Ministry of Higher Education & Scientific Research
University of Technology
Laser & Optoelectronic Eng. Department
Branch:
Class: 3rd
Subject: Engineering analysis

Final Exam
Time: 3 Hrs
Date: 5/6/2016
Examiner: Asst. Prof. Dr. Razi Al-Azawi
Code: LOPE 3201
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Attempt only (5) Questions

Q1\ Find the Laplace Transform for the following functions : (12 Marks)

a) $L\left\{\cos^2(3t)-\sin^2(3t)\right\}$, b) $L\{\cos(3t)\sinh(2t)\}$

Q2\ Solve the following : (12 Marks)

a) $L^{-1}\left\{\ln\left(1+\frac{4}{s^2}\right)\right\}$

b) $ty''-ty'+y=0 \quad with \quad y(0)=0, \; and \quad y'(0)=1$

Q3\ Check the following function ,if fulfill the Cauchy Riemann equations in the domain

$f(z)=\frac{1}{2}\ln(x^2+y^2)+i\cot^{-1}\left(\frac{x}{y}\right)$ (12 Marks)

Q4\ Find the Fourier Series for the following periodic functions (12 Marks)

$$f(x)=\begin{cases}1 & -\pi/2<x<\pi/2 \\ 0 & \pi/2<x<3\pi/2\end{cases}$$

Q5\ Find the Eigen value and Eigen vector when : (12 Marks)

$$A=\begin{bmatrix}1 & 4 & 5\\0 & 2 & 6\\0 & 0 & 3\end{bmatrix}$$

Q6\ Solve the following by use Gauss elimination method? (12 Marks)

$$-x_1+x_2+2x_3=3$$
$$3x_1-x_2+x_3=8$$
$$-x_1+3x_2+4x_3=5$$

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Q1\
a) $L\ \left\{\cos^2(3t)-\sin^2(3t)\right\}$

$\sin ce \quad \cos 2t = 1-2\sin^2 \quad then$

$\cos^2 t = \frac{1}{2}(1+\cos 2t), \sin^2 t = \frac{1}{2}(1-\cos 2t) \quad Hence$

$$L\left\{\cos^2 3t-\sin^2 3t\right\} = L\left\{\frac{1}{2}(1+\cos 6t)-\frac{1}{2}(1-\cos 6t)\right\}$$

$$=\frac{1}{2}L\left\{1\right\}+\frac{1}{2}L\left\{\cos 6t\right\}-(\frac{1}{2}L\left\{1\right\}-\frac{1}{2}L\left\{\cos 6t\right\})$$

$$=\frac{(s^2+36)+s^2}{2s(s^2+36)}-\frac{(s^2+36)-s^2}{2s(s^2+36)}$$

$$=\frac{s^2+18}{s(s^2+36)}-\frac{18}{s(s^2+36)}=\frac{s}{s^2+36}$$

b) $L\left\{\cos(3t)\sinh(2t)\right\}$

$$L(\frac{1}{2}(e^{2t}-e^{-2t})\cos(3t))$$

$$=\frac{1}{2}L(e^{2t}\cos(3t)-e^{-2t}\cos(3t))$$

$$=\frac{1}{2}\left(\quad\frac{s-2}{(s-2)^2+9}-\frac{s+2}{(s+2)^2+9}\quad\right)$$

Q2\
a)

$$L^{-1}\left\{\ln\left(1+\frac{4}{s^2}\right)\right\}$$

$$f(s)=\ln\left(1+\frac{4}{s^2}\right)\Rightarrow L\left\{F(t)\right\}$$

$$then \quad f'(s)=\frac{-8}{s(s^2+4)}=-8\left\{\frac{1}{s}-\frac{s}{s^2+1}\right\}$$

$$thus \quad \sin ce \qquad L-1\left\{f'(s)\right\}=-8(1-\cos t)=-tF(t)$$

$$\Rightarrow F(t)=\frac{8(1-\cos t)}{t}$$

b) $ty''-ty'+y=0 \quad with \quad y\left(0\right)=0, \ and \quad y'\left(0\right)=1$

Taking the Laplace transform of the two sides, we get

$$\frac{-d}{ds}\left(s^2Y(s)-sy(0)-y'(0)\right)-\frac{-d}{ds}\left(sY(s)-y(0)\right)+Y(s)=0$$

$$\frac{-d}{ds}\left(s^2Y(s)-0-1\right)+\frac{d}{ds}\left(sY(s)-0\right)+Y(s)=0$$

$$\frac{-d}{ds}\left(s^2Y(s)-1\right)+\frac{d}{ds}\left(sY(s)\right)+Y(s)=0$$

$$-\left(s^2Y'(s)+Y(s)\times(2s)\right)+\left(sY'(s)+Y(s)\right)+Y(s)=0$$

$$\left(-s^2+s\right)Y'(s)+(-2s+2)Y(s)=0 \quad\Rightarrow\quad \left(-s^2+s\right)Y'(s)=(2s-2)Y(s)$$

$$Y'(s)=\frac{dY(s)}{ds}=\frac{2s-2}{-s^2+s}Y(s) \quad\Rightarrow\quad \frac{dY(s)}{Y(s)}=\frac{2(s-1)}{-s(s-1)}ds$$

$$\int\frac{dY(s)}{Y(s)}=\int\frac{2}{-s}ds \quad\Rightarrow\quad \ln(Y(s))=-2\ln(s) \quad\Rightarrow\quad \ln(Y(s))=\ln(s^{-2})$$

$$\ln(Y(s))=\ln\left(\frac{1}{s^2}\right) \quad\Rightarrow\quad Y(s)=\frac{1}{s^2} \quad\Rightarrow\quad y(t)=t$$

