University of Technology
Department of Laser & Opto-electronic Engineering
Final Examination 2011-2012

***Subject:** Electromagnetic Fields*
***Division:** Laser& Opto-electronic Eng.*
***Examiner:** Dr. Alaa Hussein Ali*

***Class:** 2^{nd} year*
***Time:** 3 hours*
***Date:** 22/5/2012*

Answer five questions only

Q.1/A A uniform sheet of charge $\sigma = (\frac{1}{6}\pi)\ nc/m^2$, is at X=0 and a sheet $\sigma = (-\frac{1}{6}\pi)\ nc/m^2$, is at X=10m. Find $V_{AB}, V_{BC} and V_{AC}$ for A(10m,0,0), B(4m,0,0,) and C(0,0,0,).

(10 MARKS)

Q.1/B For line charge $\lambda = (\frac{10^{-9}}{2})c/m$ on the z-axis. Find V_{AB}, where A is $(2, 0.5\pi, 10) and$ B $(4, \pi, 5)$. **(10 MARKS)**

Q.2/A Given the vector field $A = 5x^2(\sin\frac{\pi x}{2})_{ax}$, find div A at x=1. **(10 MARKS)**

Q.2/B Charge in the form of a plane sheet with density $\sigma = 40\mu c/m^2$ is located at Z= - 0.5 m. A uniform line charge of $\lambda = -6\mu c/m$ lies along the Y – axis. What net flux crosses the surface of a cube 2m on an edge, centered at origin , as shown in fig(1) ?

(10 MARKS)

Q.3/A Let $A = 2_{ax} + 5_{ay} - 3_{az}$, $B = 3_{ax} - 4_{ay\,az}$, and $C = 1_{ax} + 1_{ay} + 1_{az}$. Determine

a) $A + 2B$
b) Calculate $|A - 5C|$
c) For what values of K is $|KB|$=2
d) Find $(A \times B)/(A.B)$ **(10 MARKS)**

Q.3/B Two infinite uniform sheets of charge, each with density (σ), are located at $x = \pm 1$. Determine E in all regions, as shown in fig (2). **(10 MARKS)**

Q.4 Transform $A = y_{ax} + x_{ay} + (\frac{x^2}{\sqrt{x^2 + y^2}})_{az}$ from Cartesian to cylindrical coordinates.

(20 MARKS)

Q.5/A Given $W = X^2 + Y^2 + XYZ$, compute ∇W and the direction derivative $\frac{dW}{dl}$ in the direction $3_{ax} + 4_{ay} + 12_{az}$ at (2,-1, 0). **(10 MARKS)**

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Q.5/B If $J = \frac{1}{r^3}(2\cos\theta_{ar} + \sin\theta_{a\theta})$ A/m^2, calculate the current passing through

a) A hemispherical shell of radius 20 cm.
b) A spherical shell of radius 10 cm. **(10MARKS)**

Q.6 The charge $(10^{-4}e^{-3t})$ C is removed from a sphere through a wire . Find the current in the wire at $t = 0$ and $t = 2.5s$ **(20 MARKS)**

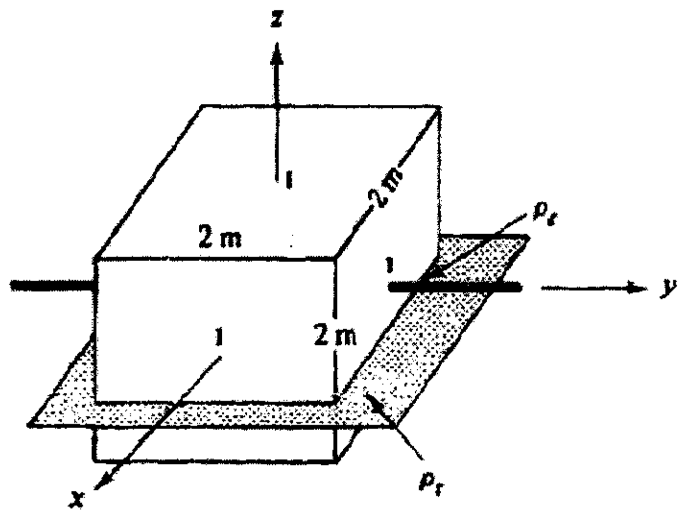

Fig (1)

