$$R_x = \sum F_x$$
$$= 2 \cos 70^\circ + 1.2 \left( \frac{4}{5} \right) = \underline{1.644 \text{ kN}}$$

$$R_y = \sum F_y = 2 \sin 70^\circ - 1.2 \left( \frac{3}{5} \right) = \underline{1.159 \text{ kN}}$$

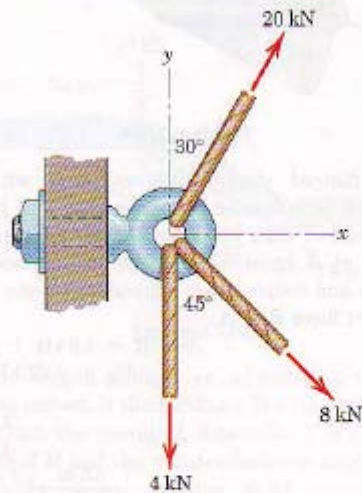
$$\sum M_A = -2 \cos 70^\circ (0.15) + 2 \sin 70^\circ (1.5 + 0.5) + 1.2 \left( \frac{4}{5} \right) (0.15) - 1.2 \left( \frac{3}{5} \right) (1.5) - 0.5 \Rightarrow \underline{2.22 \text{ kN}\cdot\text{m CCW}}$$

The force –couple system is

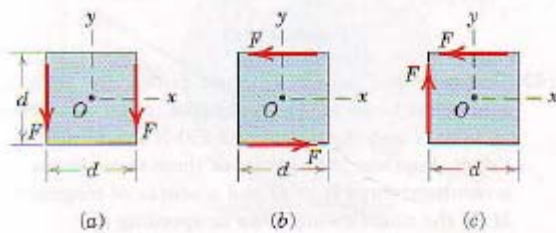
$$R = \sqrt{(1.644)^2 + (1.159)^2} \text{ kN}$$
$$M_A = 2.22 \text{ kN}\cdot\text{m CCW}$$

## Problems

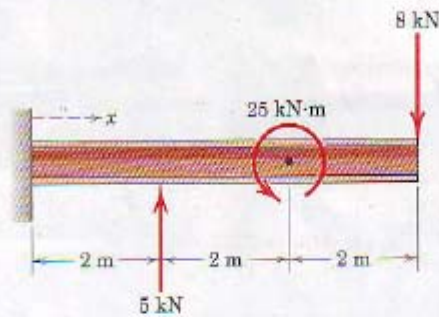
Determine the resultant  $\mathbf{R}$  of the three tension forces acting on the eye bolt. Find the magnitude of  $\mathbf{R}$  and the angle  $\theta_x$  which  $\mathbf{R}$  makes with the positive  $x$ -axis.  
*Ans.*  $R = 17.43 \text{ kN}$ ,  $\theta_x = 26.1^\circ$



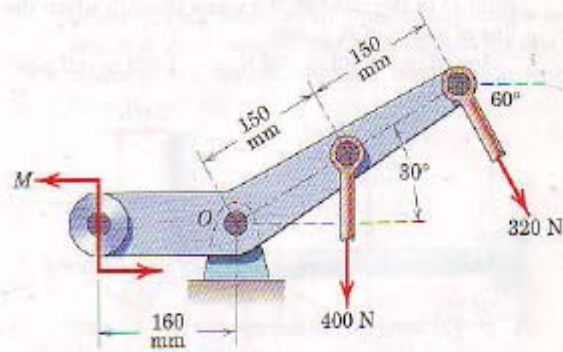
Determine the equivalent force-couple system at the center  $O$  for each of the three cases of forces being applied along the edges of a square plate of side  $d$ .



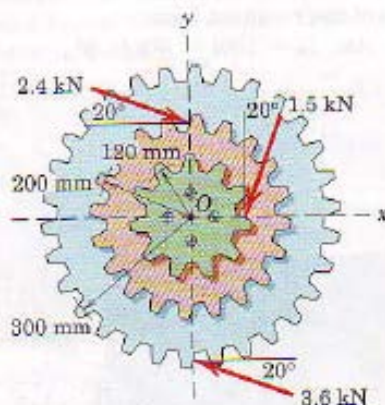
Determine and locate the resultant  $\mathbf{R}$  of the two forces and one couple acting on the I-beam.



