Engineering mechanics
"Static"

lecture 1

Force System

Before dealing with a group or system of forces, it is necessary to examine the properties of a single force in some detail, A force has been define as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a vector quantity, because its effect depends on the direction as well as on the magnitude of the action. Thus, the forces may be combined according to the parallelogram taw of vector addition.

The action of the cable tension on the bracket in Fig.1a is represented in the side view,.Fig.2b, by the force vector P of magnitude P. The effect of this action on the bracket depends on P, the angle θ, and the location of the point of application A. changing any one of these three specifications will alter the effect on the bracket, such as the forces in one of the bolts which secure the bracket to the base, or the internal the complete specification of the action of a force must include its magnitude, direction, and point application, and therefore we must treat it as a fixed vector.

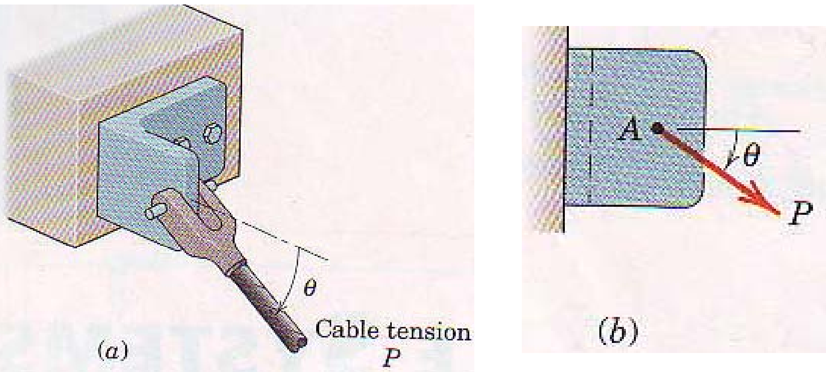


Figure 1

External and internal Effects

We can separate the action of a force on a body into two effects, External and internal , for the bracket of Fig.2 the effects of P external to the bracket are the reactive forces(not shown) exerted on the bracket by the foundation and bolts because of the action of P. forces external to a body can be either applied. forces or reactive forces. The effects of P internal to the bracket are the resulting internal forces and deformations distributed throughout the material of the bracket. The rotation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.

Principle of transmissibility

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force P action on the rigid plate in Fig.2 may be applied at A or at B or at any other point on its line of action, and the net external effects of P on the bracket will not change. The external effect are the force exerted on the plate by the bearing support at 0 and the force exerted on the plate by the roller support at C.

This conclusion is summarized by the principle of transmissibility, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts. Thus, whenever we are interested in only the resultant external effects of force, the force may be treated as a sliding vector, and we need specify only the magnitude, direction, and line of action of the force, and not its point of application.

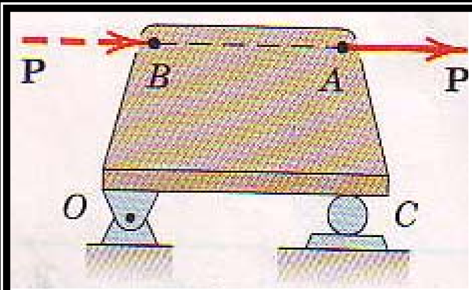


Figure 2

Force Classification

Forces are classified as either contact or body forces. A contact force Is produced by direct physical contact; an example is the force exerted on a body a supporting surface. On the other hand, a body force is generated by virtue of the position of a body within a force field such as A gravitational, electric, or magnetic field. An example of a body force is your weight.

Forces may be further classified as either concentrated or distributed. Every contact force is actually applied over a finite area and is therefore really a distributed force However, when the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an area as in the case of mechanical contact, over a volume when a body force such as weight is acting or over a line, as in the case of the weight of a suspended cable.

The weight of a body is the force of gravitational attraction distributed over its volume and may be taken as a concentrated force acting through the center of gravity. The position of the center of gravity is frequently obvious if the body is symmetric.

We can measure a force either by comparison with other known forces, using a mechanical balance, or by the calibrated movement of an elastic element. AIl such comparisons or calibrations have

as their basis a primary standard. The standard unit of force in SI units is the Newton (N) and in the U.S. customary system is the pound (lb).

action and Reaction

According to Newton's third law, the action of a force is always accompanied by an equal and apposite reaction. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first isolate the body in question and then identify the force exerted on that body (not the force exerted by the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

Concurrent Forces

Two or more forces are said to be concurrent at a point if their lines of action intersect at that point. The forces F1 and F2 shown in Fig.3a have a common point of application and are concurrent at the point A. Thus, they can he added using the parallelogram law in their common plane to obtain their sum or resultant R, as shown in Fig. 3a. The resultant lies in the same plane as Fl and F2.

Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Fig. 3b. By the principle of transmissibility, we may move them along their lines of action and complete their vector sum R at the point of concurrent A, as shown in Fig. 3b. We can replace F1 and F2 with the resultant R without altering the external effects on the body upon which they act.

We can also use the triangle law to obtain R, but we need to move the line of action of one of the forces, as shown in Fig.3c. If we add the same two forces, as shown in Fig. 3d, we correctly preserve the magnitude and direction of R, but we lose the correct line of action, because R obtained in this way does not pass through A. Therefore this two of combination should be avoided.

We can express the sum of the two forces mathematically by the vector equation

$$R=F1+F2$$

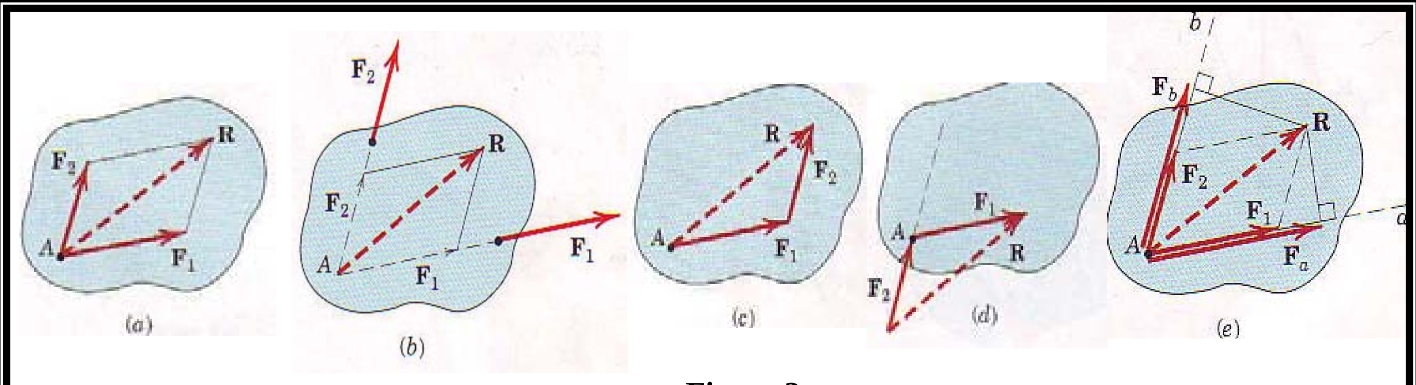


Figure 3

Vector Components

In addition to combining forces to obtain their resultant, we often need to replace a force by its vector components in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force R in Fig. 3a may be replaced by, or .resolved into, two vector components F1 and F2 with the specified directions by completing the parallelogram as shown to obtain the magnitudes of F1and F2.

The relationship between a force and its vector components along given axes must not be contused with the relationship between a force and its perpendicular projections onto the same axes. Fig.3e shows the perpendicular projections Fa and Fb of the given force R onto axes a and b, which are parallel to the vector components F1 and F2 of Fig.3a. Figure 3e shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections Fa and Fb is not the vector R, because the parallelogram law of vector addition must be used to form the sum. The components and projections of R are equal only when the axes a and b are perpendicular.

A Special Case of Vector Addition

To obtain the resultant when the two forces Fl and F2 are parallel as in Fig. 4, we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces F and -F of convenient magnitude, which taken together produce no external effect on the body. adding F1 and F to produce R1 , and combining with the sum R2 of F2 and F yield the resultant R, which is correct in magnitude, direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient point of concurrency because they are almost parallel.

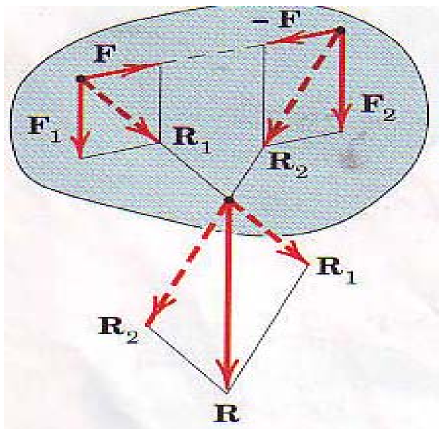


Figure 4

Rectangular Components

The most common two dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector F of Fig. 5 may be written as

F=Fx +Fy

Where Fx and Fy are vector components of F in the x- and y-direction.

For the force vector of Fig. 5, the x and y scalar components arc both positive and are related to the magnitude and direction of F by

$$\boxed{\begin{array}{ll} F_x = F\cos\theta & F = \sqrt{F_x{}^2 + F_y{}^2} \\ F_y = F\sin\theta & \theta = \tan^{-1}\dfrac{F_y}{F_x} \end{array}} \qquad \text{...... Eqs.1}$$

Determining the Components of a Force

Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the x-axis, and the origin of coordinate need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. Figure 6 suggests a few typical examples of vector resolution in two dimensions.

