DESIGN STUDY OF VARIOUS SYNTHETIC APERTURE CONFIGURATION

A Thesis Submitted to the Laser and Optoelectronics Engineering Department, University of Technology in a Partial Fulfillment of the Requirements for the Degree of Master of Science in Optoelectronics Engineering

By

Kamal Hussein Kazem Al-Lamy

Supervisor by

Dr. Ali H. Al-Hamdani

2008
دراسة تصميمية لمختلف أشكال الفتحة المركبة

رسالة مقدمة إلى قسم هندسة الليزر والبصريات الالكترونية الجامعة التكنولوجية كجزء من متطلبات نيل درجة الماجستير علوم في هندسة البصريات الالكترونية

تقدم بها
كمال حسين كاظم اللامي

بإشراف
الدكتور علي هادي عبد المنعم الحمداني
ABSTRACT

This research is a theoretical study to distribute the intensity in a point object image for multi synthetic circular apertures of an optical system, the mathematical relationships were derived, a numerical programmes was written for this purpose. A comparison between the present results and a previous researches results of single aperture was done.

There are many functions that are of efficient effects on the distribution of intensity in optical system, in the present research main concerning depend on the study of the effect of point spread function (PSF). A circular aperture was used in this work, because other types may cause error in calculation.

The most three important main factors which are affect in point spread function are (apertures number, radial distance, magnification). The effects of these factors of a diffraction-limited optical system or an optical system with focus error was studied, and all the functions which are necessary for this purpose have been derived and calculate the relationships of it by using the numerical integration method to Gauss and then programming it by using Q.basic language and compare the results that has been obtained with the analytical results of previous researches, to be sure that the relationships derived and the program are correct.

In the present work, the optimum balance was studied briefly and the mathematical relationships that concerning it was derived for a diffraction-limited optical system and another system contain a focus error with multi synthetic circular apertures, to prove that their is a
difference between the values of balance multi synthetic circular apertures optical system and one circular aperture optical system.

The mean conclusion of this research is that the circular synthetic apertures have a high resolving power compared with the single circular aperture, and the aberration effect on central intensity decrease when the number of circular synthetic aperture increasing compared with the single circular aperture of the same optical system.
APPENDIX [1]

SPECIAL PROGRAM STEPS FOR CALCULATION OF POINT SPREAD FUNCTION FOR ROW OF SYNTHETIC APERTURES

REM *** POINT SPREAD FUNCTION***
REM *** INCOHERENT LIGHTING***
REM *** 2D PLOT (u,v)***
CLS : COLOR 2'
CONST PI = 3.141592
DIM I, J AS INTEGER
5 DIM T(20), W(20) AS SINGLE
RESTORE 70
FOR I = 1 TO 10
READ T(I), W(I)
T(21 - I) = -T(I)
W(21 - I) = W(I)
NEXT I
6 DIM SUMR, SUMI, SUMR1, SUMI1 AS SINGLE
DIM SR, SI AS SINGLE
DIM XI, YJ, RJ AS SINGLE
DIM XP(20), YP(20) AS SINGLE
DIM U, V AS SINGLE
NORMFACT = PI
10 PRINT "************************************************************************
PRINT " *** GENERAL PROGRAM OF PSF N-SYNTHETIC APERTURE WITH ABERRATIONS"***
PRINT "************************************************************************
INPUT " ENTER FILE NAME FOR OUTPUT DATA ":", NA$
NA$ = NA$ + ".dot"
OPEN NA$ FOR OUTPUT AS #1
INPUT " NUMBER OF SUB-APERTURES [N] ":", N
INPUT " RADIAL DISTANCE OF EACH APERTURE [D] ":", D
INPUT " Focal number ":", F
REM INPUT " SPHERICAL ABERRATION FACTORS [W40,W60] ":", W40, W60
REM INPUT " Umin,Umax,Step ":", UMIN, UMAX, STP
REM INPUT " Vmin,Vmax,Step ":", VMIN, VMAX, STP
15 INPUT " CONTINUE [Y] RETURN TO INPUT [N] ":", YN$
SELECT CASE YN$
CASE "Y"
GOTO 18
CASE "y"
GOTO 18
CASE "N"
CLS : CLOSE #1: GOTO 10
CASE "n"
CLS : CLOSE #1: GOTO 10
CASE ELSE
GOTO 15
END SELECT
****************************** DEFINING XJ & YJ
18 U = UMIN
20 V = VMIN
FOR THETA = 0 TO 2 STEP .005
21 m = 1 / SQR(D / N): ss = .1
**********************
FOR J = 1 TO N
XP(J) = D * m * SIN(2 * PI * J / N)
YP(J) = D * m * COS(2 * PI * J / N)
NEXT J
CALCULATING SUMMATION TERM

SR = 0; SI = 0
FOR J = 1 TO N
SR = SR + COS(2 * PI * (U * XP(J) + V * YP(J)))
SI = SI + SIN(2 * PI * (U * XP(J) + V * YP(J)))
NEXT J