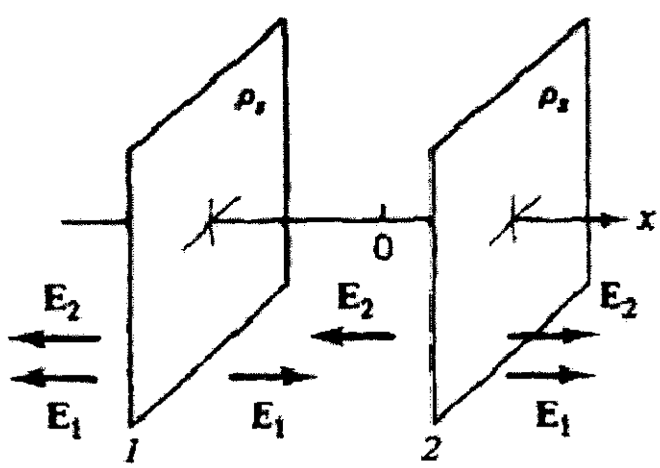

Fig (2)

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Q.1/A

$$V_{AB} = -\int_B^A E \cdot dl$$

$$= -\int \frac{\sigma}{2\epsilon_0}\, dx$$

$$= -\int_4^{10} \frac{-\pi * 10^{-9}}{12\epsilon_0}\, dx$$

$$= 29.6\,[10-4]$$

$$V_{AB} = 177.65 \text{ volt}$$

$$V_{BC} = -\int_C^B \frac{\sigma}{2\epsilon_0}\, dx$$

$$= \int_0^4 \frac{\pi * 10^{-9}}{12\epsilon_0}\, dx$$

$$= 118.4 \text{ Volt}$$

Therefore

$$V_{AC} = V_{AB} - V_{BC}$$

$$= 177.65 - 118.4$$

$$= 59.25 \text{ Volt}$$

Q.1/B

$$V_{AB} = -\int_B^A E \cdot dl \,, \quad \text{where } E = \frac{\lambda}{2\pi\epsilon_0 r} \, a_r$$

$$\therefore V_{AB} = -\int_B^A \frac{10^{-9}}{2(2\pi\epsilon_0 r)} \, dr$$

$$= -9 \, [\ln r]$$

$$= -9 \, [\ln]_4^2$$

$$= 6.24 \text{ Volt}$$

Q.2/A

$$\operatorname{div} A = \frac{\partial}{\partial x}\left(5x^2 \sin \frac{\pi x}{2}\right)$$

$$= 5x^2 \left(\cos \frac{\pi x}{2}\right) \frac{\pi}{2} + 10x \sin \frac{\pi x}{2}$$

$$= \frac{5}{2} \pi x^2 \cos \frac{\pi x}{2} + 10x \sin \frac{\pi x}{2}$$

$$\therefore \operatorname{div} A \Big|_{x=1} = 10$$

Q.2/ B:

$$Q = (4\,m^2)(40\,\mu c/m^2)$$

$$= 160\,\mu c$$

The charge enclosed from the Plane

$$Q = (2m)(-6\,\mu c/m)$$

$$= -12\,\mu c$$

The charge from the line

Thus $Q_{enc} = Q = 160 - 12 = 148\,\mu c$.

Q.3/A

a) $\bar{A}+2\bar{B} = (2,5,-3)+(6,-8,0)$

$= 8a_x - 3a_y - 3a_z$

b) $\bar{A}-5\bar{C} = (2,5,-3)-(5,5,5)$

$= -3a_x - 8a_z$

$|\bar{A}-5\bar{C}| = \sqrt{9+64}$

$= 8.544$

c) $KB = 3Ka_x - 4Ka_y$

$|KB| = \sqrt{9K^2+16K^2}$

$= \mp 5K = |KB|$, when $|KB| = 2$

from $5K = 2$

$\therefore K = \mp 0.4$

d) $\bar{A}\cdot\bar{B} = (2,5,-3)\cdot(3,-4,0)$

$= 6 - 20 + 0$

$= 14$

$\bar{A}\times\bar{B} = \begin{vmatrix} 2 & 5 & -3 \\ 3 & -4 & 0 \end{vmatrix}$

$= (-12, -9, -23) \times -1$

$= (12a_x + 9a_y + 23a_z)$

$\frac{\bar{A}\times\bar{B}}{\bar{A}\cdot\bar{B}} = \left(\frac{12}{14}, \frac{9}{14}, \frac{23}{14}\right) = 0.8971a_x + 0.6412a_y + 1.6412a_z$

