

حل المسألة (التي هي الأولى)

Q.1B)

The transmissivity T of a transparent material 20 mm thick to normally incident light is 0.85. If the index of refraction of this material is 1.6, compute the thickness of material that will yield a transmissivity of 0.75. All reflection losses should be considered.

Solution

We are asked to compute the thickness of material to yield a transmissivity of 0.75 given that T is 0.85 when $l = 20$ mm, $n = 1.6$, and for normally incident radiation. The first requirement is that we calculate the value of β for this material using Equations 21.13 and 21.19. The value of R is determined using Equation 21.13 as

$$R = \frac{(n_s - 1)^2}{(n_s + 1)^2}$$

$$= \frac{(1.6 - 1)^2}{(1.6 + 1)^2} = 5.33 \times 10^{-2}$$

Now, it is necessary to compute the value of β using Equation 21.19. Dividing both sides of Equation 21.19 by $I_0(1 - R)^2$ leads to

$$\frac{I_T}{I_0(1 - R)^2} = e^{-\beta l}$$

And taking the natural logarithms of both sides of this expression gives

$$\ln \left[\frac{I_T}{I_0(1 - R)^2} \right] = -\beta l$$

and solving for β we get

$$\beta = -\frac{1}{l} \ln \left[\frac{I_T}{I_0(1 - R)^2} \right]$$

Since the transmissivity is T is equal to I_T/I_0 , then the above equation takes the form

$$\beta = -\frac{1}{l} \ln \left[\frac{T}{(1 - R)^2} \right]$$

$$= -\frac{1}{20} \ln \left[\frac{0.85}{(1 - 5.33 \times 10^{-2})^2} \right] = 2.65 \times 10^{-3} \text{ mm}^{-1}$$

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$$l = -\frac{1}{\beta} \ln \left[\frac{T}{(1 - R)^2} \right]$$

$$= -\frac{1}{2.65 \times 10^{-3}} \ln \left[\frac{0.75}{(1 - 5.33 \times 10^{-2})^2} \right] = 67.3 \text{ mm}$$

$$= 6.73 \text{ cm}$$

CHAPTER 18

ELECTRICAL PROPERTIES

PROBLEM SOLUTIONS

Ohm's Law

Electrical Conductivity

Q2 (a) Compute the electrical conductivity of a 5.1-mm (0.2-in.) diameter cylindrical silicon specimen 51 mm (2 in.) long in which a current of 0.1 A passes in an axial direction. A voltage of 12.5 V is measured across two probes that are separated by 38 mm (1.5 in.).

(b) Compute the resistance over the entire 51 mm (2 in.) of the specimen.

Solution

This problem calls for us to compute the electrical conductivity and resistance of a silicon specimen.

(a) We use Equations 18.3 and 18.4 for the conductivity, as

$$\sigma = \frac{1}{\rho} = \frac{Il}{VA} = \frac{Il}{V\pi\left(\frac{d}{2}\right)^2}$$

And, incorporating values for the several parameters provided in the problem statement, leads to

$$\sigma = \frac{(0.1 \text{ A})(38 \times 10^{-3} \text{ m})}{(12.5 \text{ V})\pi\left(\frac{5.1 \times 10^{-3} \text{ m}}{2}\right)^2} = 14.9 (\Omega \cdot \text{m})^{-1}$$

(b) The resistance, R , may be computed using Equations 18.2 and 18.4, as

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{l}{\sigma\pi\left(\frac{d}{2}\right)^2}$$

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$$R = 168.52$$

Q3

Calculate the activation energy for vacancy formation in aluminum, given that the equilibrium number of vacancies at 500°C (773 K) is $7.57 \times 10^{23} \text{ m}^{-3}$. The atomic weight and density (at 500°C) for aluminum are, respectively, 26.98 g/mol and 2.62 g/cm³.

Solution

Upon examination of Equation 4.1, all parameters besides Q_v are given except N , the total number of atomic sites. However, N is related to the density, (ρ_{Al}), Avogadro's number (N_A), and the atomic weight (A_{Al}) according to Equation 4.2 as

$$\begin{aligned} N &= \frac{N_A \rho_{\text{Al}}}{A_{\text{Al}}} \\ &= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.62 \text{ g/cm}^3)}{26.98 \text{ g/mol}} \\ &= 5.85 \times 10^{22} \text{ atoms/cm}^3 = 5.85 \times 10^{28} \text{ atoms/m}^3 \end{aligned}$$

Now, taking natural logarithms of both sides of Equation 4.1,

$$\ln N_v = \ln N - \frac{Q_v}{kT}$$

and, after some algebraic manipulation

$$\begin{aligned} Q_v &= -kT \ln \left(\frac{N_v}{N} \right) \\ &= - (8.62 \times 10^{-5} \text{ eV/atom} \cdot \text{K}) (500^\circ\text{C} + 273 \text{ K}) \ln \left[\frac{7.57 \times 10^{23} \text{ m}^{-3}}{5.85 \times 10^{28} \text{ m}^{-3}} \right] \\ &= 0.75 \text{ eV/atom} \end{aligned}$$

Qu1

The outer surface of a steel gear is to be hardened by increasing its carbon content. The carbon is to be supplied from an external carbon-rich atmosphere, which is maintained at an elevated temperature. A diffusion heat treatment at 850°C (1123 K) for 10 min increases the carbon concentration to 0.90 wt% at a position 1.0 mm below the surface. Estimate the diffusion time required at 650°C (923 K) to achieve this same concentration also at a 1.0-mm position. Assume that the surface carbon content is the same for both heat treatments, which is maintained constant. Use the diffusion data in Table 5.2 for C diffusion in α -Fe.