CALCULATING INTEGRATION

SUMR1 = 0; SUMI1 = 0
FOR J = 1 TO 20
YJ = T(J) * m
RJ = SQR((D / N) - YJ^2)
SUMR = 0; SUMI = 0
FOR I = 1 TO 20
XI = RJ * T(I); W20 = (D / (16 * F)) * ((1 - ss) / ss) * (THETA)^2
Z = 2 * PI * (W20 * (XI^2 + YJ^2) + (U * XI + V * YJ))
SUMR = SUMR + W(I) * COS(Z)
SUMI = SUMI + W(I) * SIN(Z)
NEXT I
PRINT "---"
SUMR1 = SUMR1 + SUMR * RJ * W(J)
SUMI1 = SUMI1 + SUMI * RJ * W(J)
NEXT J
FR = (SUMR1 * SR - SUMI1 * SI) * m / NORMFACT
FI = (SUMR1 * SI + SUMI1 * SR) * m / NORMFACT
PSF = (FR^2 + FI^2)
PRINT
PRINT PSF, W20
PRINT #1, PSF, W20
NEXT THETA
IF V < VMAX THEN
V = V + STP
GOTO 21
END IF
IF U < UMAX THEN
U = U + STP
GOTO 20
END IF
CLOSE #1
PRINT "END OF PROGRAM ?"
END

70 DATA .9931286,.01761401,.96397192
DATA .04060143,.9122344,.06267205
DATA .8391169,.08327674,.74633190
DATA .10193012,.6360536,.11819453
DATA .5108670,.1316886,.37370608
DATA .1420961,.2277858,.1491730
DATA .07652652,.1527534
SPECIAL PROGRAM STEPS FOR CALCULATION OF POINT SPREAD
FUNCTION FOR ROW OF SYNTHETIC APERTURES WHEN CHANGING THE
RADIAL DISTANCE & MAGNIFICATION

REM *** POINT SPREAD FUNCTION***
REM *** INCOHERENT LIGHTING***
CLS: 'COLOR 2
CONST PI = 3.141592
DIM I, J AS INTEGER

5 DIM T(20), W(20) AS SINGLE
RESTORE 70
FOR I = 1 TO 10
READ T(I), W(I)
T(21 - I) = -T(I)
W(21 - I) = W(I)
NEXT I

6 DIM SUMR, SUMI, SUMR1, SUMI1 AS SINGLE
DIM SR, SI AS SINGLE
DIM XI, YJ, RJ AS SINGLE
DIM xp(20), yp(20) AS SINGLE
DIM U, V AS SINGLE
NORMFACT = PI

10 PRINT
"********************************************************************************
PRINT " *** GENERAL PROGRAM OF PSF N-SYNTHETIC APERTURE WITH
PRINT " ABERRATIONS***
PRINT "********************************************************************************

INPUT " ENTER FILE NAME FOR OUTPUT DATA ":", NA$
NA$ = NA$ + " .dat"
OPEN NA$ FOR OUTPUT AS #1
INPUT " NUMBER OF SUB-APERTURES [N] ":", N
INPUT " RADIAL DISTANCE OF EACH APERTURE [D] ":", D
INPUT " FOCUS ERROR FACTOR [W20] ":", W20
INPUT " SPHERICAL ABERRATION FACTORS [W40, W60] ":", W40, W60
INPUT " Umin, Umax, Step ":", UMIN, UMAX, STP
INPUT " Vmin, Vmax, Step ":", VMIN, VMAX, STP

15 INPUT " CONTINUE [Y] RETURN TO INPUT [N] ":", YN$
SELECT CASE YN$
CASE "Y"
GOTO 18
CASE "y"
GOTO 18
CASE "N"
CLS : CLOSE #1: GOTO 10
CASE "n"
CLS : CLOSE #1: GOTO 10
CASE ELSE
GOTO 15
END SELECT

******************************************************************************
18 U = UMIN
20 V = 0
21 M = 1 / SQR(N)
******************************************************************************
DEFINING XJ & YJ
FOR J = 1 TO N
xp(J) = D * M * SIN(2 * PI * J / N)
yp(J) = D * M * COS(2 * PI * J / N)
NEXT J
******************************************************************************
CALCULATING SUMMATION TERM
APPENDIX [2]

SR = 0: SI = 0
FOR J = 1 TO N
SR = SR + COS(2 * PI * (U * xp(J) + V * yp(J)))
SI = SI + SIN(2 * PI * (U * xp(J) + V * yp(J)))
NEXT J

********************************************************************************
CALCULATING INTEGRATION
SUMR1 = 0: SUMI1 = 0
FOR J = 1 TO 20
YJ = T(J) * M
RJ = SQR((1 / N) - YJ ^ 2)
SUMR = 0: SUMI = 0
FOR I = 1 TO 20
XI = RJ * T(I)
RR = XI ^ 2 + YJ ^ 2
Z = 2 * PI * (W20 * RR + W40 * RR ^ 2 + W60 * RR ^ 3 + (U * XI + V * YJ))
SUMR = SUMR + W(I) * COS(Z)
SUMI = SUMI + W(I) * SIN(Z)
NEXT I
PRINT;"--"
SUMR1 = SUMR1 + SUMR * RJ * W(J)
SUMI1 = SUMI1 + SUMI * RJ * W(J)
NEXT J
FR = (SUMR1 * SR - SUMI1 * SI) * M / NORMFACT
FI = (SUMR1 * SI + SUMI1 * SR) * M / NORMFACT
PSF = (FR ^ 2 + FI ^ 2)
PRINT "U ="; U, "F(U) ="; PSF
PRINT #1, U, "          ", PSF
IF U < UMAX THEN
U = U + STP
GOTO 20
END IF
CLOSE #1
PRINT "END OF PROGRAM ? 
70 DATA .9931286,.01761401,.96397192
DATA .04060143,.9122344,.06267205
DATA .8391169,.08327674,.74633190
DATA .10193012,.6360536,.11819453
DATA .5108670,.1316886,.37370608
DATA .1420961,.2277858,.1491730
DATA .07652652,.1527534
SLEEP
PRINT " J         xp                yp"
PRINT "----------------------------------"
FOR J = 1 TO N
PRINT J; TAB(5); xp(J); TAB(25); yp(J)
NEXT J
Examination Committee Certification

We certify that we have read the thesis entitled "Design Study of Various Synthetic Aperture Configuration ", and as an examination committee examined the student "Kamal Hussein Kazem" in its content, and that in our opinion it is adequate for the partial fulfillment of the requirements for the degree of Master of Science in Optoelectronics Engineering.