Memorization of Eqs.1 is not a substitute for understanding the parallelogram law and for correctly projecting a vector onto a reference axis. A neatly drawn sketch always he1p6 to clarify the geometry and avoid error.

Rectangular components arc convenient for finding the sum or resultant R of two forces which are concurrent. Consider two forces F1and F2 which are originally concurrent at a point O. Figure 7 shows the line of action of F2 shifted from O to the tip of F1accoding to the triangles rule of Fig. 3 In adding the force vectors F1 and F2, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\mathbf{j}$$

From which we conclude that

$$\begin{aligned} R_x &= F_{1_x} + F_{2_x} = \Sigma F_x \\ R_y &= F_{1_y} + F_{2_y} = \Sigma F_y \end{aligned} \qquad \text{......2}$$

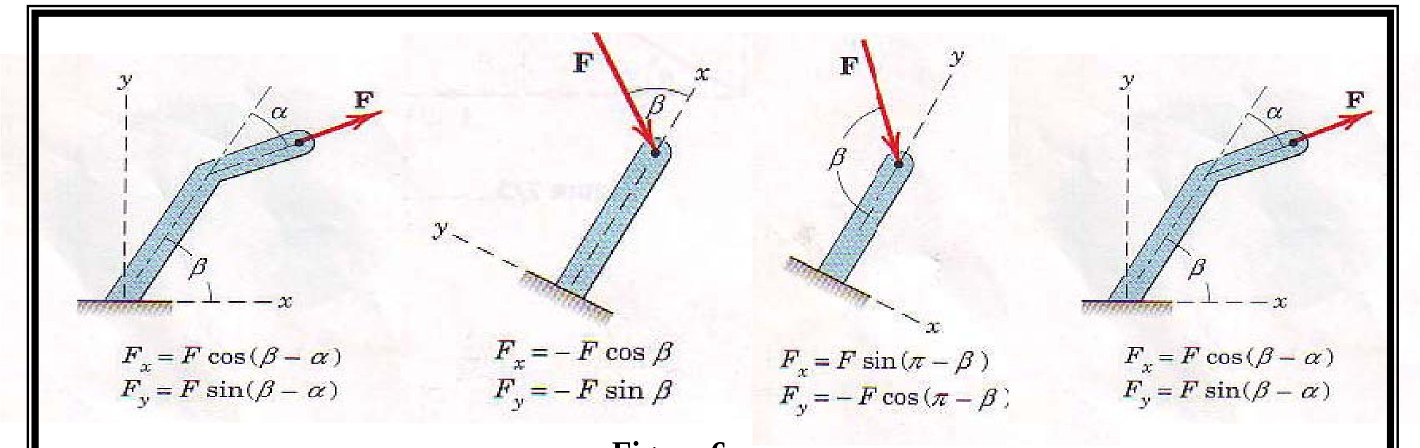


Figure 6

The term ΣFx means "the algebraic sum of the x scalar components". For" the example- shown In Fig. 7, note that the scalar component F_{2y} would be negative.

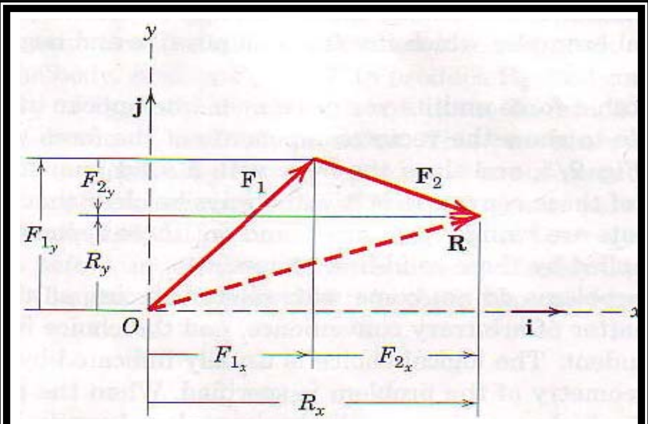
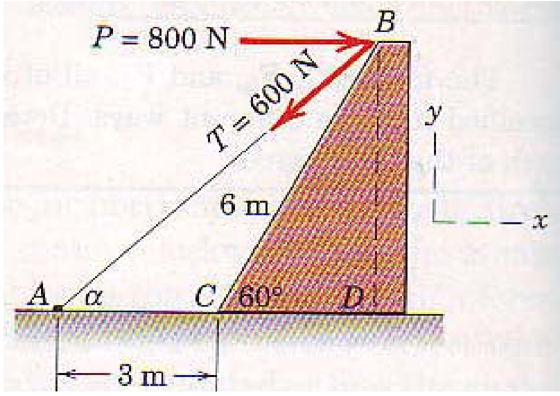


Figure 7

Examples

Example 1

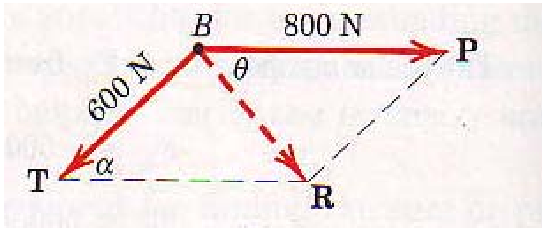
Combine the two forces p and T, which act on the fixed structure at B, into a single equivalent force R.



Graphical solution

The parallogram for the vector addition of forces T and P is constructed as shown in Fig.a . the approxmate scale used here is 1cm=400n; a scale of 1cm = 100 N would be more suitable for regular- size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallogram. From the given figure

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60^\circ}{3 + 6\cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$



(a)

Measurment of the length R and direction θ of the resultant force R yield the approximate results

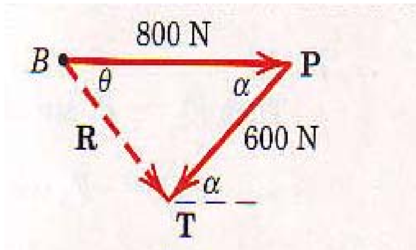
$$R = 525 \text{ N} \qquad \theta = 49^\circ$$

Geometric solution

The triangle for the vector addition of T and P is shown in Fig, b. the angle α is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9^\circ = 274{,}300$$

$$R = 524 \text{ N}$$



(b)

frome the law sines, we may determine the angle θ which orients R. thuse,

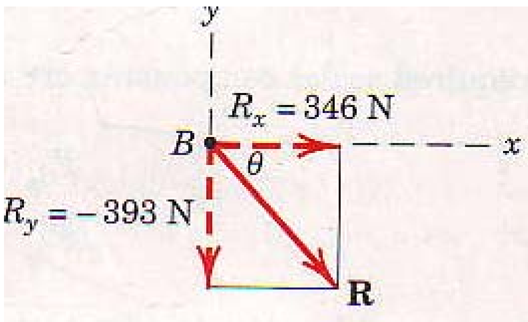
$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9^\circ} \qquad \sin\theta = 0.750 \qquad \theta = 48.6^\circ$$

Algebric solution

By using the x-y coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600\cos 40.9^\circ = 346 \text{ N}$$

$$R_y = \Sigma F_y = -600\sin 40.9^\circ = -393 \text{ N}$$



(c)

The magintude and direction of the resultant force R as shown in Fig, c are then

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ N}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6^\circ$$

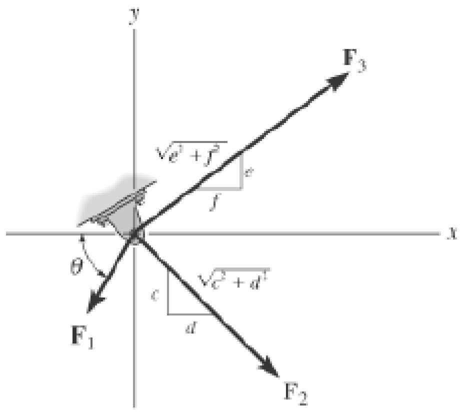
Examples 2:

Determine the magnitude of the resultant force and its direction measured clockwise from the positive *x* axis.

Units Used:

$kN = 10^3$ N

Given:
$F1$ = 20 kN
$F2$ = 40 kN
$F3$ = 50 kN
θ = 60 deg
c = 1
d = 1
e = 3
f = 4



Solution:

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = F_3\left(\frac{f}{\sqrt{e^2+f^2}}\right) + F_2\left(\frac{d}{\sqrt{c^2+d^2}}\right) - F_1\cos(\theta)$$

$$F_{Rx} = 58.28\,\text{kN}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = F_3\left(\frac{e}{\sqrt{e^2+f^2}}\right) - F_2\left(\frac{c}{\sqrt{c^2+d^2}}\right) - F_1\sin(\theta)$$

$$F_{Ry} = -15.6\,\text{kN}$$

$$F = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$F = 60.3\,\text{kN}$$

$$\theta = \operatorname{atan}\left(\frac{|F_{Ry}|}{F_{Rx}}\right)$$

$$\theta = 15\,\text{deg}$$

Example 3

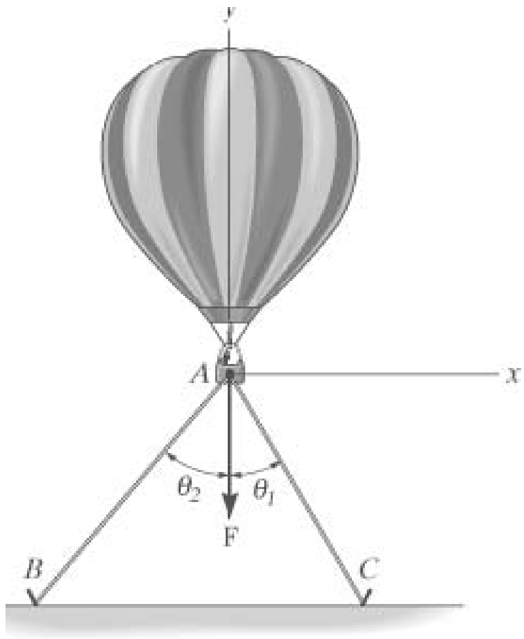
A resultant force **F** is necessary to hold the ballon in place. Resolve this force into components along the tether lines *AB* and *AC*, and compute the magnitude of each component.