Q.3./ B.

only parts of the two sheets of charge as in fig.
Both sheets result in E fields that are directed along X, independent of the distance. Then

$$\bar{E}_1 + \bar{E}_2 = \begin{cases} -\left(\frac{\tilde{\sigma}}{\epsilon_0}\right) a_x & \text{for } x < -1 \\ 0 & \text{for } -1 < x < 1 \\ \left(\frac{\tilde{\sigma}}{\epsilon_0}\right) a_x & \text{for } x > 1 \end{cases}$$

Q.4

$$x = r\cos\phi, \quad y = r\sin\phi, \quad r = \sqrt{x^2+y^2}$$

$$A = r\sin\phi\, a_x + r\cos\phi\, a_y + r\cos^2\phi\, a_z$$

Now, the projections of the Cartesian unit vectors on a_r, a_ϕ and a_z are obtained

$a_x \cdot a_r = \cos\phi$ $\quad$ $a_x \cdot a_\phi = -\sin\phi$ $\quad$ $a_x \cdot a_z = 0$

$a_y \cdot a_r = \sin\phi$ $\quad$ $a_y \cdot a_\phi = \cos\phi$ $\quad$ $a_y \cdot a_z = 0$

$a_z \cdot a_r = 0$ $\quad$ $a_z \cdot a_\phi = 0$ $\quad$ $a_z \cdot a_z = 1$

Therefore

$$a_x = \cos\phi\, a_r - \sin\phi\, a_\phi$$
$$a_y = \sin\phi\, a_r + \cos\phi\, a_\phi$$
$$a_z = a_z$$

and $$A = 2r\sin\phi\cos\phi\, a_r + (r\cos^2\phi - r\sin^2\phi)\, a_\phi + r\cos^2\phi\, a_z$$

5/A.

$$\nabla W = \frac{\partial W}{\partial x} a_x + \frac{\partial W}{\partial y} a_y + \frac{\partial W}{\partial z} a_z$$

$$= (2x + yz)_{a_x} + (2y + xz)_{a_y} + (xy)_{a_z}$$

At $(2,-1,0)$: $\nabla W = 4_{a_x} - 2a_y - 2a_z$

Hence $$\frac{dW}{dl} = \nabla W \cdot a_l = (4, -2, -2) \cdot \frac{(3, 4, 12)}{13}$$

$$= -\frac{20}{13}$$

Q.5/B

$I = \int J \cdot dS$, where $dS = r^2 \sin\theta \, d\phi \, d\theta \, a_r$ in this case

a) $$I = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} \frac{1}{r^3} 2\cos\theta \, r^2 \sin\theta \, d\phi \, d\theta \Big|_{r=0.2}$$

$$= \frac{2}{r} 2\pi \int_{\theta=0}^{\frac{\pi}{2}} \sin\theta \, d(\sin\theta) \Big|_{r=0.2}$$

$$= \frac{4\pi}{0.2} \frac{\sin^2\theta}{2} \Big|_0^{\frac{\pi}{2}} = 10\pi = 31.4 \text{ Amp/m}$$

b) The only difference here is that we have $0 \leq \theta \leq \pi$ instead of $0 \leq \theta \leq \frac{\pi}{2}$ and $r = 0.1$, Hence

$$I = \frac{4\pi}{0.1} \left. \frac{\sin^2\theta}{2} \right|_0^\pi$$

$$= 0$$

Alternatively, for this case

$$I = \oint J \cdot ds' = \int \nabla \cdot J \, ds' = 0$$

since $\nabla \cdot J = 0$

Q.6

$$I = \frac{dQ}{dt}$$

$$= -3 * 10^{-4} e^{-3t}$$

a) $I(t=0) = -0.3$ m Ampere.

b) $I(t=2.5) = -0.3 e^{-7.5}$

$= -166$ n Ampere