Signature:
Name: Dr. Adnan S. Al-Aithawi
Title: Professor
(Chairman)
Date : / 5 / 2008

Signature:
Name: Dr. Waleed Y. Hussein
Title: Lecturer
(Member)
Date : / 5 / 2008

Signature:
Name: Dr. Ali H. Al-Hamdani
Title: Assist Professor
(Supervisor)
Date : / 5 / 2008

Approved by the Department of Laser and Optoelectronics Engineering, University of Technology.

Signature:
Name: Dr. Mohammed H. Ali.
Title: Assist Professor
Address: Head of Laser and Optoelectronics Eng.
Dep. University of Technology.
Date : / 5 / 2008
## CONTENTS

<table>
<thead>
<tr>
<th>Seq.</th>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Acknowledgment</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>Abstract</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>List of Abbreviations and Symbols</td>
<td>IV</td>
</tr>
<tr>
<td></td>
<td>Contents</td>
<td>VII</td>
</tr>
<tr>
<td></td>
<td><strong>Chapter One</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>General Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Lens Testing</td>
<td>2</td>
</tr>
<tr>
<td>1.2.1</td>
<td>Qualitative Test</td>
<td>2</td>
</tr>
<tr>
<td>1.2.2</td>
<td>Quantitative Test</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2.1</td>
<td>Visible Test</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2.2</td>
<td>The Photometer Test</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>Diffraction</td>
<td>6</td>
</tr>
<tr>
<td>1.4</td>
<td>Aberration</td>
<td>7</td>
</tr>
<tr>
<td>1.4.1</td>
<td>Monochromatic Aberration</td>
<td>8</td>
</tr>
<tr>
<td>1.4.1.1</td>
<td>Spherical Aberration</td>
<td>9</td>
</tr>
<tr>
<td>1.4.1.2</td>
<td>Coma</td>
<td>10</td>
</tr>
<tr>
<td>1.4.1.3</td>
<td>Astigmatism</td>
<td>12</td>
</tr>
<tr>
<td>1.4.1.4</td>
<td>Field Curvature</td>
<td>12</td>
</tr>
<tr>
<td>1.4.1.5</td>
<td>Distortion</td>
<td>13</td>
</tr>
<tr>
<td>1.4.2</td>
<td>Chromatic Aberrations</td>
<td>14</td>
</tr>
<tr>
<td>1.5</td>
<td>Strehl Ratio</td>
<td>15</td>
</tr>
<tr>
<td>1.6</td>
<td>Aberrations Balancing</td>
<td>17</td>
</tr>
<tr>
<td>1.7</td>
<td>Resolution of Optical System</td>
<td>19</td>
</tr>
<tr>
<td>1.8</td>
<td>Focus Error</td>
<td>20</td>
</tr>
<tr>
<td>1.9</td>
<td>Depth of Focus</td>
<td>21</td>
</tr>
<tr>
<td>1.10</td>
<td>Literature Survey</td>
<td>21</td>
</tr>
<tr>
<td>1.11</td>
<td>Aim of The Work</td>
<td>23</td>
</tr>
</tbody>
</table>
## Chapter Two

### Point Spread Function

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>24</td>
</tr>
<tr>
<td>2.2</td>
<td>Point Spread Function (PSF)</td>
<td>24</td>
</tr>
<tr>
<td>2.2.1</td>
<td>PSF for Circular Aperture</td>
<td>29</td>
</tr>
<tr>
<td>2.2.1.1</td>
<td>PSF for A Diffraction-Limited System</td>
<td>29</td>
</tr>
<tr>
<td>2.2.1.2</td>
<td>PSF with Focus Error</td>
<td>31</td>
</tr>
</tbody>
</table>

## Chapter Three

### PSF for Multi Subapertures

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>33</td>
</tr>
<tr>
<td>3.2</td>
<td>Derivation of PSF Equation for Row of Circular Synthetic Apertures</td>
<td>34</td>
</tr>
<tr>
<td>3.2.1</td>
<td>PSF for A diffraction-Limited System for Row of Synthetic Apertures</td>
<td>41</td>
</tr>
<tr>
<td>3.2.2</td>
<td>PSF with Focus Error for Row of Synthetic Apertures</td>
<td>43</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Optimum Balance for Multi Aperture System</td>
<td>43</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Optimum Balance for Multi Aperture System with Focus Error</td>
<td>44</td>
</tr>
</tbody>
</table>