Solution

In order to compute the diffusion time at 650°C to produce a carbon concentration of 0.90 wt% at a position 1.0 mm below the surface we must employ Equation 5.6b with position (x) constant; that is

$$Dt = \text{constant}$$

Or

$$D_{850}t_{850} = D_{650}t_{650}$$

In addition, it is necessary to compute values for both D_{850} and D_{650} using Equation 5.8. From Table 5.2, for the diffusion of C in α -Fe, $Q_d = 80,000$ J/mol and $D_0 = 6.2 \times 10^{-7}$ m²/s. Therefore,

$$\begin{aligned} D_{850} &= (6.2 \times 10^{-7} \text{ m}^2/\text{s}) \exp \left[-\frac{80,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(850 + 273 \text{ K})} \right] \\ &= 1.17 \times 10^{-10} \text{ m}^2/\text{s} \end{aligned}$$

$$\begin{aligned} D_{650} &= (6.2 \times 10^{-7} \text{ m}^2/\text{s}) \exp \left[-\frac{80,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(650 + 273 \text{ K})} \right] \\ &= 1.83 \times 10^{-11} \text{ m}^2/\text{s} \end{aligned}$$

Now, solving the original equation for t_{650} gives

$$\begin{aligned} t_{650} &= \frac{D_{850}t_{850}}{D_{650}} = \frac{1.17 \times 10^{-10} \text{ m}^2/\text{s} \times 10 \text{ min}}{1.83 \times 10^{-11} \text{ m}^2/\text{s}} \\ &= 63.9 \text{ min} \end{aligned}$$

$$= \frac{51 \times 10^{-3} \text{ m}}{[14.9 (\Omega\text{-m})^{-1}] (\pi) \left(\frac{5.1 \times 10^{-3} \text{ m}}{2} \right)^2} = 168 \ \Omega$$

Q51

Consider the brass alloy for which the stress-strain behavior is shown in Figure 6.12. A cylindrical specimen of this material 6 mm (0.24 in.) in diameter and 50 mm (2 in.) long is pulled in tension with a force of 5000 N (1125 lb_f). If it is known that this alloy has a Poisson's ratio of 0.30, compute: (a) the specimen elongation, and (b) the reduction in specimen diameter.

Solution

(a) This portion of the problem asks that we compute the elongation of the brass specimen. The first calculation necessary is that of the applied stress using Equation 6.1, as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left(\frac{d_0}{2} \right)^2} = \frac{5000 \text{ N}}{\pi \left(\frac{6 \times 10^{-3} \text{ m}}{2} \right)^2} = 177 \times 10^6 \text{ N/m}^2 = 177 \text{ MPa} \quad (25,000 \text{ psi})$$

From the stress-strain plot in Figure 6.12, this stress corresponds to a strain of about 2.0×10^{-3} . From the definition of strain, Equation 6.2

$$\Delta l = \epsilon l_0 = (2.0 \times 10^{-3})(50 \text{ mm}) = 0.10 \text{ mm} \quad (4 \times 10^{-3} \text{ in.})$$

(b) In order to determine the reduction in diameter Δd , it is necessary to use Equation 6.8 and the definition of lateral strain (i.e., $\epsilon_x = \Delta d/d_0$) as follows

$$\begin{aligned} \Delta d &= d_0 \epsilon_x = -d_0 \nu \epsilon_z = -(6 \text{ mm})(0.30)(2.0 \times 10^{-3}) \\ &= -3.6 \times 10^{-3} \text{ mm} \quad (-1.4 \times 10^{-4} \text{ in.}) \end{aligned}$$

Q6:

The metal iridium has an FCC crystal structure. If the angle of diffraction for the (220) set of planes occurs at 69.22° (first-order reflection) when monochromatic x-radiation having a wavelength of 0.1542 nm is used, compute (a) the interplanar spacing for this set of planes, and (b) the atomic radius for an iridium atom.

Solution

(a) From the data given in the problem, and realizing that $69.22^\circ = 2\theta$, the interplanar spacing for the (220) set of planes for iridium may be computed using Equation 3.13 as

$$d_{220} = \frac{n\lambda}{2 \sin \theta} = \frac{(1)(0.1542 \text{ nm})}{(2) \left(\sin \frac{69.22^\circ}{2} \right)} = 0.1357 \text{ nm}$$

(b) In order to compute the atomic radius we must first determine the lattice parameter, a , using Equation 3.14, and then R from Equation 3.1 since Ir has an FCC crystal structure. Therefore,

$$a = d_{220} \sqrt{(2)^2 + (2)^2 + (0)^2} = (0.1357 \text{ nm})(\sqrt{8}) = 0.3838 \text{ nm}$$

And, from Equation 3.1

$$R = \frac{a}{2\sqrt{2}} = \frac{0.3838 \text{ nm}}{2\sqrt{2}} = 0.1357 \text{ nm}$$