Given:

$F = 350$ lb

$\theta_1 = 30$ deg

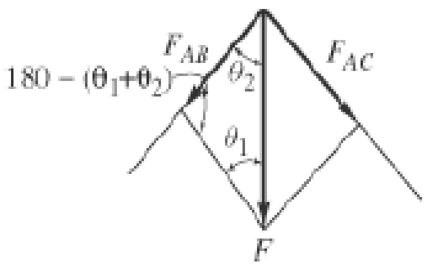
$\theta_2 = 40$ deg

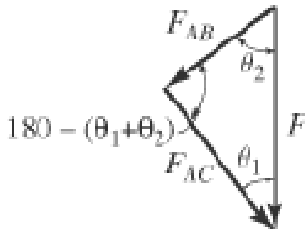


Solution:

$$\frac{F_{AB}}{\sin(\theta_1)} = \frac{F}{\sin[180\ \text{deg} - (\theta_1 + \theta_2)]}$$

$$F_{AB} = F\left[\frac{\sin(\theta_1)}{\sin[180\ \text{deg} - (\theta_1 + \theta_2)]}\right]$$





$F_{AB} = 186\ \text{lb}$

$$\frac{F_{AC}}{\sin(\theta_2)} = \frac{F}{\sin[180\ \text{deg} - (\theta_1 + \theta_2)]}$$

$$F_{AC} = F\left[\frac{\sin(\theta_2)}{\sin[180\ \text{deg} - (\theta_1 + \theta_2)]}\right]$$

$F_{AC} = 239\ \text{lb}$

Problems

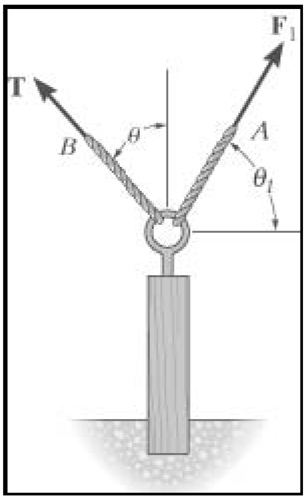
The post is to be pulled out of the ground using two ropes *A* and *B*. Rope *A* is subjected to force *F1* and is directed at angle θ1 from the horizontal. If the resultant force acting on the post is to be *FR*, vertically upward, determine the force **T** in rope *B* and the corresponding angle θ.

Given:

FR = 1200 lb

F1 = 600 lb

θ*1* = 60 deg



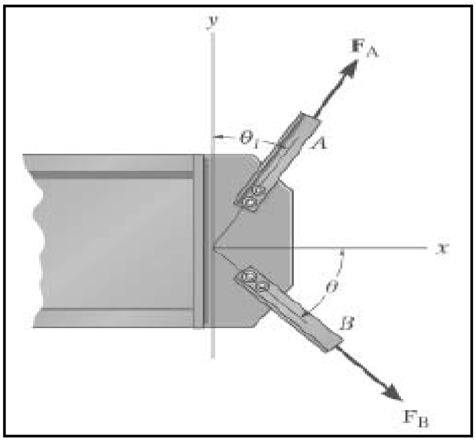
The plate is subjected to the forces acting on members *A* and *B* as shown. Determine the magnitude of the resultant of these forces and its direction measured clockwise from the positive *x* axis. Given:

FA = 400 lb

FB = 500 lb

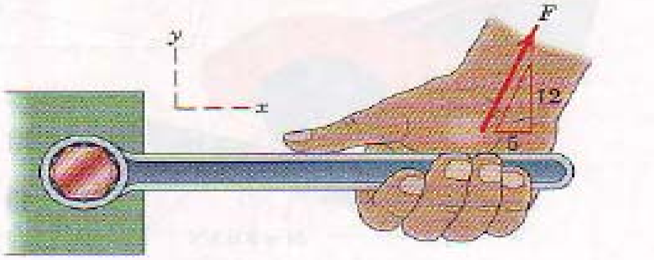
θ1 = 30 deg

θ = 60 deg

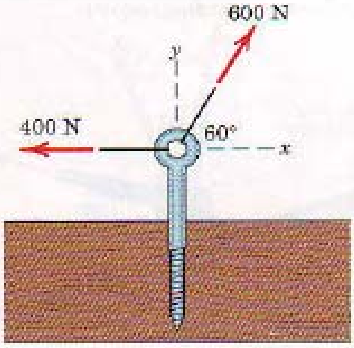


The y-component of the force $\mathbf{F}$ which a person exerts on the handle of the box wrench is known to be 320 N. Determine the x-component and the magnitude of $\mathbf{F}$.

Ans. $F_x = 133.3$ N, $F = 347$ N

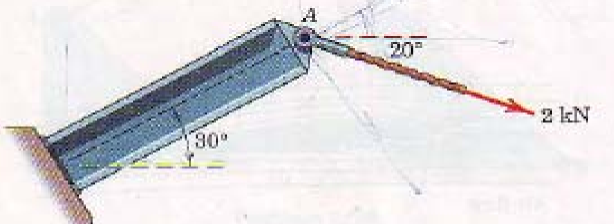


Determine the resultant $\mathbf{R}$ of the two forces shown by (*a*) applying the parallelogram rule for vector addition and (*b*) summing scalar components.

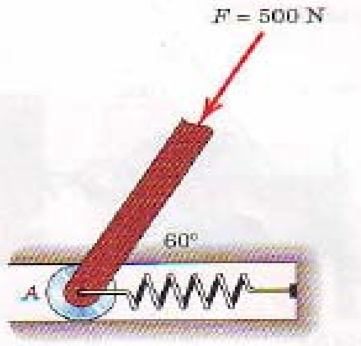


To satisfy design limitations it is necessary to determine the effect of the 2-kN tension in the cable on the shear, tension, and bending of the fixed I-beam. For this purpose replace this force by its equivalent of two forces at A, F_t parallel and F_n perpendicular to the beam. Determine F_t and F_n.

Ans. $F_t = 1.286$ kN, $F_n = 1.532$ kN

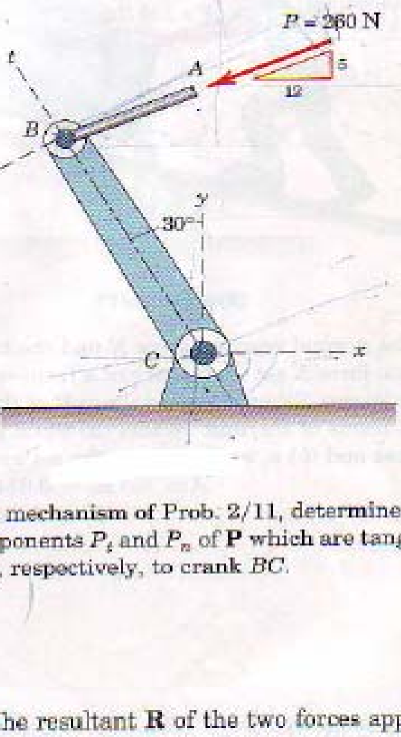


Determine the magnitude F_s of the tensile spring force in order that the resultant of $\mathbf{F}_s$ and $\mathbf{F}$ is a vertical force. Determine the magnitude R of this vertical resultant force.



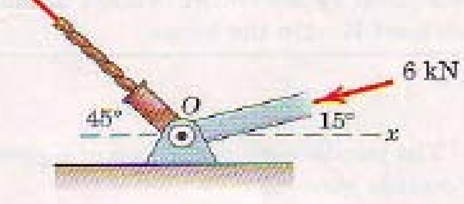
In the design of a control mechanism, it is determined that rod AB transmits a 260-N force $\mathbf{P}$ to the crank BC. Determine the x and y scalar components of $\mathbf{P}$.

Ans. $P_x = -240$ N
$P_y = -100$ N

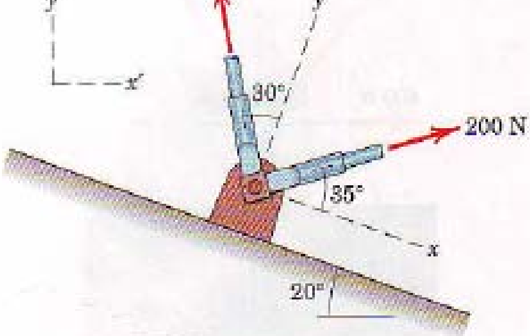


For the mechanism of Prob. 2/11, determine the scalar components P_t and P_n of $\mathbf{P}$ which are tangent and normal, respectively, to crank BC.

The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant $\mathbf{R}$ of the two forces and the angle θ which $\mathbf{R}$ makes with the positive x-axis.