## Chapter Four

### Numerical Calculation, Results & Discussion

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>46</td>
</tr>
<tr>
<td>4.2</td>
<td>Field Angle</td>
<td>47</td>
</tr>
<tr>
<td>4.3</td>
<td>Mathematical Relation Between Field Angel &amp; PSF</td>
<td>47</td>
</tr>
<tr>
<td>4.4</td>
<td>Gauss Method</td>
<td>49</td>
</tr>
<tr>
<td>4.5</td>
<td>Numerical Evaluation of PSF for A diffraction-Limited System (One Circular Aperture)</td>
<td>50</td>
</tr>
<tr>
<td>4.6</td>
<td>Numerical Evaluation of PSF with Focus Error (One Circular Aperture)</td>
<td>51</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.7</td>
<td>Effect of the Field Angle on Resolving Power for (One Circular Aperture)</td>
<td>53</td>
</tr>
<tr>
<td>4.8</td>
<td>Numerical Evaluation of PSF for a diffraction-limited System (Multi Circular Aperture)</td>
<td>54</td>
</tr>
<tr>
<td>4.9</td>
<td>Numerical Evaluation of PSF with Focus Error (Multi Circular Aperture)</td>
<td>56</td>
</tr>
<tr>
<td>4.10</td>
<td>Effect of the Field Angle on Resolving Power for (Multi-Circular Aperture)</td>
<td>57</td>
</tr>
<tr>
<td>4.11</td>
<td>Effect of Apertures Number on Focus Error</td>
<td>59</td>
</tr>
<tr>
<td>4.12</td>
<td>Conclusions</td>
<td>110</td>
</tr>
<tr>
<td>4.13</td>
<td>Future Work</td>
<td>112</td>
</tr>
</tbody>
</table>

References 113

Appendix [1] 118

Appendix [2] 120
1.1 Introduction

The production and industrialization of the optical system passes through several stages, the optical design is the first one, after this stage is completed, the optical components industrialization will be the next stage and then, the evaluation and the testing of these components will be the last stage before the optical system is being used.

The optical design include specification of the radii of the surfaces curvature, the thickness, the air spaces, the diameters of the various components, the type of glass to be used and the position of the stop aperture. These parameters are known as (degrees of freedom). The designer can change them to maintain the desired system.

The image that is formed by these optical systems will be approximately corrected from the aberrations. But there is no ideal image correspond to the object dimensions because of the wave nature of the light, which almost affects with several factors like the type of illumination that be used (incoherent, coherent and partial coherent), the object shape (Point, Line or Edge) and the aperture shape [1].

There are several factors that affect the evaluation of the image quality which is formed by the optical system, from these important factors is the, measured spread function (Point, Line and Edge) which represents descriptions of the intensity distribution in image plane for an object (Point, Line and Edge). The spread function depends on diffraction that produces by the lens aperture and the amount of the aberrations and its type in lens or in the optical system [1,2,3].
The point spread function is an important parameter that is used for identification the efficiency of the optical system, where several functions are derived from the point spread function which are in differential or integral relations with it [4].

1.2 Lens Testing

There are generally three basic reasons for carrying out series of test on lenses:

1. To determine if the lens are suitable for a given purpose.
2. To determine whether a lens which has been constructed fulfills the design characteristics.
3. To study the limitation on accuracy of image and the relation among various methods of assessing image quality [4].

There are two ways of testing the lenses and optical systems which are:

1.2.1 Qualitative Test

The qualitative test gives the ability to know the type of aberrations in the tested lens, without measuring it. The star test is classified under the qualitative test ways. In the star test a collimator used to produce a plane wave which is fall directly on the tested lens. The image formed by the tested lens is examined through a microscope as show in figure (1-1). The lens rotates around it axis through the test, to examine the disinterring aberrations and asymmetric aberrations in the point image. If the tested lens is perfect the observer sees bright circular surround by several rings rapidly diminishing brightness which called Airy pattern. This process of examination enable the deduction of some aberrations which reach to \((1/10)\) from the wavelength that are used [5].
In this process human eye is used which is practically a good detector for asymmetric and for the change in the form, but it can not show the exact difference value of the intensity and the distance between the fringes [5].

1.2.2 Quantitative Test

The Quantitative test is divided into two types:

1.2.2.1 Visible Test

It is the test that contains all the required measurements that are designed on the basic principle of interference between the wave fronts coming from the lens through using ideal wave of mono wavelength from point source (the ideal wave is considered as a reference to the wave coming from the testing lens) [6].

The instrument that used for this purpose is Twyman-Green interferometer which is widely used in examination the lens and prisms, it is a good instrument to know the amount of moving away from the ideal state, starting from \((1/20)\) \(\lambda\) parts of the wavelength to little wavelength \((3\lambda)\). When the wavelength moving away about hundreds of wavelength the interferometer will be useless [2,6,7].
The Twyman-Green interferometer is essentially a variation of the Michelson interferometer. It is an instrument of great importance in the domain of modern optical testing as shown in figure (1-2) [6].

This device is setup to examine lenses. This spherical mirror M1 has its center of curvature tested is free of aberrations (which is usually plane mirror), the emerging reflected light returning to the beam splitter will again be plane wave. In case, an astigmatism, coma, or spherical aberrations deform, the waveform, fringe pattern will manifest these distortions and can be seen and photographed. When M2 is replaced by plane mirror, a number of other elements (primes, optical flats) can be equally tested as well [7].

1.2.2.2 The Photometer Test

This type of examination includes the measurement of special function that explains the lens efficiency, its ideality, and the amount of aberrations that is presented in it [8].
Some of these functions e.g. point spread functions (PSF), line spread function (LSF), disk spread function (DSF) and other spread function that give good description of the intensity distribution in the image plane of an object by the optical system that to be examined. The spread function depends on the aperture lens diffraction and the aberrations type and the amount of aberrations in the lens or in the optical components [8].

