University of Technology
Department of Laser & Opto-electronic Engineering
Final Examination 2011-2012

Subject: Physical Optics
Division: Optoelectronics
Examiner: Ali Hadi

Class: year
Time: 3 hours
Date: 25/5 /2012

Answer five questions only

Q1/ a- A beam of light is right circularly polarized and incident at an angle 45 degree on glass surface (n=1.5). Use the reflection matrix to find the polarization of the reflected light for internal reflection? (5 degree)

b- Find the axial intensity distribution in the image plane of optical system with circular aperture suffer from focus error (W_{20}= 0.0, 0.2 λ, 0.5 λ and 0.7 λ)? (5 degree)

Q2-a

8.1 Describe completely the state of polarization of each of the following waves:

(a) $\vec{\mathbf{E}} = \hat{\mathbf{i}}E_0 \cos(kz - \omega t) - \hat{\mathbf{j}}E_0 \cos(kz - \omega t)$

(b) $\vec{\mathbf{E}} = \hat{\mathbf{i}}E_0 \sin 2\pi(z/\lambda - \nu t) - \hat{\mathbf{j}}E_0 \sin 2\pi(z/\lambda - \nu t)$

(c) $\vec{\mathbf{E}} = \hat{\mathbf{i}}E_0 \sin(\omega t - kz) + \hat{\mathbf{j}}E_0 \sin(\omega t - kz - \pi/4)$

(d) $\vec{\mathbf{E}} = \hat{\mathbf{i}}E_0 \cos(\omega t - kz) + \hat{\mathbf{j}}E_0 \cos(\omega t - kz + \pi/2)$.

(5 degree)

Q2b /- Find the reflectance for both TE and TM polarizations at an angle of incidence of 45 degrees for water? (5 degree)

Q3/- A beam of light is totally reflected in a 45-90-45-degree glass prism (n = 1.5) as shown in Figure 1. The wavelength of the light is 500 nm. At what distance from the surface is the amplitude of the evanescent wave lie of its value at the surface? By what factor is the intensity of the evanescent wave reduced at a distance of 1 mm from the surface? (10 degree)

Q4/ Find the optimum balance for spherical aberration $W_{40}r^4$ in optical system with unit circular aperture? (5 degree)

b- Derive in step leading to the phase difference in total reflection formula? (5 - degree)

Q5/a- Show by direct calculation. Using Mueller matrices, that a beam of horizontal plane-state light passing through a quarter wave plate with its fast axis horizontal emerges unchanged. (5 degree)

b- Compute the Fourier series that corresponds to the following wave.

$$E(x) = \begin{cases} E_0 \cos k_p x & \text{when } -L \leq x \leq L \\ 0 & \text{when } |x| > L \end{cases}$$

(5-degree)

Q6/ a- write the equations which represent the fraunhofer diffraction pattern of systems with ,1- slit aperture, 2- rectangular aperture, 3- circular aperture? Compare between resolution of each aperture?

(5 degree)

b- There are different parameters could be used to evaluate and analysis the image of an optical system, explain the application for only five parameters?

(5 degree)

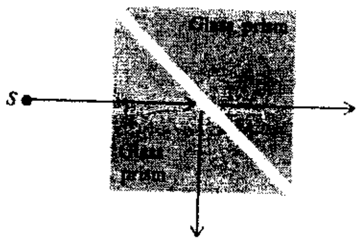

Figure 1

الفصل الأول
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٢ محاضرات [illegible]
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Q_1 b)

$$I = \left[\operatorname{sinc}\left(\pi \frac{W_{20}}{\lambda}\right)\right]^2$$

if $W_{20} = 0 \Rightarrow I = 1$

if $W_{20} = 0.125\lambda \Rightarrow I = 0.8$

$W_{20} = 0.5\lambda \Rightarrow I = 0.405$

$W_{20} = 0.7\lambda \Rightarrow I = 0\cdots$

(a) Right circular polarized light $= \begin{bmatrix} 1 \\ -i \end{bmatrix}$ — (1)