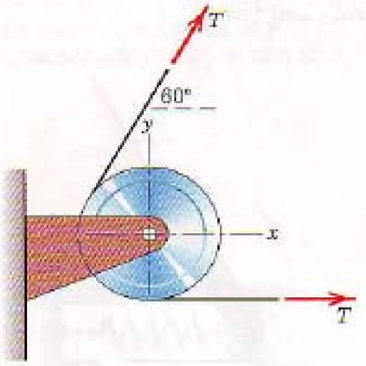


Determine the resultant $\mathbf{R}$ of the two forces applied to the bracket. Write $\mathbf{R}$ in terms of unit vectors along the x- and y-axes shown.



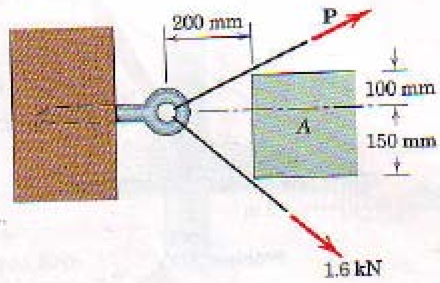
If the equal tensions T in the pulley cable are 400 N, express in vector notation the force **R** exerted on the pulley by the two tensions. Determine the magnitude of **R**.

Ans. $\mathbf{R} = 600\mathbf{i} + 346\mathbf{j}$ N, $R = 693$ N

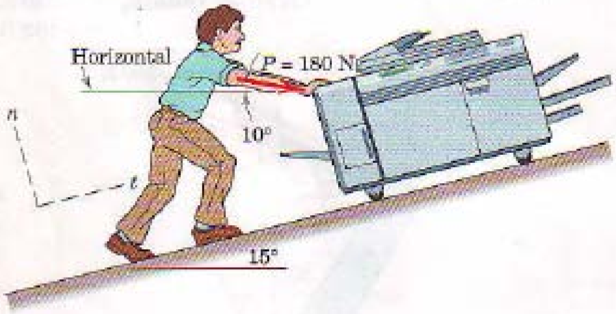


It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction A prevents direct access, so that two forces, one 1.6 kN and the other **P**, are applied by cables as shown. Compute the magnitude of **P** necessary to ensure a resultant **T** directed along the spike. Also find T.

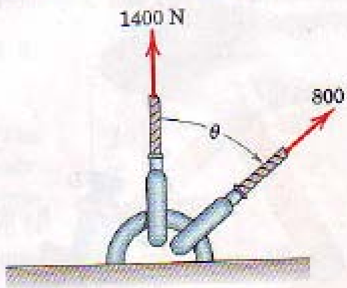
Ans. $P = 2.15$ kN
$T = 3.20$ kN



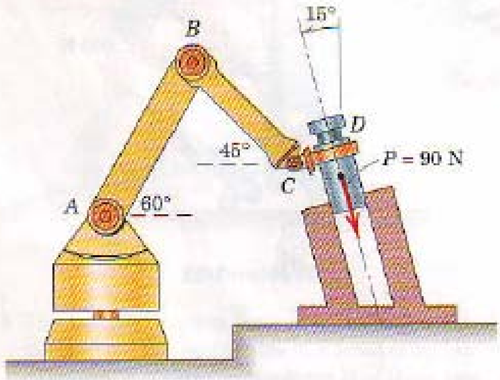
While steadily pushing the machine up an incline, a person exerts a 180-N force **P** as shown. Determine the components of **P** which are parallel and perpendicular to the incline.



At what angle θ must the 800-N force be applied in order that the resultant **R** of the two forces has a magnitude of 2000 N? For this condition, determine the angle β between **R** and the vertical.



In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force P on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts *on* the robot along axes (*a*) parallel and perpendicular to the arm AB, and (*b*) parallel and perpendicular to the arm BC.



Lecture 2

Moment

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the tine of action of the force. This rotational tendency is known as the moment M of the force. Moment is also refereed to as torque.

As a familiar example of the concept of moment, consider. the pipe wench of Fig. a. One effect of the force applied perpendicular to the handle of the wench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

Moment about a Point

Figure b shows a two-dimensional body acted on by a force F in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, which is the perpendicular distance from the axis to the line of l action of the force. Therefore ,the magnitude of the moment is defined as

$$\boxed{M = Fd}$$

The moment is a vector M perpendicular to the plane of the body. The sense of M depends on the direction in which F tends to rotate the body The right-hand rule, Fig.1c, is used to identify this sense. We represent the moment of F about O-O as a vector pointing in the direction of the thumb, with the finger curled in the direction of the relational tendency.

The moment M obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are Newton-meters (N.m), and in the U.S. customary system are pound-feet (ob-ft).

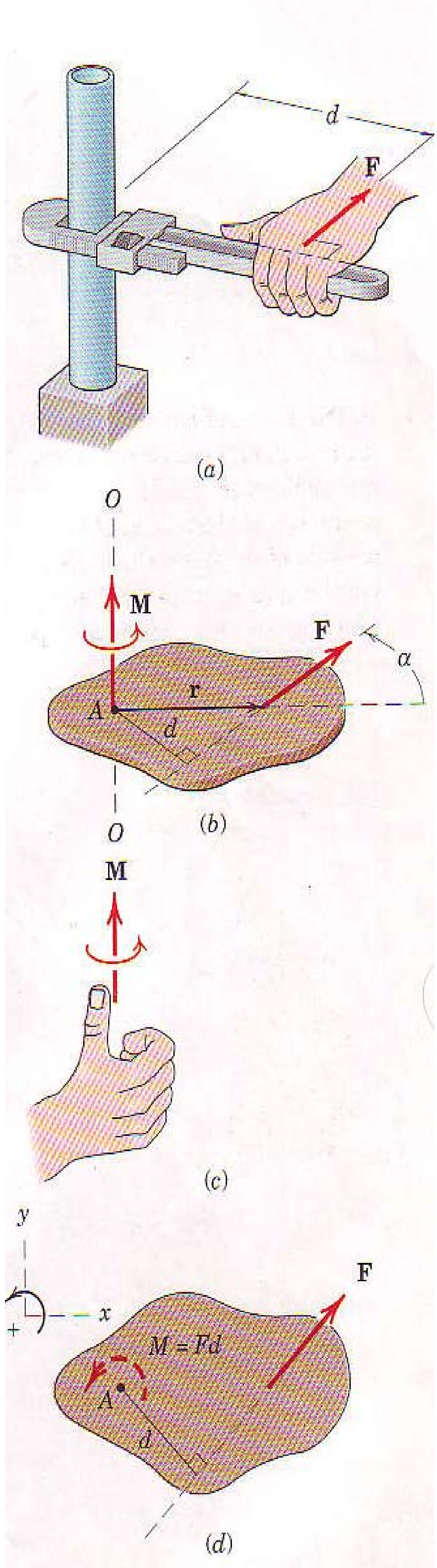


Figure 1

When dealings with forces which all act in a given plane, we customarily speak of the moment about a point. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force F about point A in Fig.d has the magnitude M =Fd and is counterclockwise.

Moment directions may be accounted for by using a stated sign convention. such as a plus sign (+) for counterclockwise moment and a minus sign! (+) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig.d, the moment of F about point A (or about the z-axis passing through point A) is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.

Varignon's theorem

One of the useful principles of mechanics is Varignon's theorem, which states that the moment of a force about any point is equal to the sum of the moment of the components of the force about the same point.

To prove this theorem, consider the force R acting in the plane of the body shown in Fig. 2a. The forces P and Q represent any two nonrectangular components of R. The moment of R about point O is

$$\mathbf{M_o = r \times R}$$

Because R=P + Q, we may write

$$\mathbf{r \times R = r \times (P+Q)}$$

Using the distributive law for cross products, we have

$$\mathbf{M_o = r \times R = r \times P + r \times Q} \qquad \ldots\ldots\ldots\ldots$$

which says that the moment of R about O equals the sum of the moments about O of its components P and Q. This proves the theorem.

Varignon's theorem need not be restricted to the case of two component, but it applies equally well to three or more. Thus we could have used any number of concurrent components of R in the foregoing proof

figure 2b illustrates the usefulness of Varignon's theorem. The moment of R about point O is Rd. However, if d is more difficult to determine than p and q, we can resolve R into the components P and Q, and compute the moment as

$$\mathbf{M_o = Rd = -pP + qQ}$$

where we take the clockwise moment sense to be positive. Sample Problem 1 shows how Varignon's theorem can help us to calculate moments.