There is another important function which is used to examine the optical system like the optical transfer function (OTF). It is define as the ability of the optical system to transfer the different frequency from the object plane to the image plane as shown in figure (1-3). Another important functions used to evaluate the image specificity is the contrast transfer function (contrast) which is given by [5]:

\[
\text{contrast} = \frac{I_{\text{MAX}} - I_{\text{MIN}}}{I_{\text{MAX}} + I_{\text{MIN}}}
\]

where
\[ I_{\text{MAX}} \] represents the maximum intensity.
\[ I_{\text{MIN}} \] represents the minimum intensity.

\[ \text{Figure (1-3)} \]
\[ \text{The optical transfer function /8/} \]
1.3 Diffraction

Diffraction is a phenomena or effect resulting from the interaction of the radiation wave with the limiting edges of the aperture stop of optical system [5, 9]. Diffraction is a natural property of light arising from its wave nature, possesses fundamental limitation on any optical system. It is always present, although its effects may be notice if the system has significant aberrations. When an optical system is essentially free from aberrations, its performance is limited solely by diffraction, and it is referred to as diffraction-limited. The image of a point source formed by diffraction-limited optics is blurring, which appears as a bright central disk surrounded by several alternately bright dark rings [5,10,11]. This diffraction blur or Airy disk, named in honor of Lord George Biddel Airy; is one of those who analyzed the diffraction process. The energy distribution and the appearance of Airy disk are shown in figure (1-4).

![Figure (1-4)](image)

Airy disk, energy distribution and appearance [5]

If the lens aperture is circular, approximately (84%) of the energy from an image point energy is spread over the central disk and the rest energy is lost in surrounding rings of the Airy pattern [5].
The angular diameter of Airy disk ($B_{\text{ang}}$) which is assumed to be the diameter of the first dark ring is \[5, 10, 12\].

$$B_{\text{ang}} = 2.44 \frac{\lambda}{D} \quad (1-2)$$

The Airy disk diameter $B_{\text{diff}}$ is:

$$B_{\text{diff}} = B_{\text{ang}} f = 2.44 \frac{\lambda f}{D} = 2.44 \frac{\lambda f}{\text{NO.}} \quad (1-3)$$

Where $\lambda$ is light wavelength that is used. The angular diameter is expressed in radians if $\lambda$ and $D$ are in the same units. Since the blur size is proportional to the wavelength as indicated in equation (1-3) the diffraction effect can often become the limiting factor for optical system \[5\].

### 1.4 Aberration

For a perfect lens and monochromatic point source the wave aberrations ($Wa$) measure the optical path difference ($OPD$) of each ray compared with that of the principle ray \[13\].

The wave aberration polynomial in polar coordinates is \[14\].

$$Wa = W(\sigma^2, r^2, r \cos \phi) \quad (1-4)$$

$$Wa = \sum_i \sum_m \sum_j W_{mj} \sigma^i r^m \cos \phi \quad (1-5)$$

Where $(i, m, j)$ represent the power of $(\sigma, r, \cos \phi)$ respectively \[14\].

Where

- $r$ represent the radius distance $B', E'$ in exit plane figure (1-5)
- $\phi$ the angle between the two variable $x, r$.
- $\sigma$ represent the amount of principle ray high on the optical axis in the image plane.
1.4.1 Monochromatic Aberration

The most important aberrations in the majority of application are Seidel aberrations. The aberrations of any ray are expressed in terms of five sums ($S_1$ to $S_5$) called Seidel sums [15].

Seidel was the first one who studied this type of aberration. If a lens is to be free of all defects all five of these sums would be of equal zero. No optical system can be made to satisfy all these conditions once. Therefore it is customary to treat each sum separately, thus, if for a given axial object point the Seidel sum $S_1=0$, there is no spherical aberration at the corresponding image point. If both $S_1=0$ and $S_2=0$, the system will also be free of coma. If, in addition to $S_1=0$ & $S_2=0$ the sums $S_3=0$ and $S_4=0$ as well the images will be free of astigmatism and field curvature. Finally if $S_5=0$, there would be no distortion of the image. These aberrations are also known as the five monochromatic aberrations because they exist for any specified colour and refractive index [16].
1.4.1.1 Spherical Aberration

In paraxial region (with monochromatic light) all rays originating from an axial point again pass through a single point after traversing the system. This is not generally true for larger angle of divergence; different zones of the aperture have different focal length, depending on their distance from the axis. This difference called spherical aberration when the separation of these foci is taken as a measurement of the aberration, it is referred to as longitudinal, where the accompanying spread in the image point is referred to as transverse aberration [4]. The primary spherical aberration seen in figure (1-6).

![Figure (1-6) Spherical aberration [5]](image)

The algebraic formulation of spherical aberration which is even and rotationally symmetric.

\[ W = \sum_{m=0}^{m=\text{even}} W_m r^m = W_{40} r^4 + W_{60} r^6 + ... \]  

(1-6)

In Cartesian coordinates where

\[ x' = r\sin\phi \]
\[ y' = r\cos\phi \]

\[ W(x', y') = W_{40} (x'^2 + y'^2)^2 + W_{60} (x'^2 + y'^2)^3 + ... \]  

(1-7)
Where

\[ W(x', y') \] represents the aberration polynomial.
\[(x', y')\] is the exit pupil coordinates.

The spherical aberration may be minimized for a single lens when the deviation is nearly equal at the two refracting surfaces. The radii of curvature minimize the spherical aberration for an infinitely object where only spherical aberration is considered and the thickness of the lens is neglected as follows:

\[
\begin{align*}
    r_1 &= \frac{2(n + 2)(n - 1)f}{n(2n - 1)} , \\
    r_2 &= \frac{2(n + 2)(n - 1)f}{4 + n - 2m^2}
\end{align*}
\]  \hspace{1cm} (1-8)

where
\[ f \] is the focal length.
\[ n \] is the refraction index of the material.

The spherical aberration can be eliminated by combining two lenses of different glass and opposite parity power [17].

1.4.1.2 Coma

Coma is the first of the lens aberration that appears as the conjugate points moved away from the optical axis [18]. Parallel input beams approaching the lens at an oblique angle is shown in figure (1-7).
The ray at the upper edge of the lens has higher angle of incidence with the curved surface than the ray at the lower edge. The deflection of the upper ray will be greater, and it will intersect the chief ray closer to the lens than the ray from the lower edge [13].