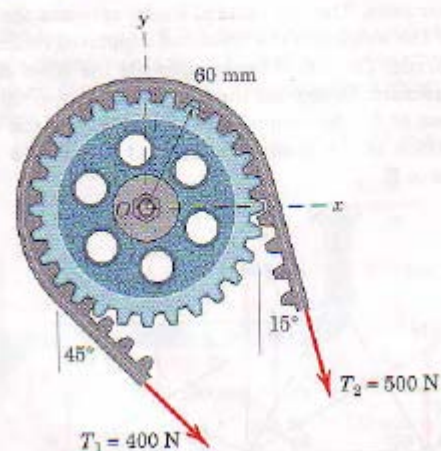
If the resultant of the two forces and couple  $M$  passes through point  $O$ , determine  $M$ .  
*Ans.*  $M = 148.0 \text{ N}\cdot\text{m}$  CCW



Determine the  $x$ - and  $y$ -axis intercepts of the line of action of the resultant of the three loads applied to the gearset.  
*Ans.*  $x = 1.637 \text{ m}$ ,  $y = -0.997 \text{ m}$



As part of a design test, the camshaft-drive sprocket is fixed and then the two forces shown are applied to a length of belt wrapped around the sprocket. Find the resultant of this system of two forces and determine where its line of action intersects both the  $x$ - and  $y$ -axes.



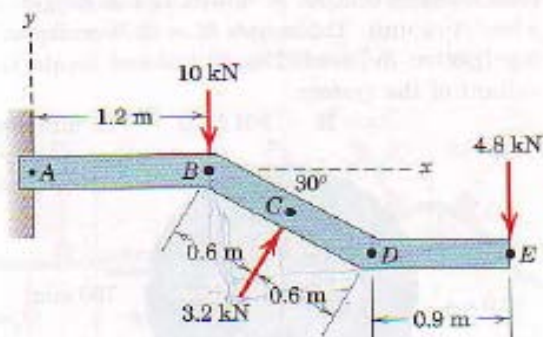


Replace the three forces which act on the bent bar by a force-couple system at the support point A. Then determine the  $x$ -intercept of the line of action of the stand-alone resultant force  $\mathbf{R}$ .

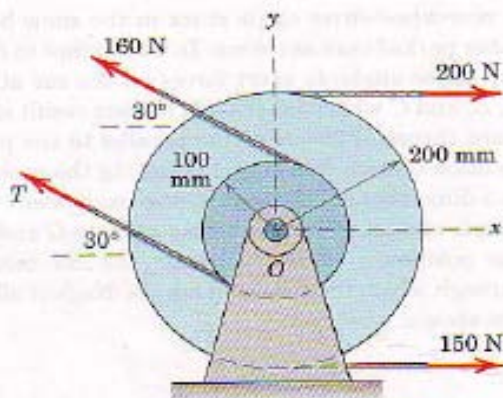
$$\text{Ans. } \mathbf{R} = 1.6\mathbf{i} - 12.03\mathbf{j} \text{ kN}$$

$$M_A = 21.8 \text{ kN}\cdot\text{m CW}$$

$$x = 1.814 \text{ m}$$



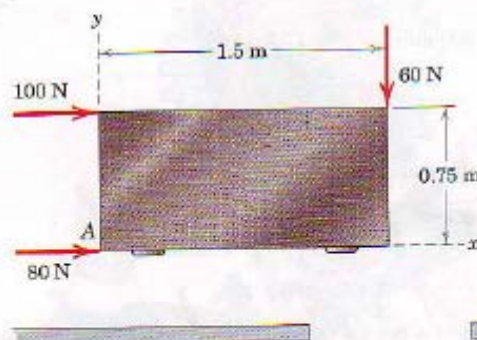
Two integral pulleys are subjected to the belt tensions shown. If the resultant  $\mathbf{R}$  of these forces passes through the center  $O$ , determine  $T$  and the magnitude of  $\mathbf{R}$  and the counterclockwise angle  $\theta$  it makes with the  $x$ -axis.



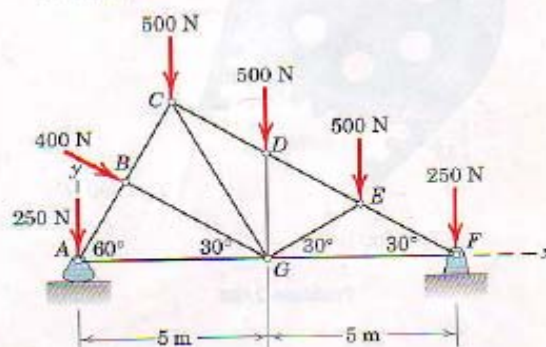
While sliding a desk toward the doorway, three students exert the forces shown in the overhead view. Determine the equivalent force-couple system at point A. Then determine the equation of the line of action of the resultant force.