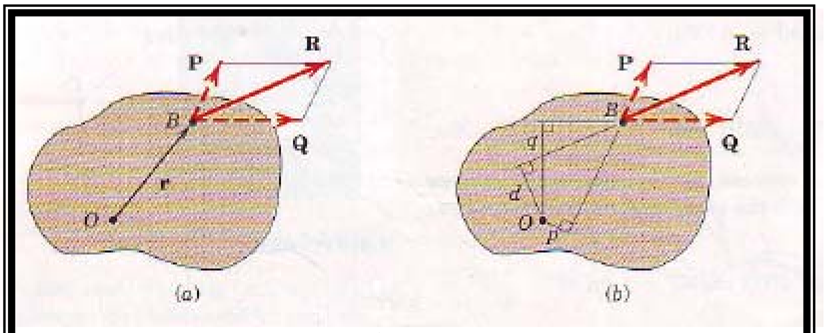
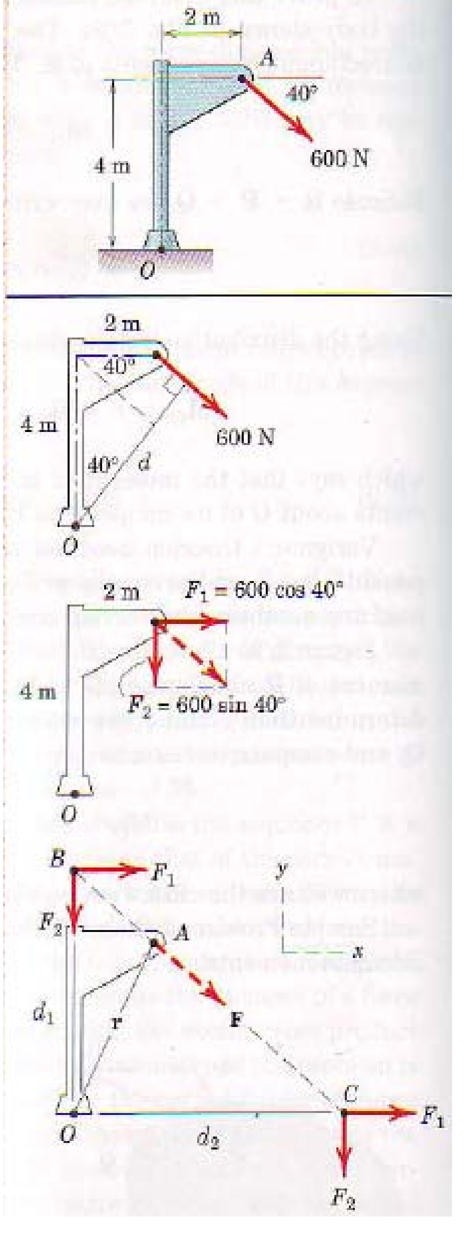


Figure 2

Examples

Example 1

Calculate the magnitude of the moment about the base point O of the 600N force in five different way



Solution

(I) The moment arm to the 600-N force is

$$d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35\ \text{m}$$

(1) By M = rd the moment is clockwise and his the magnitude

$$M_o = 600(4.35) = 2610\ \text{N.m}$$

(II) Replace the force by its rectangular components at A

$$F1 = 600\cos 40^\circ = 460\ \text{N}, \quad F2 = 600\sin 40^\circ = 386\text{N}$$

(2)By Varignon's theorem, the moment becomes

$$M_o = 460(4) + 386(2) = 2610\text{N}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F2. The moment arm of F1 becomes

$$d1 = 4 + 2\tan 40^\circ = 5.68\ \text{m}$$

and the moment is

$$M_o = 460(5.68) = 2610\ \text{N.m}$$

(3) (IV) Moving the force to point C eliminates the moment of the component F1. The moment am of F2 becomes

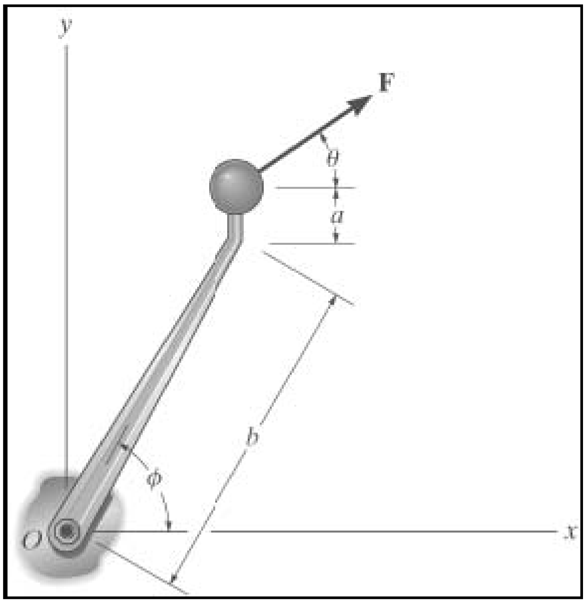
$$d2 = 2 + 4\cos 40^\circ = 6.77\ \text{m}$$

and the moment is

$$M_o = 386(6.77) + 2610\ \text{N.m}$$

Example 2

Determine the angle θ (0 <= θ <= 90 deg) so that the force **F** develops a clockwise moment *M* about point *O*.



Given:

F = 100 N   φ = 60 deg

M 20 = N.m a = 50 mm

θ = 30 deg b = 300 mm

Solution:

Initial Guess θ = 30 deg

Given

$M=F\cos(\theta)(a+b\sin(\varphi))-F\sin(\theta)(b\cos(\varphi))$

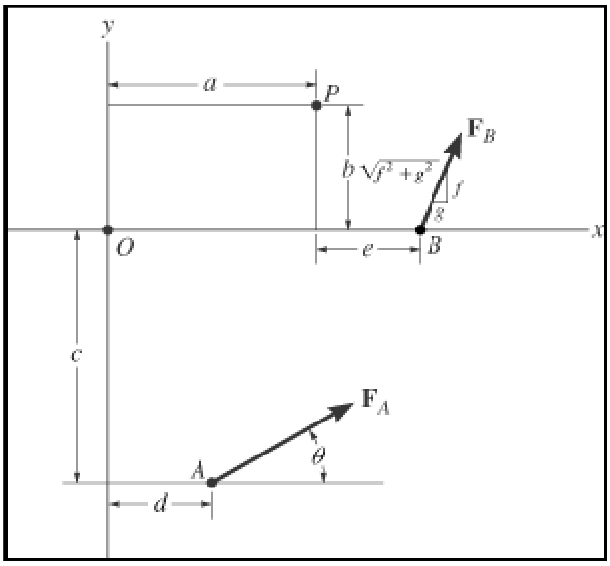
θ = Find(θ) θ = 28.6 deg

Example 3

Determine the magnitude and directional sense of the moment of the forces

(1) about point O.

(2) about point P.



Given:

F_B = 260 N e = 2 m

a = 4 m f = 12

b = 3 m g = 5

c = 5 m   θ = 30 deg

d = 2 m F_A = 400 N

Solution:

(1)

$$M_O = F_A\sin(\theta)d + F_A\cos(\theta)c + F_B\frac{f}{\sqrt{f^2+g^2}}(a+e)$$

M_O = 3.57 kN·m   (positive means counterclockwise)

$$M_P = F_B\frac{g}{\sqrt{f^2+g^2}}b + F_B\frac{f}{\sqrt{f^2+g^2}}e - F_A\sin(\theta)(a-d) + F_A\cos(\theta)(b+c)$$

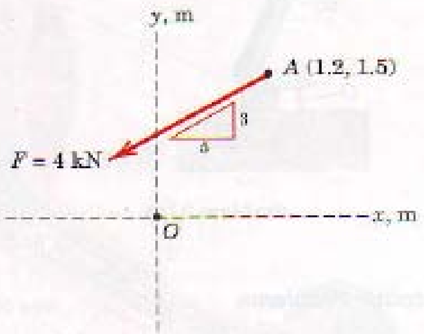
(2)

M_P = 3.15 kN·m   (positive means counterclockwise)

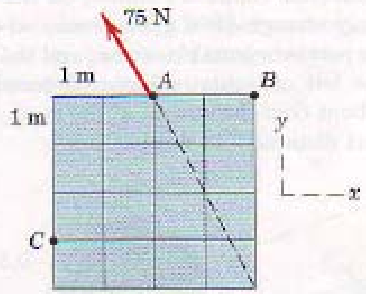
Problems

The 4-kN force **F** is applied at point A. Compute the moment of **F** about point O, expressing it both as a scalar and as a vector quantity. Determine the coordinates of the points on the x- and y-axes about which the moment of **F** is zero.

Ans. $M_O = 2.68$ kN·m CCW, $\mathbf{M}_O = 2.68\mathbf{k}$ kN·m
$(x, y) = (-1.3, 0)$ and $(0, 0.78)$ m

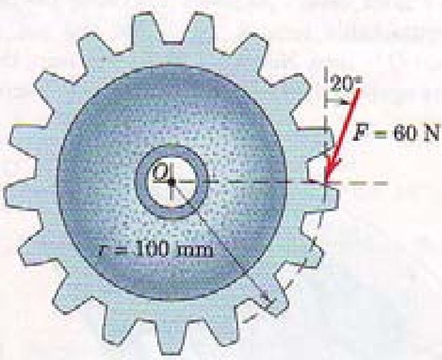


The rectangular plate is made up of 1-m squares as shown. A 75-N force is applied at point A in the direction shown. Determine the moment of this force about point B and about point C.



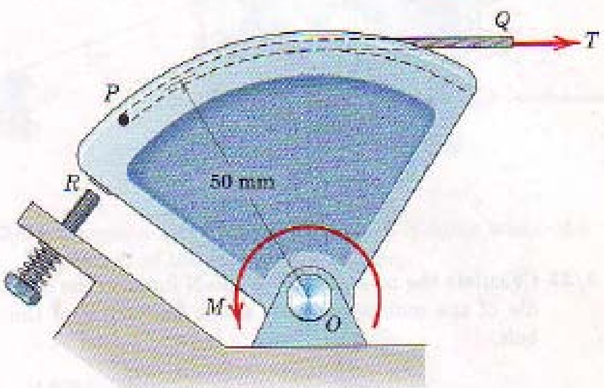
A force **F** of magnitude 60 N is applied to the gear. Determine the moment of **F** about point O.