Coma is un rotated axis and given as [14]:

\[
W = \sum_{m=\text{odd}} W_m \sigma r^m \cos \phi + \sigma r^3 \cos \phi + \ldots \quad (1-9)
\]

In Cartesian coordinates the above equation will become as:

\[
W(x', y') = W_{31} (x'^2 + y'^2) y' + \sigma W_{51} (x'^2 + y'^2)^2 y' + \ldots \quad (1-10)
\]

If the axis is rotated by angle \( \psi \) then the coordinates becomes [14]:

\[
X' = x' \cos \psi - y' \sin \psi \\
Y' = x' \sin \psi + y' \cos \psi
\]

then the equation (1-10) takes the form

\[
W(x', y') = W_{31} (x'^2 + y'^2)(x' \sin \psi + y' \cos \psi) + \sigma W_{51} (x'^2 + y'^2)^2 (x' \sin \psi + y' \cos \psi) + \ldots \quad (1-12)
\]

Coma can be controlled by the lens curvature and for an object of no limits the condition to eliminate coma completely are given in [4, 19]:

\[
r_1 = \frac{(n^2 - 1)f}{n^2}, \quad r_2 = \frac{(n^2 - 1)f}{(n^2 - n - 1)}
\]

(1-13)
1.4.1.3 Astigmatism

The word \textit{(Astigmatism)} is derived from the Greek A-means not, and stigma means spot or point \cite{12}. When a narrow beam of light is obliquely incident on reflecting surface, astigmatism is introduced and the image of the point source is formed by small lens aperture becomes a pair of focal lines. Astigmatism in Cartesian is given by \cite{14}:

\[ W(x', y') = W_{22} (x^2 \sin^2 \psi + y^2 \cos^2 \psi + xy \sin^2 \psi) \] \hspace{1cm} (1-14)

Astigmatism is off-axis and asymmetric aberration and it is controlled by lens curvature, by choosing the refractive indexes of the lens components, by which the location of the iris and the refractive surfaces are selected, Astigmatism shown in figure (1-8).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure18.png}
\caption{Astigmatism \cite{5}}
\end{figure}

1.4.1.4 Field Curvature

When a plane surface, normal to the optical axis is imaged by a lens, in which all the above aberrations have been eliminated, the image will not be plane but will lie on curved surface, This image defect is known as curvature of the field, \textit{figure}(1-9)\cite{13}.
Field curvature is given by

\[ W = W_0 \sigma^2 r^2 + \ldots \]  (1-15)

To minimize the aberration relatively strong negative element of low-index glass can be combined with positive elements of high-index glass. The positive and negative elements must be axially separated to provide the lens with useful amount of positive power [13].

**1.4.1.5 Distortion**

Distortion is produced when the chief rays intersect the image surface at heights different from those predicated by the paraxial approximation. The case of pincushion and barrel distortion is shown in figure (1-10) [4, 13].
Distortion is given by:

\[ W = 3 W_{11} \sigma^3 \cos \phi + \ldots \ldots \]  

\[ (1-16) \]

Distortion is sensitive to lens shaping, spacing and iris position and its elimination may require skilful manipulation of these Parameters \[13\].

### 1.4.2 Chromatic Aberrations

The presence of material dispersion causes the refractive index to vary with wavelength \[20\]. There are however other aberrations that arise when this optics is used to transform light containing multiple wavelengths. Chromatic aberrations caused by variation in the index of refraction of the lens material with wavelength. The first two chromatic errors are variation of the paraxial image plane position and image height with wavelength. These are known respectively as longitudinal axial and lateral chromatic aberration. \textit{Figure (1-11)} illustrates these two types of aberrations.
Chromatic aberration is given by [14]:

\[ W_i = W_{i,1} \beta_r \cos \theta + W_{20} r^2 \]  \hspace{1cm} (1-17)

1.5 Strehl Ratio

The ratio of the intensity at the Gaussian image point in the presence of aberration and the intensity that would be obtained if no aberrations are present, is called, the Strehl ratio, or Strehl intensity. Strehl ratio \( (S.R) \) is proportional to the on-axis intensity [21].

Strehl ratio \( S.R \) is expressed as:

\[ S.R = \frac{I(z)}{I_0} \]  \hspace{1cm} (1-18)

\( I(z) \) is the misfocus intensity.
Chapter one

General Introduction

\[ F(u', v') = n.f \int_{y} \int_{x} e^{ikW(x', y')} e^{2\pi i (u'x' + v'y')} dx' dy' \]  \hspace{1cm} (1-19)

where

\((u', v')\) represents the radius coordinates of the image.

\(K\) represents the wave number \(\frac{2\pi}{\lambda}\).

\(n.f\) represents the normalizing factor which is used to make \(F(0, 0) = 1\) [22].

