Q: a Consider a S.I fiber with $a = 25\,\mu m$, $n_1 = 1.45$ and $\Delta = 0.9\%$. IF this fiber is operated at 1550 nm, calculate V-parameter

b Calculate the required Δ if a fiber with an $8\,\mu m$ and a $125\,\mu m$ cladding at wavelength of 1300 nm. Assume the core index is 1.46.

Solution

a
$$V = \frac{2\pi a n_1}{\lambda}\sqrt{2\Delta} = \frac{2\pi(4\times10^{-6})\times1.45}{1550\times10^{-9}}\sqrt{2(0.009)}$$

$$= 13.46$$

b for S.M Fiber $2 < V < 2.405$, we will arbitrarily choose $V = 2.1$

$$V = \frac{2\pi a n_1}{\lambda}\sqrt{2\Delta}$$

$$\sqrt{2\Delta} = \frac{V\lambda}{2\pi a n_1}$$

$$= \frac{2.1(1300\times10^{-9})}{2\pi(4\times10^{-6})(1.46)}$$

$$= 7.44\times10^{-3}$$

$$\therefore \Delta = 0.277\%$$

Q: Derive the expression for the material dispersion in fiber

Solution: The arrival time τ of light after traversing a length L of fiber is

$$\tau = \frac{L}{v_g} \quad (1)$$

Where v_g is the group velocity of the fiber, given by:

$$v_g = \frac{1}{\frac{d\beta}{d\omega}} \quad (2)$$

we have then;

$$\tau = L\frac{d\beta}{d\omega} = L\frac{d\beta}{d\lambda}\cdot\frac{d\lambda}{d\omega} \quad (5)$$

Since $\lambda = \frac{c}{\nu} = \frac{2\pi c}{\omega}$, we find;

$$\frac{d\lambda}{d\omega} = -\frac{2\pi c}{\omega^2} = -\frac{1}{\omega}\,\frac{2\pi c}{\omega} = -\frac{\lambda}{\omega} \quad (6)$$

Substituting Eq(6) into Eq(5), we obtain:

$$\tau = L\frac{d\beta}{d\lambda}\left(-\frac{\lambda}{\omega}\right) = -\frac{L\lambda}{\omega}\frac{d\beta}{d\lambda} = -\frac{L\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \quad (9)$$

we know $\beta = \frac{2\pi n(\lambda)}{\lambda}$, so

$$\tau = -\frac{L\lambda^2}{2\pi c}\cdot\frac{d\beta}{d\lambda}$$

$$= -\frac{L\lambda^2}{2\pi c}\left[-\frac{2\pi n}{\lambda^2} + \frac{2\pi n'}{\lambda}\right]$$

$$= -\frac{L}{c}\left[-n + \lambda n'\right] = +\frac{L}{c}\left[n(\lambda) - \lambda\frac{dn(\lambda)}{d\lambda}\right] \quad (10)$$

The pulse spread $\Delta\tau$ due to a source linewidth of $\Delta\lambda$ is;

$$\frac{\Delta\tau}{\Delta\lambda} = \frac{d\tau}{d\lambda} = \frac{L}{c}\left[\frac{dn}{d\lambda} - \lambda\frac{d^2n}{d\lambda^2} - \frac{dn}{d\lambda}\right] = -\frac{L\lambda}{c}\,\frac{d^2n}{d\lambda^2} \quad (11)$$

Multiplying by $\Delta\lambda$;

$$\Delta\tau = -\frac{L\lambda\Delta\lambda}{c}\frac{d^2n}{d\lambda^2} = -\frac{L\Delta\lambda}{c\lambda}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right) \quad (12)$$

$\longleftrightarrow$

Q: Calculate the coupling efficiency that can be expected in coupling a 50/125 SI (emitting) fiber with an NA of 0.15 to a 62.5/125 GI ($g=2$) receiving fiber with an NA $=0.2$.

Solution

Since $NA_r > NA_e$

$$\eta_{NA} = 1$$

Since $a_e < a_r$ $\quad \eta_r = 1$

Since $g_e (=\infty) > g_r$

$$\eta_g = \frac{g_r(g_e+2)}{g_e(g_r+2)}$$

$$= \frac{g_r(1+\frac{2}{g_e})}{(g_r+2)}$$

$$= \frac{2(1+\frac{2}{\infty})}{(2+2)} = 0.5$$

The total efficiency:

$$\eta = \eta_{NA} \cdot \eta_r \cdot \eta_g$$

$$= (1)(1)(0.5)$$

$$= 0.5$$

The total Losses are:

$$-10 \log(0.5) = 3dB$$

Q: An optical beam is found to be $-10dB$ from its on-axis value of power at a measure angle of $75°$. a. If the angular power depence $P(\theta) = P_0 \cos^n\theta$ find the value of n. b. Calculate the full angle beam divergence of this source.