The reflection matrix is $\equiv \begin{bmatrix} -r_p & 0 \\ 0 & r_s \end{bmatrix}$ — (2)

$\therefore$ Jones vector is $\begin{bmatrix} -r_p & 0 \\ 0 & r_s \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A' \\ B' \end{bmatrix}$ — (3)

where

(4) $$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-(\frac{1}{1.5})^2\cos 45 + \sqrt{(\frac{1}{1.5})^2 - \sin^2 45}}{(1.5)^2\cos 45 + \sqrt{(1.5)^2 - \sin^2 45}}$$

(5) $$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\cos 45 - \sqrt{(\frac{1}{1.5})^2 - \sin^2 45}}{\cos 45 + \sqrt{(\frac{1}{1.5})^2 - \sin^2 45}}$$

substitute (4, 5) in (3) we find the reflectance.

$$\begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$\frac{n_2}{n_1} > 1$ external

$\frac{n_2}{n_1} < 1$ internal

$\boxed{\frac{1}{1.5} = n}$

Q2) a- $E_0 = $ is constant

$$E = E_0 \cos(kz - \omega t)(i - j) \equiv \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

linearly polarized (−45)

b) linear = (−45)

c) $E = i E_0 \sin(\omega t - kz)\{ i + j \, e^{i - \frac{\pi}{4}})$

$= (\quad) (i + j(\cos(+45) - i \sin 45)$

$e^{i\frac{\pi}{2}} = i$, $e^{i\frac{\pi}{4}} = \cos 45 + i \sin 45 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ $(i + j\, 0.707 - 0.707 i)$ ellips

d) Circular polarized

(−) right circular pol

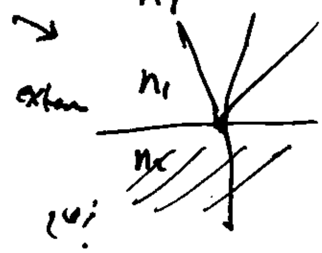
(+) left circular pola

Q2b/ $R_s = (r_s)^2 = \dfrac{\cos 45 - \sqrt{(1.33)^2 - \sin^2 45}}{\cos 45 + \sqrt{(1.33)^2 - \sin^2 45}}$ for TE

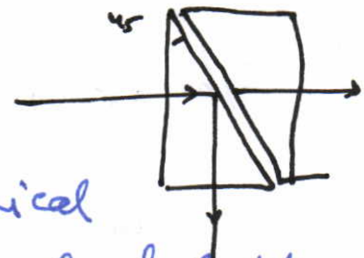
$R_p = (r_p)^2 = \dfrac{-(1.33)^2 \cos 45 + \sqrt{(1.33)^2 - \sin^2 45}}{n^2 \cos 45 + \sqrt{(1.33)^2 - \sin^2 45}}$ for TM

internal

$n > 1$ $\quad \frac{n_2}{n_1} =$



Q3/ $\lambda = 500$ mm



When the incident angle exceeds the critical angle, totally reflection is occur for the incident light, but their is still an electromagnetic wave field in the region ~~behind~~ beyond the boundary. This field is known as evanescent wave. This can be understood by considering the wavefunction electric field vector of the transmitted wave $E_{trans} = E'' e^{i(k''\cdot r - wt)}$ —①

but $k''\cdot r = k'' x \sin\phi - k'' y \cos\phi$

$= k'' x \sin\phi - i k'' y \sqrt{\frac{\sin^2\theta}{n^2} - 1}$ —②

from Snell law $\cos\phi = \sqrt{1 - \frac{\sin^2\phi}{n^2}} = i\sqrt{\frac{\sin^2\theta}{n^2} - 1}$ —③

this equation mean that $\cos\phi$ is imaginary in case of total internal reflection.