By using polar coordinates

\[ x' = r \sin \phi \hspace{1cm} y' = r \cos \phi \]
\[ u' = p' \sin \psi \hspace{1cm} v' = p' \sin \psi \]  \hspace{1cm} (1-20)

The equation (1-19), for circular aperture with radius is \(r = 1\) and its area equals \(\pi\) in the image plane can be written as:

\[ F(p', \psi) = \frac{1}{\pi} \int_{0}^{2\pi} \int_{0}^{1} e^{i k W(r, \phi)} e^{2\pi i p' r \cos(\psi - \phi)} r dr d\phi \]  \hspace{1cm} (1-21)

To calculate the axial intensity, let \(p' = 0\) and \(W(r, \phi) = W_{20} r^2\), then the equation (1-21) will be in the form [23]:

\[ F(0) = \frac{1}{\pi} \int_{0}^{2\pi} \int_{0}^{1} e^{2\pi i W_{20} r^2 / \lambda} r dr d\phi \]  \hspace{1cm} (1-22)

By solving the integral:

\[ F(0) = \frac{-i \lambda}{2\pi W_{20}} (e^{i 2\pi W_{20} / \lambda} - 1) \]  \hspace{1cm} (1-23)

So that the intensity equals:

\[ G(0) = \left| F(0) \right|^2 = \left| \frac{-i \lambda}{2\pi W_{20}} (e^{i 2\pi W_{20} / \lambda} - 1) \right|^2 \]  \hspace{1cm} (1-24)

After long series of simplification the intensity given as:

\[ G(0) = \frac{\sin^2(\frac{\pi W_{20}}{\lambda})}{(\frac{\pi W_{20}}{\lambda})^2} = \text{sinc}^2(\frac{\pi W_{20}}{\lambda}) \]  \hspace{1cm} (1-25)
If the focus error coefficient $W_{20} = \frac{\lambda}{4}$ the equation (1-25) abbreviates to:

$$G(0) = \frac{8}{\pi^2} \approx 0.8$$

Strehl intensity ratio of 0.8 is good tolerance which is equivalent to the Rayleigh quarter-wavelength criterion [23], for defocused system Marshall has shown that decrees in the Strehl ratio resulting from presence of aberrations is determined by the total variance ($V$) of the wave front aberration $W$, which is define as [24]:

$$V = \overline{W^2} - \overline{W}^2$$

(1-26)

Where the bars denote mean value over the pupil domain and for circular aperture of normalized area equal $\pi$, the variance is:

$$V = \frac{1}{\pi} \int_0^{2\pi} \left[ \int_0^{2\pi} W(r, \phi) r dr d\phi \right]^2 - \frac{1}{\pi^2} \left[ \int_0^{2\pi} \int_0^{2\pi} W(r, \phi) r dr d\phi \right]^2$$

(1-27)

Marshal shows that for small aberrations, ($S.R$) is given by:

$$S.R = \left( 1 - \frac{2\pi^2 V}{\lambda^2} \right)^2$$

(1-28)

When the exit pupil contains annular aperture ($\varepsilon$), then the intensity equals to [25] :-

$$S.R = \left[ 1 - \left( \frac{2\pi^2}{\lambda^2} \right) V \right]^2 \left( 1 - \varepsilon^2 \right)^2$$

(1-29)

1.6 Aberration Balancing

To improve optical system performance, optical designers make sure that the total aberration contribution from all surfaces taken together sums to nearly zero. Normally, such process requires computerized analysis and optimization [26]. We will use the way that Marshal used it, by obtaining the minimum value to mean square derivation consequently, and get the maximum intensity central value according to Strehl criterion [24].
Let takes the primary spherical aberration for axial image:

\[ W(r, \phi) = W_{20} r^2 + W_{40} r^4 \]  

(1-30)

For circular pupil \((x^2+y^2=1)\) and area \((A = \pi)\) Substitution in

**equation** of variance \((1-27)\) as follows:

\[
V = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 |W(r, \phi)|^2 r dr d\phi - \frac{1}{\pi^2} \left[ \int_0^{2\pi} \int_0^1 W(r, \phi)r dr d\phi \right]^2
\]

solving the above equation for primary spherical aberration get:

\[
V = \frac{4}{45} W_{40}^2 + \frac{1}{12} W_{20}^2 + \frac{1}{6} W_{20} W_{40}
\]

(1-31)

The Intensity value becomes maximum when the variance at minimum is:

\[
\frac{\partial V}{\partial W_{20}} = \frac{1}{6} W_{20} + \frac{1}{6} W_{40}
\]

(1-32)

\[
\frac{\partial V}{\partial W_{20}} = 0 \text{ when } W_{20} = -W_{40}
\]

(1-33)

Substitute \(W_{20} = -W_{40}\) in the **equation (1-31)** and entrance Strehl condition, then:

\[
V = \frac{1}{180} W_{40}^2 \leq \frac{\lambda^2}{180}
\]

Then the tolerance value for primary spherical aberration with focus error is:

\(W_{20} \leq 1\lambda, \ W_{40} \leq -1\lambda\)

The best focus occurs when there are optimum balance values (third order aberration and focus error) in the optical system [26].
1.7 Resolution of Optical System

Consider an optical system which images are two equally bright point source of light, each point, is imaged as an Airy disk with the encircling rings, and if the points are close, the diffraction patterns will overlap. When the separation is such that it is just possible to determine that there are two points and not one, the points which are said to be resolved \[27\]. The most widely used value for the limiting resolution of an optical system is Rayleigh's criterion. Rayleigh suggested that the image formed by an aberration free system of two self luminous points of the same brightness may be regarded as resolved if the central maximum of one image falls on the first minimum of the other.