Solution:

a:
$$P(\theta) = P_0 \cos^n\theta$$

$$\frac{P(\theta)}{P_0} = \cos^n\theta$$

$$-10dB = \cos^n 75$$

$$10^{-\frac{10}{10}} = 0.1 = \cos^n 75°$$

$$\text{Log}(0.1) = n \,\text{Log}\cos 75$$

$$n = \frac{-1}{-0.587} = 1.703$$

b-
$$\frac{P(\theta)}{P_0} = \cos^{1.703}\theta$$

$$\frac{1}{2} = (\cos\theta)^{1.703}$$

$$\cos\theta = (0.5)^{1/1.703} = 0.665$$

$$\theta = 48.3° \text{ (half-angle divergence)}$$

$$2\theta = 2 \times 48.3 = 96.6 \text{ (full-angle divergence)}$$

$\Leftrightarrow$

Q. show that the power coupled into a step-index optical fiber from a source (with $r_s < a$) with a radinance of $B(\theta) (= B_o \cos^m\theta$ is given by:

$$P_f = \frac{2\pi A_s B_o [1 - \cos^{m+1}\theta_{max}]}{m+1}$$

Solution

$$P_f = \int_0^{r_s}\int_0^{2\pi}\left(\int_0^{2\pi}\int_0^{\phi_{max}} B_o \cos^m\theta \sin\theta \, d\theta \, d\phi\right) d\phi_s \, r \, dr \quad (1)$$

$$\int_0^{\phi_{max}} B_o \cos^m\theta \sin\theta \, d\theta = -\frac{B_o \cos^{m+1}\theta}{m+1}\Bigg|_0^{\theta_{max}}$$

$$= B_o\left[\frac{1 - \cos^{m+1}\theta_{max}}{m+1}\right] \quad (2)$$

$$\int_0^{2\pi} d\phi = 2\pi \quad (3)$$

sub eq (2) & (3) in (1):

$$\therefore P_f = \int_0^{r_s}\int_0^{2\pi}\left(2\pi B_o\left[\frac{1 - \cos^{m+1}\theta_{max}}{m+1}\right]\right) d\phi_s \, r \, dr \quad (4)$$

$$\int_0^{2\pi} d\phi_s = 2\pi \quad (5)$$

$$\int_0^{r_s} r \, dr = \frac{r^2}{2}\Bigg|_0^{r_s} = \frac{r_s^2}{2} \quad (6)$$

But $A_s = \pi r_s^2$

Then $\int_0^{r_s}\int_0^{2\pi} d\phi_s \, r \, dr = A_s$

Then $P_f = \dfrac{2\pi A_s B_o [1 - \cos^{m+1}\theta_{max}]}{m+1}$

$\longleftrightarrow$

Q: Consider a Silicon APD with $R = 0.65\ A/W$, $I_d = 1\,nA$, $R_L = 1000\,\Omega$, $\Delta f = 100\,MHz$, $x = 0.3$, and gain $M = 100$. Calculate the SNR for input power (a) $P = 100\,nW$ and (b) $P = 10\,\mu W$, for $T = 300^\circ K$

Solution:

(a)

$$SNR = \frac{R^2 P^2}{2qM^x(RP + I_d)\Delta f + \frac{4k_BT}{R_L}\Delta f M^{-2}}$$

$$= \frac{(0.65)^2(100\times10^{-9})^2}{2\times1.6\times10^{-19}\,100^{0.3}(0.65\times100\times10^{-9} + 1\times10^{-9})\times100\times10^{6} + \frac{4\times1.38\times10^{-23}\times300}{1000}\times100\times10^{6}\times100^{-2}}$$

$SNR \simeq 493 \simeq 27\,dB$

(b)

$$SNR = \frac{(0.65)^2(10\times10^{-6})^2}{2\times1.6\times10^{-19}\times100^{0.3}(0.65\times10\times10^{-6} + 1\times10^{-9})\times100\times10^{6} + \frac{4\times1.38\times10^{-23}\times300}{1000}\times100\times10^{6}\times100^{-2}}$$

$SNR \simeq 51000 \simeq 47\,dB$