Therfore $E_{tran} = E' e^{-\alpha|y|} e^{i(k_1 x - wt)}$ —④

where $\alpha = k'\sqrt{\frac{\sin^2\theta}{n^2} - 1}$; $k_1 = \frac{k''\sin\theta}{n}$ —⑤

The factor $e^{-\alpha|y|}$ shows that the evanescent wave amplitude drops off very rapidly as we proceed away from the boundary into the rarer medium

from equ. ④ $\frac{E_{tran}}{E'} = e^{-\alpha|y|} e^{i(k_1 x - wt)}$

$= e^{-k''\sqrt{\frac{\sin\theta}{n} - 1}\,|y|} e^{i\left(\frac{k''\sin\theta}{n} x - wt\right)}$

$1 = e$

$|y| = ?$

Q4) The optimum balance for $W_{40} r^4$

$$V = \frac{1}{\pi}\underbrace{\int_0^{2\pi}\int_0^1 (W_{40} r^4)^2 r\,dr\,d\theta}_{A} - \underbrace{\left[\frac{1}{\pi}\int_0^1\int_0^{2\pi} W_{40} r^4 \,r\,dr\,d\theta\right]^2}_{B}$$

$V = A - B$

$$A = \frac{2\pi}{\pi} \int_0^1 W_{40}^2 \cdot \frac{r^{10}}{10}\Big|_0^1 = \frac{2 W_{40}^2}{10} = \frac{W_{40}^2}{5}$$

$$B = \left[\frac{2\pi}{\pi} W_{40} \frac{r^6}{6}\Big|_0^1\right]^2 = \frac{W_{40}^2}{(3)^2} = \frac{W_{40}^2}{9}$$

$$\therefore V = W_{40}^2\left[\frac{1}{5} - \frac{1}{9}\right] = \frac{4}{45} W_{40}^2$$

but $V = \frac{\lambda^2}{180}$

$$\therefore \frac{4}{45} W_{40}^2 = \frac{\lambda^2}{180} \Rightarrow$$

$$W_{40}^2 = \frac{45\,\lambda^2}{4 \times 180}$$

$$\boxed{W_{40} = \pm 0.25\,\lambda}$$

Q4b/ In the case of total reflection, the complex values for the coeff. of reflection are

$$r_s = \frac{\cos\theta - i\sqrt{\sin^2\theta - n^2}}{\cos\theta + i\sqrt{\sin^2\theta - n^2}} \quad \text{— (1)}$$

$$r_p = \frac{-n^2\cos\theta + i\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta + i\sqrt{\sin^2\theta - n^2}} \quad \text{— (2)}$$

let $a e^{i\alpha} = \cos\theta + i\sqrt{\sin^2\theta - n^2}$

$b e^{i\beta} = n^2\cos\theta + i\sqrt{\sin^2\theta - n^2}$

suppose r_s and r_p are both unity, we can write

$$r_s = e^{-i\delta_s} = \frac{a e^{-i\alpha}}{a e^{+i\alpha}}$$

$$r_p = -e^{-i\delta_p} = -\frac{b e^{-i\beta}}{b e^{+i\beta}}$$

where δ_s, δ_p are phase shift for the TE & TM

we have $\delta_s = 2\alpha$

as $\tan\alpha = \tan\frac{\delta_s}{2}$ $\Rightarrow$

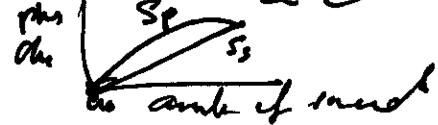
$\tan\beta = \tan\frac{\delta_p}{2}$

$$\tan\frac{\delta_s}{2} = \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}$$

$$\tan\frac{\delta_p}{2} = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}$$

$\Delta = \delta_p - \delta_s$

$$\tan\frac{\Delta}{2} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$



Q5/ a) $\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

↓ unchanged

b) phase 80 ~~Hath~~

Q6) $I = \left(\frac{2J_1(z)}{z}\right)^2$ circ

$I = (\text{sinc}\,\alpha)^2$ slit

$= (\text{sinc}\,\alpha)^2(\text{sinc}\,\beta)^2$ rect

b) ① PSF
② MTF
③ H.W
④ E.C
⑤ XE
6) ZE
7) spiral [illegible]
8) [illegible]
9) [illegible] ratio.

[illegible] 14 70% 80% 89

[illegible] 6

+1 +2

+3 → 99%

95% 100%