Ans. $M_O = 5.64$ N·m CW

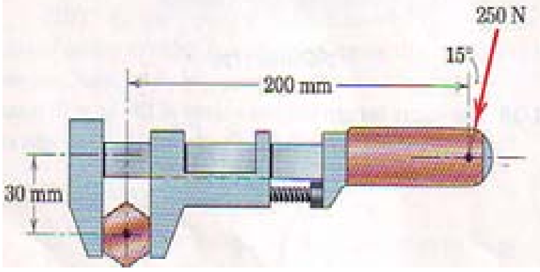


The throttle-control sector pivots freely at O. If an internal torsional spring exerts a return moment $M = 2$ N·m on the sector when in the position shown, for design purposes determine the necessary throttle–cable tension T so that the net moment about O is zero. Note that when T is zero, the sector rests against the idle-control adjustment screw at R.

Ans. $T = 40$ N

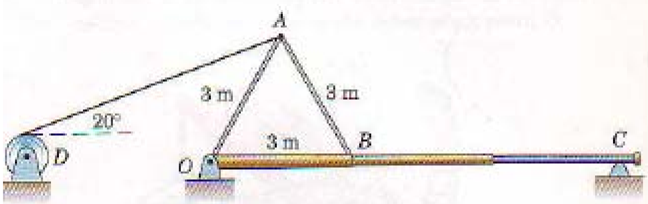


Calculate the moment of the 250-N force on the handle of the monkey wrench about the center of the bolt.



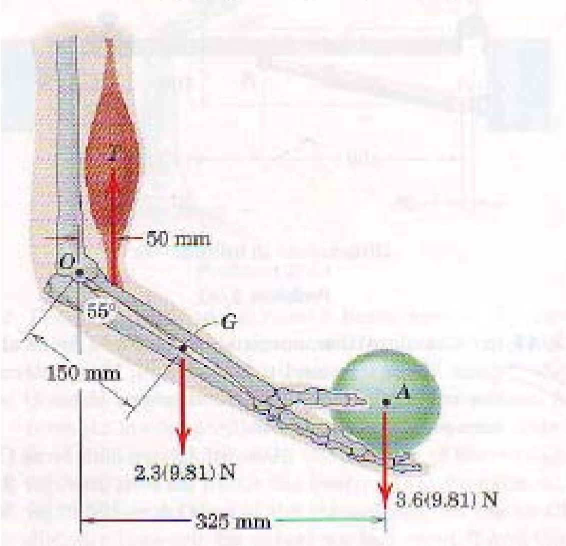
In order to raise the flagpole OC, a light frame OAB is attached to the pole and a tension of 3.2 kN is developed in the hoisting cable by the power winch D. Calculate the moment M_O of this tension about the hinge point O.

Ans. $M_O = 6.17$ kN·m CCW

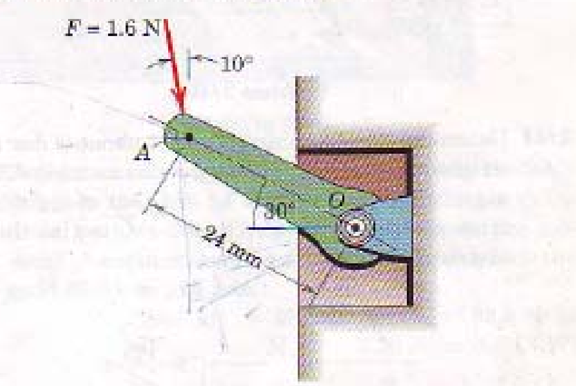


Elements of the lower arm are shown in the figure. The mass of the forearm is 2.3 kg with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the 3.6-kg homogeneous sphere. What must the biceps tension force T be so that the overall moment about O is zero?

Ans. M_O = 14.25 N·m CW, T = 285 N

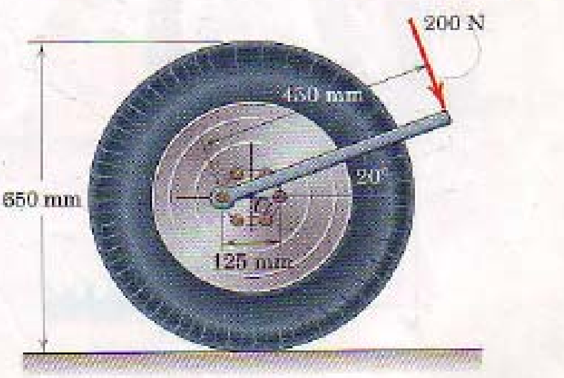


Compute the moment of the 1.6-N force about the pivot O of the wall-switch toggle.

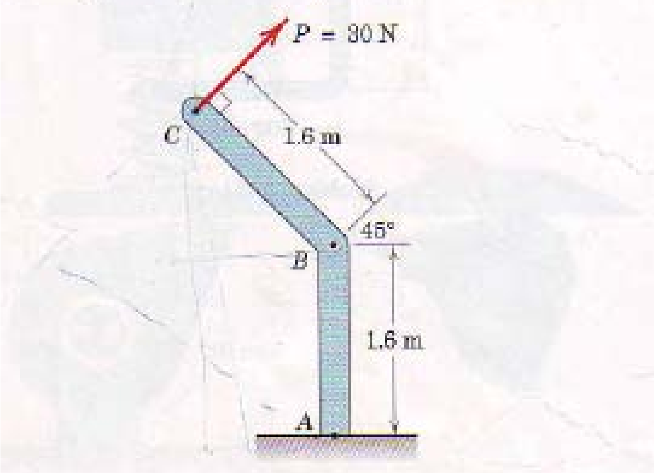


A force of 200 N is applied to the end of the wrench to tighten a flange bolt which holds the wheel to the axle. Determine the moment M produced by this force about the center O of the wheel for the position of the wrench shown.

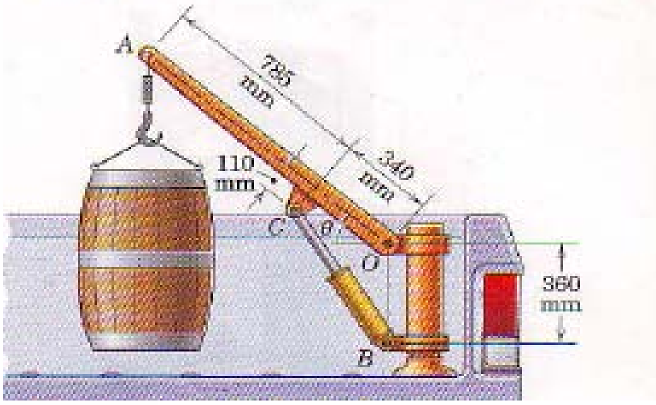
Ans. M = 78.3 N·m CW



The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.

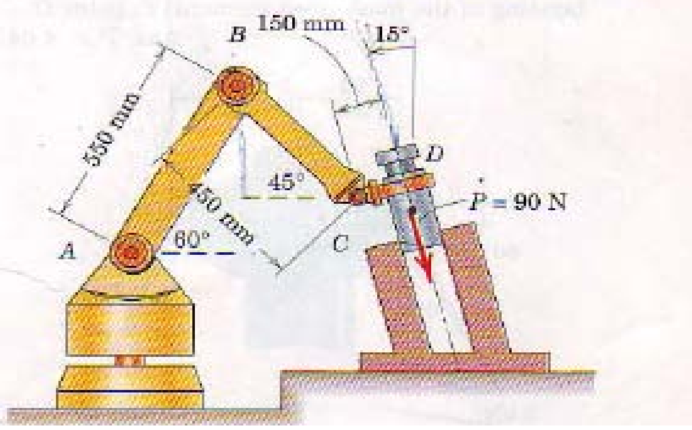


The small crane is mounted along the side of a pickup bed and facilitates the handling of heavy loads. When the boom elevation angle is θ = 40°, the force in the hydraulic cylinder BC is 4.5 kN, and this force applied at point C is in the direction from B to C (the cylinder is in compression). Determine the moment of this 4.5-kN force about the boom pivot point O.



Design criteria require that the robot exert the 90-N force on the part as shown while inserting a cylindrical part into the circular hole. Determine the moment about points A, B, and C of the force which the part exerts on the robot.

Ans. M_A = 68.8 N·m, M_B = 33.8 N·m
M_C = 13.50 N·m (all CCW)



Lecture 3

Couples

The moment produced by two equal, opposite, and noncollinear forces is called a couple. couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces F and -F a distance d apart, as shown in lfig.1a . These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple M. This couple has a magnitude

$$\mathbf{M= F(a+b)-Fa}$$

Or $$\mathbf{M= Fd}$$

Its direction is counterclockwise when viewed from above for the case Illustrate. Note especially that the magnitude of the couple is dependent of the distance a which locates the forces with respect to the moment center 0. lt follows that the moment of a couple has the same value for all moment centers.