Q3\

$$f(z)=\frac{1}{2}\ln(x^2+y^2)+i\cot^{-1}\left(\frac{x}{y}\right)$$

Here,

$$u(x,y)=\frac{1}{2}\ln(x^2+y^2) \qquad and \qquad v(x,y)=\cot^{-1}\frac{x}{y}$$

are both defined and C^∞, when $y\neq 0$. Assuming this we get

$$\frac{\partial u}{\partial x}=\frac{x}{x^2+y^2}, \qquad \frac{\partial v}{\partial y}=-\frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\left(-\frac{x}{y^2}\right)=\frac{x}{x^2+y^2}=\frac{\partial u}{\partial x},$$

$$\frac{\partial u}{\partial y}=\frac{y}{x^2+y^2}, \qquad \frac{\partial v}{\partial x}=-\frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{1}{y}=-\frac{y}{x^2+y^2}=-\frac{\partial u}{\partial y},$$

proving that Cauchy-Riemann's equations are fulfilled for $y\neq 0$.

Q4\ $f(x)=\begin{cases}1 & -\pi/2<x<\pi/2\\ 0 & \pi/2<x<3\pi/2\end{cases}$

$$a_0=1/2$$
$$b_n=0$$

$$when \quad n=even, a_n=0$$

$$when \quad n=odd=1,5,9,......\Rightarrow a_n=2/n\pi$$
$$n=3,7,11,........\Rightarrow a_n=-2/n\pi$$

$$\frac{1}{2}+\frac{2}{\pi}\left(\cos(x)-\frac{1}{3}\cos(3x)+\frac{1}{5}\cos(5x)-\ldots\right)$$

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Q5\ To find Eigen value :

Det|A-λI|=0 $\Rightarrow\begin{vmatrix}1-\lambda & 4 & 5\\ 0 & 2-\lambda & 6\\ 0 & 0 & 3-\lambda\end{vmatrix} \quad \Rightarrow(1-\lambda)(2-\lambda)(3-\lambda)=0$

$$\Rightarrow\lambda_1=1 \ ,\lambda_2=2 \ ,\lambda_3=3$$

To find Eigen vector

$at \qquad \lambda_1 = 1$

$$\begin{bmatrix} 0 & 4 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4X_2 + 5X_3 = 0$$
$$X_2 + 6X_3 = 0$$
$$2X_3 = 0$$

$$\therefore X_3 = 0, X_2 = 0 \Rightarrow X_1 = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = X_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$at \qquad \lambda_2 = 2$

$$\begin{bmatrix} -1 & 4 & 5 \\ 0 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -X_1 + 4X_2 + 5X_3 &= 0 \\ 6X_3 &= 0 \\ X_3 &= 0 \end{aligned}\right\} X_3 = 0$$

$$X_1 = 4X_2$$

$$X_2 = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 4X_2 \\ X_2 \\ 0 \end{pmatrix} = X_2 \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$$

$at \qquad \lambda_3 = 3$

$$\begin{bmatrix} -2 & 4 & 5 \\ 0 & -1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2X_1+4X_2+5X_3=0$$
$$-X_2+6X_3=0 \Rightarrow X_2=6X_3$$
$$-2X_1+29X_3=0 \Rightarrow X_1=\frac{29}{2}X_3$$

$$X_3=\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}=\begin{pmatrix} \frac{29}{2}X_3 \\ 6X_3 \\ X_3 \end{pmatrix}=X_3\begin{pmatrix} \frac{29}{2} \\ 6 \\ 1 \end{pmatrix}$$

Q6\

$$-x_1+x_2+2x_3=3$$
$$3x_1-x_2+x_3=8$$
$$-x_1+3x_2+4x_3=5$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 3 & -1 & 1 & 8 \\ -1 & 3 & 4 & 5 \end{array}\right](R_1*-1)$$

$$\left[\begin{array}{ccc|c} -1 & -1 & -2 & -3 \\ 3 & -1 & 1 & 8 \\ -1 & 3 & 4 & 5 \end{array}\right]\begin{array}{l} R_2=R_2-3R_1 \\ R_3=R_3+R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & -3 \\ 0 & 2 & 7 & 7 \\ 0 & 2 & 2 & 2 \end{array}\right]R_2\div 2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & -3 \\ 0 & 1 & \frac{7}{2} & \frac{17}{2} \\ 0 & 2 & 2 & 2 \end{array}\right]R_3=R_3-2R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & -3 \\ 0 & 1 & \frac{7}{2} & \frac{17}{2} \\ 0 & 0 & -5 & -15 \end{array}\right] R_3 \div -5$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & -3 \\ 0 & 1 & \frac{7}{2} & \frac{17}{2} \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 = R_1 + 2R_3 \\ R_2 = R_2 - \frac{7}{2}R_3 \end{array}$$

$$z = 3$$
$$y + \frac{7}{2}z = \frac{17}{2} \Rightarrow y = \frac{17}{2} - \frac{21}{2} = -2$$
$$x = 1$$