$$\text{Ans. } \mathbf{R} = 180\mathbf{i} - 60\mathbf{j} \text{ N}, M_A = -165 \text{ kN}\cdot\text{m}$$

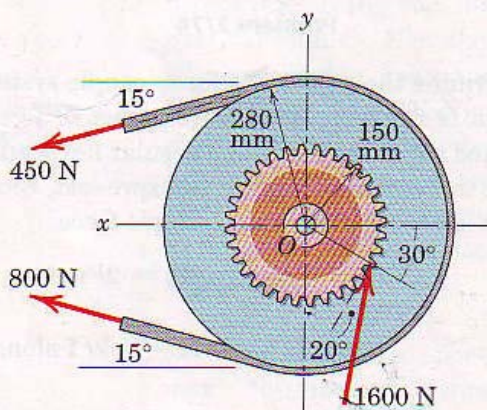
$$y = -\frac{1}{3}x + \frac{11}{12} \text{ m}$$



The asymmetric roof truss is of the type used when a near normal angle of incidence of sunlight onto the south-facing surface ABC is desirable for solar energy purposes. The five vertical loads represent the effect of the weights of the truss and supported roofing materials. The 400-N load represents the effect of wind pressure. Determine the equivalent force-couple system at A. Also, compute the  $x$ -intercept of the line of action of the system resultant treated as a single force  $\mathbf{R}$ .

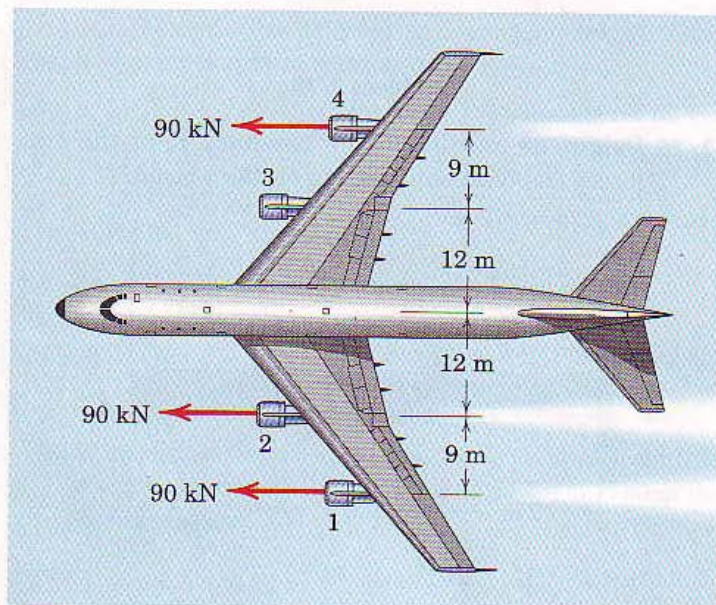


The gear and attached V-belt pulley are turning counterclockwise and are subjected to the tooth load of 1600 N and the 800-N and 450-N tensions in the V-belt. Represent the action of these three forces by a resultant force  $\mathbf{R}$  at  $O$  and a couple of magnitude  $M$ . Is the unit slowing down or speeding up?



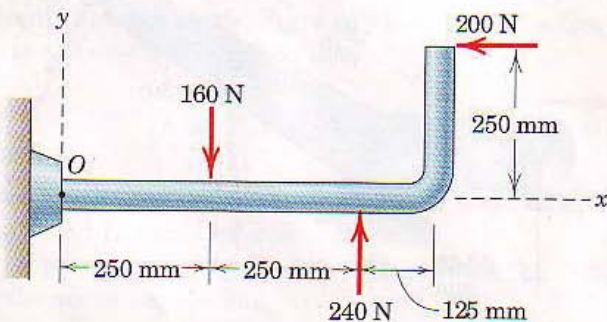


A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two-dimensional problem.



Replace the three forces acting on the bent pipe by a single equivalent force  $\mathbf{R}$ . Specify the distance  $x$  from point  $O$  to the point on the  $x$ -axis through which the line of action of  $\mathbf{R}$  passes.

Ans.  $\mathbf{R} = -200\mathbf{i} + 80\mathbf{j}$  N,  $x = 1.625$  m (off pipe)



The pedal-chainwheel unit of a bicycle is shown in the figure. The left foot of the rider exerts the 160-N force, while the use of toe clips allows the right foot to exert the nearly upward 80-N force. Determine the equivalent force-couple system at point  $O$ . Also, determine the equation of the line of action of the system resultant treated as a single force  $\mathbf{R}$ . Treat the problem as two-dimensional.