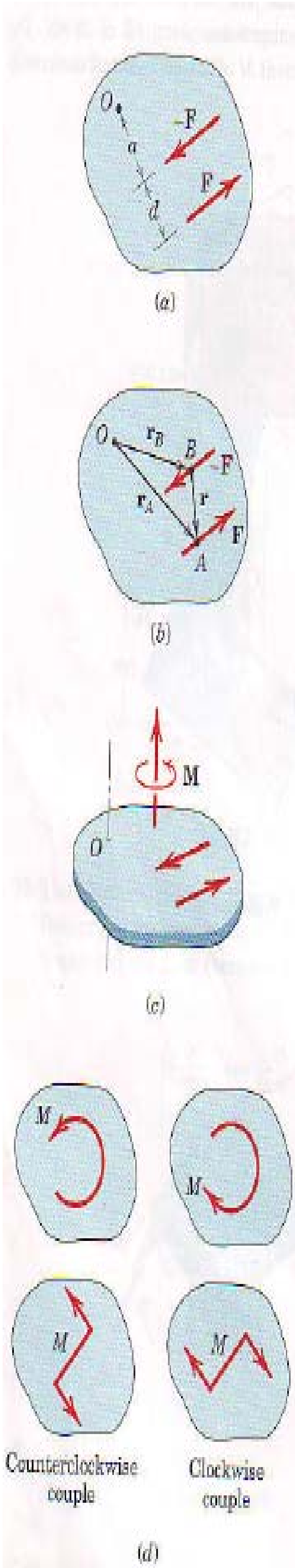


Figure 1

Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross product Eq. the combined moment about point 0 of the couple of Fig. 1b is

$$\mathbf{M = r_A \times F + r_B \times (-F) = (r_A - r_B) \times F}$$

where r_A and r_B are position vector which run from point O to arbitrary points A and B on the tines of action of F and -F, respectively. Because $r_A - r_B = r$, we can express M as

$$\mathbf{M = r \times F}$$

Here again, the moment expression contains no reference to the moment center 0 and, therefore, is the same for all moment centers. Thus, we may represent M by a free vector, as shown in Fig. 1c, where the direction of M is normal to the plane of the couple and sense of M is established by the right-hand rule.

Because the couple vector M is always perpendicular to the plane of the forces which constitute the couple, in two dimensional analysis we can represent the sense of couple vector as clockwise or counterclockwise by one of the convention shown in fig.1d. later, when we deal with couple vectors in three-dimensional problems, we will make full use of vector notation to represent them, and the mathematics will automatically account for their sense.

Equivalent Couples

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2 shows four different configurations of the same couple M. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

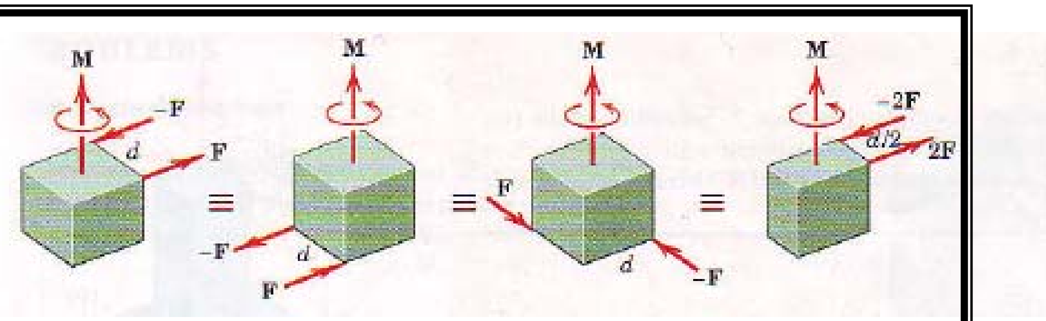


Figure 2

Force-Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 3, where the given force F acting at point A is replaced by an equal force F at some point B and the counterclockwise couple M = Fd. The transfer is seen in the middle figure, where the equal and opposite forces F and -F are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple M = Fd, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig.3 is referred to as a force-couple system.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force-couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

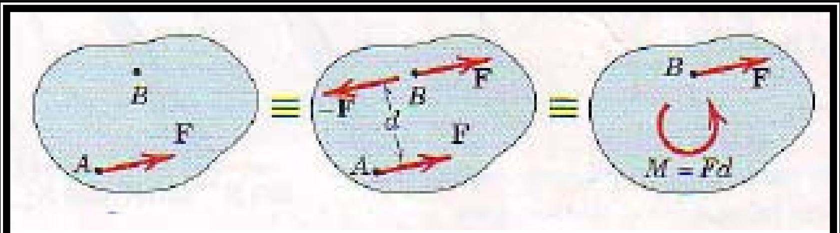
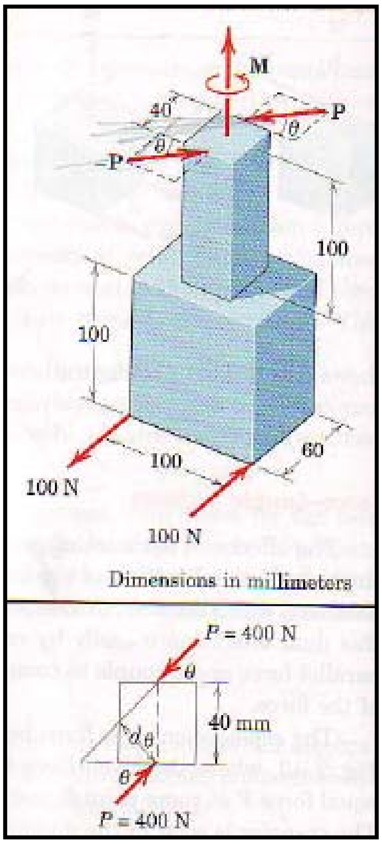


Figure 3

Examples

Example 1

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces P and –P, each of which has a magnitude of 400 N. Determine the proper angle θ.



Solution

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$$\mathbf{[M = F\,d]}$$

$$\mathbf{M = 100(0.1) = 10\ N.m}$$

The forces P and –P produce a counterclockwise couple

$$\mathbf{M = 400(0.040)\cos\theta}$$

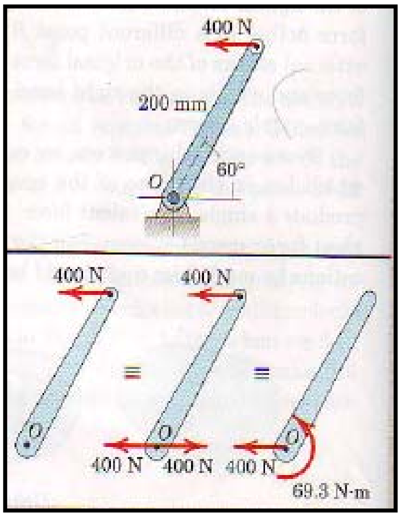
Equating the two expression gives

$$\mathbf{10 = 400(0.040)\cos\theta}$$

$$\mathbf{\theta = \cos(10/16) = 51.3^{\circ}}$$

Example 2

Replace the horizontal 400-n force acting on the lever by an equivalent system consisting of a force at O and a couple.



Solution

We apply two equal and opposite 400-N forces at o and identify counterclockwise couple

$$\mathbf{[M = F\,d]}$$

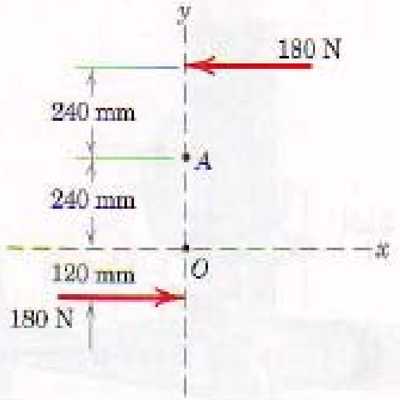
$$\mathbf{M = 400(0.200\sin 60^{\circ}) = 69.3\ N.m}$$

Thus, the original force is equivalent to the 400-n forces at 0 and the 69.3 N.m couple as shown in third of the three equivalent figures

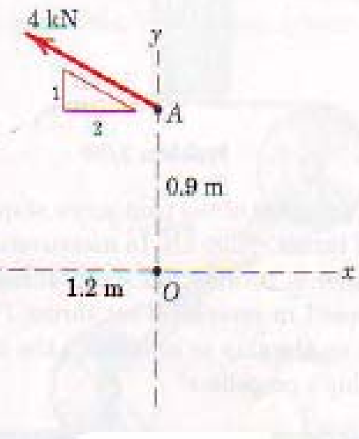
PROBLEMS

Compute the combined moment of the two 180-N forces about (*a*) point *O* and (*b*) point *A*.

Ans. (*a*) M_O = 108 N·m CCW
(*b*) M_A = 108 N·m CCW

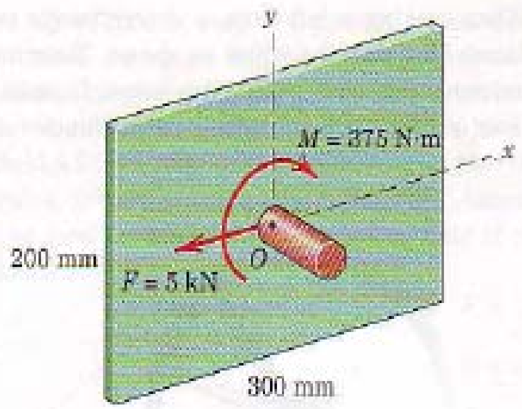


Replace the 4-kN force acting at point *A* by a force-couple system at (*a*) point *O* and (*b*) point *B*.

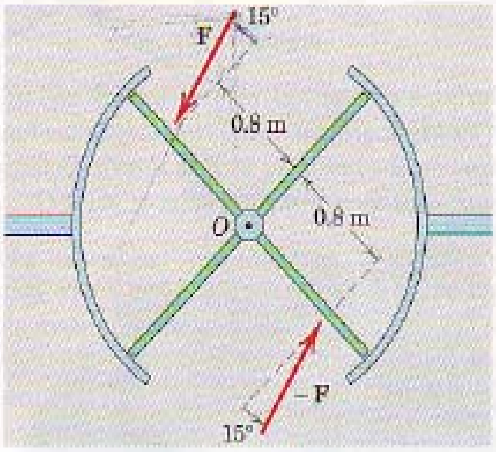


The indicated force-couple system is applied to a small shaft at the center of the rectangular plate. Replace this system by a single force and specify the coordinate of the point on the *y*-axis through which the line of action of this resultant force passes.

Ans. y = −75 mm

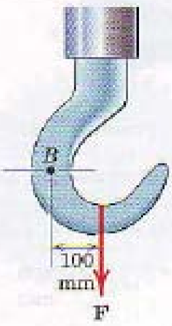


The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert forces of equal magnitudes as shown. If the resulting moment about the door pivot axis at *O* is 25 N·m, determine the force magnitude *F*.

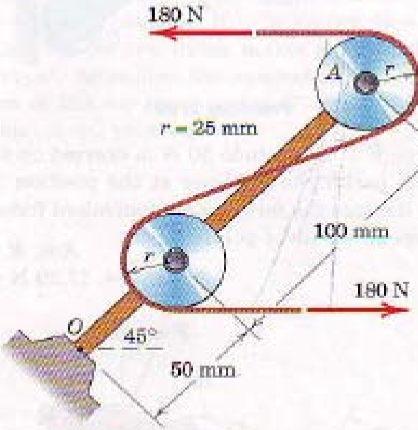


In the design of the lifting hook the action of the applied force **F** at the critical section of the hook is a direct pull at B and a couple. If the magnitude of the couple is 4000 N·m, determine the magnitude of **F**.

Ans. $F = 40$ kN

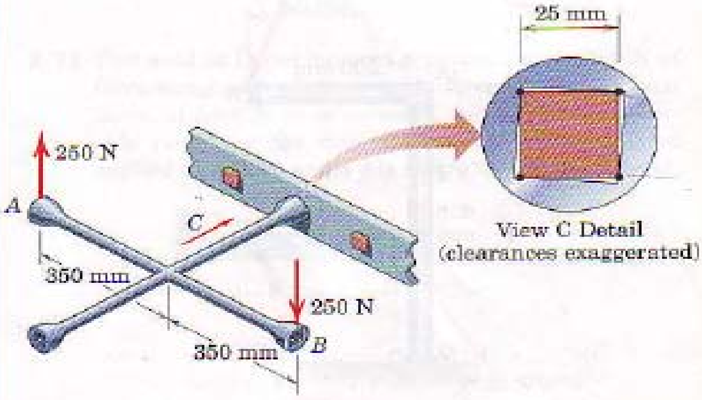


The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.

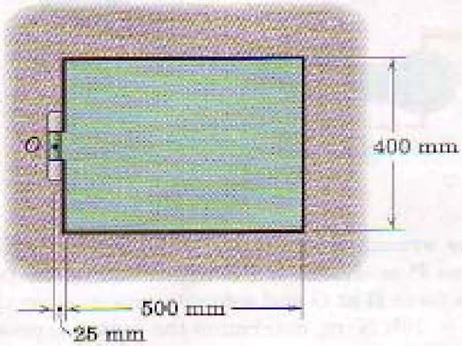


A lug wrench is used to tighten a square-head bolt. If 250-N forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 25-mm bolt head so that their external effect on the bolt is equivalent to that of the two 250-N forces. Assume that the forces are perpendicular to the flats of the bolt head.

Ans. $F = 3500$ N

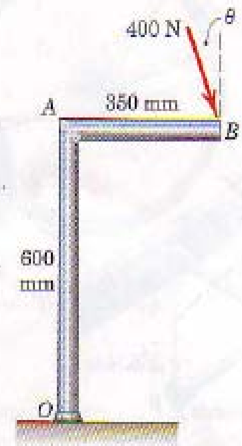


The inspection door shown is constructed of sheet steel which is 3 mm thick. Determine the force–couple system located at the hinge center O which is equivalent to the weight of the door. State any assumptions.

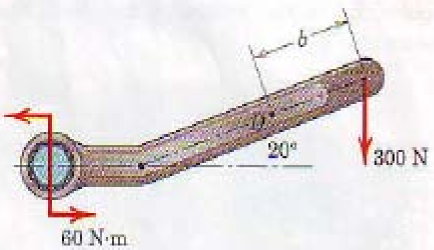


A 400-N force is applied to the welded slender bar at an angle $\theta = 20°$. Determine the equivalent force–couple system acting on the weld at (*a*) point *A* and (*b*) point *O*. For what value of θ would the results of parts (*a*) and (*b*) be identical?

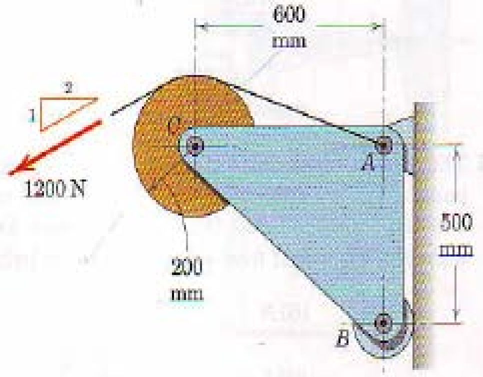
Ans. (*a*) $F = 400$ N, $M_A = 131.6$ N·m CW
(*b*) $F = 400$ N, $M_O = 214$ N·m CW
$\theta = 0$ or $180°$



Replace the couple and force shown by a single force **F** applied at a point *D*. Locate *D* by determining the distance *b*.

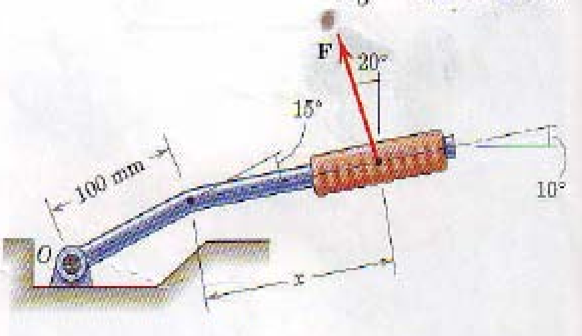


Calculate the moment of the 1200-N force about pin *A* of the bracket. Begin by replacing the 1200-N force by a force–couple system at point *C*.



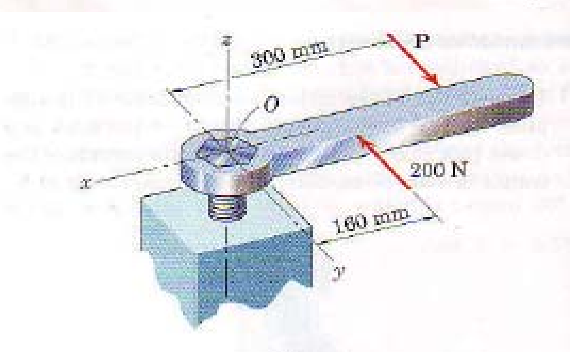
A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point *O*.

Ans. $R = 50$ N
$M_O = 17.29$ N·m CCW

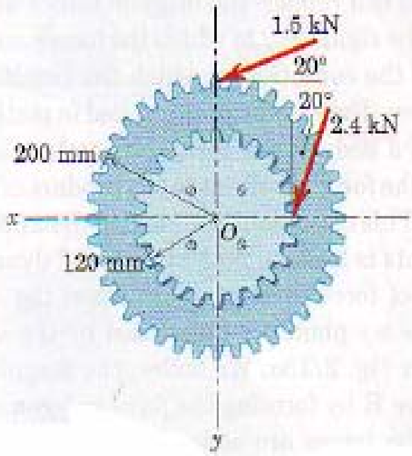


The wrench is subjected to the 200-N force and the force **P** as shown. If the equivalent of the two forces is a force **R** at *O* and a couple expressed as the vector $\mathbf{M} = 20\mathbf{k}$ N·m, determine the vector expressions for **P** and **R**.

Ans. $\mathbf{P} = 40\mathbf{j}$ N
$\mathbf{R} = -160\mathbf{j}$ N

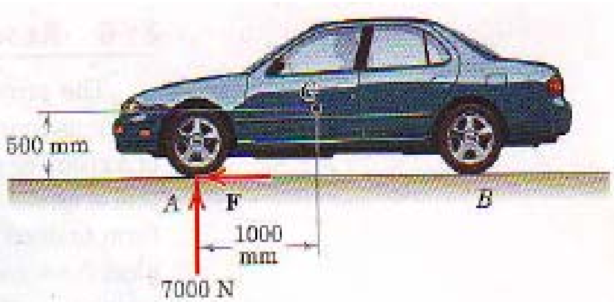


The figure represents two integral gears subjected to the tooth-contact forces shown. Replace the two forces by an equivalent single force **R** at the rotation axis O and a corresponding couple **M**. Specify the magnitudes of **R** and **M**. If the gears were to start from rest under the action of the tooth loads shown, in what direction would rotation take place?



The combined drive wheels of a front-wheel-drive automobile are acted on by a 7000-N normal reaction force and a friction force **F**, both of which are exerted by the road surface. If it is known that the resultant of these two forces makes a 15° angle with the vertical, determine the equivalent force–couple system at the car mass center G. Treat this as a two-dimensional problem.

Ans. $R = 7250$ N
$M_G = 7940$ N·m CW



The weld at O can support a maximum of 2500 N of force along each of the n- and t-directions and a maximum of 1400 N·m of moment. Determine the allowable range for the direction θ of the 2700-N force applied at A. The angle θ is restricted to $0 \le \theta \le 90°$.