This minimum resolvable separation known as \textit{(limit of resolution)} which is given by \[28\].

\[
S = \frac{0.61\lambda}{n'\sin\alpha'} = 1.22\lambda\left(f / NO.\right) \tag{1-34}
\]

This represents Airy disk \[27\]. If slit aperture has been used Rayleigh's criterion for resolving power the limit of resolution will be in the new form:

\[
S=1.0\lambda\left(f'/\text{NO.}\right) \tag{1-35}
\]

Dawes criterion is used only for circular and annular aperture and gives the separation between point image centers for circular aperture \[27\]:

\[
S=1.02\lambda\left(f'/\text{NO.}\right) \tag{1-36}
\]

Both Raleigh's and Dawes criterions apply only for incoherent source .But Sparrow gives special criterion for resolving power .He showed that the second intensity derivation between half distance of point image center equal zero. This criterion applies on coherent and incoherent source, according to that the separation distance for circular aperture \[29\]:
Chapter one General Introduction

\[ S = 0.947 \lambda (f/NO.) \]  
Incoherent  \( (1-37) \)

\[ S = 1.464 \lambda (f/NO.) \]  
Coherent  \( (1-38) \)

For slit aperture:-

\[ S = 0.829 \lambda (f/NO.) \]  
Incoherent  \( (1-39) \)

\[ S = 1.325 \lambda (f/NO.) \]  
Coherent  \( (1-40) \)

Therefore the resolution power of slit aperture performs more than the circular aperture for the two type of light source (coherent and incoherent) \( [27] \), figure (1-12) represents the criterion discussed.

\[ W(x', y') = W_{20}r^2 = W_{20}(x'^2 + y'^2) \]  \( (1-41) \)

1.8 Focus Error

There are two types of focus error, longitudinal focus shift and transverse focus shift. The following equations represent the longitudinal focal shift and transverse focal respectively:

\[ W(x', y') = W_{20}r^2 = W_{20}(x'^2 + y'^2) \]  \( (1-41) \)
It is possible to reduce these two types of focus error through the search about a best image place; therefore this type of error is not affected.

### 1.9 Depth of Focus

The concept *(depth of focus)* rests on the assumption that for an optical system, there exits blur *(due to defocusing)* of small enough size such that it will not adversely affect that performance of the system. The depth of focus is the amount by which the image may be shifted longitudinally with respect to some reference plane and introduces no more than the acceptable blur. This amount of the shifted image which is corresponding to being out of focus its one quarter wavelength. The depth of focus *(d)* which is corresponding to an optical path difference of \( \pm \frac{1}{4} \lambda \) is \[30].

\[
\delta = \pm 2 \lambda \left( \frac{f}{\text{NO.}} \right)^2 \tag{1-43}
\]

### 1.10 Literature Survey

Many scientists and researchers are interested in the study of the intensity of the image that is formed by the optical system and find the ways and instruments which work on improving the image of optical system and specifying the qualification of the optical systems through specifying goodness of image that is formed by the optical system. Lord George Biddel Airy [31] studied the intensity distribution in image plane of a point source for free optical system. Airy [31] is the first to analyze the diffraction process using circular aperture .Rayleigh [23] studied the effect of wavefront on the point spread function (PSF) . Barakat [33] a counts from the researchers who made their researches on point spread function. There are another researchers who made their researches on optical systems contained aberrations.O'nill [34] and Hopkins [35] were the first of them.
died the effect of object illumination (coherent, incoherent illumination) on the intensity distribution of the image that is formed. Mahajan [26] works on the creation and development of simple pattern to calculate PSF for optical systems that contain asymmetric aberrations and illuminated with incoherent source. O'Neill [40] explains a process which evaluates the intensity distribution of a point image for annular aperture by using 2D-fourier transform. Poon [41] uses Fourier approximation in calculation the intensity distribution around the geometric focus. Gauss [42] was the first to give equivalence between the 1st orders spherical aberration and the 2nd orders spherical aberration in presence of the focus error. Strehl [43] studied the effect of the 1st orders aberrations on the image of a point source and evaluate the optimum balance values. Dufieux [3] studied Fourier transforms technique for optical systems. Hopkins [44] used the simple canonical coordinates in calculation the diffraction pattern in image plane. Barakat [45] calculates the intensity for optical systems contains circular aperture in presence of the spherical aberration and he uses Gauss rule in the numerical analysis. And in another research Barakat [46] calculates the PSF for optical systems contains aberrations.

There are many researches in the field of image apodization by using different kinds of filters. O'Neil [34] and Marechal [47] made their researches on the filters that used coherent light. Luneburg [48] used the optical systems with circular exit pupil and concluded that homogeneous energy distribution on the circular exit pupil in limits of the paraxial rays gives perfect results. There are other researches [49, 50] who used only the paraxial rays. Forskin [51] studied the non-paraxial state. Rao [52] studies the effect of filters forms (pupil's transparency form) on the edge spread function (ESF).
Reddy [53] indicates that the resolution limits increased with presence Lanczos filters. Rao [54] notices that if the optical systems contain Bartlett filters the secondary peaks will be disappeared.

In 1997 Rao [55] studied the modulation transfer function (MTF) by using different filters types. J.campos, J.C.Escalerd and M.J, Yzuel [56] studied the effect of aperture form on characteristic's symmetric image. In the field of calculation the total object energy in image plane Wolf [28] gives a study for calculating the total illumination by using Lommel function. Mahajan [57] calculates the total illumination function for obstruction diffraction- limited system.

### 1.11 Aim of The Work

The aim of this thesis to study the computational image evaluation of multi-circular apertures optical system and this includes the study of:

1. Improvement Resolving Power of optical system design by using multi-circular synthetic apertures.

2. Obtain on values best for (number of subapertures, radial distance and magnification) for this thesis.

3. Compared between system contain from single circular aperture and another system multi circular synthetic aperture.
Chapter Three  
Point Spread Function for Multi Subapertures

3.1 Introduction

The Michelson interferometer is considered as an example to applied synthetic aperture, because it consists of two mirrors [60]. As modern practical proven about synthetic aperture is (infrared aperture syntheses), which used in the (multiple mirrors telescope).

(NASA) performed the multiple apertures technique, figure(3-1) show one of (NASA) aspects for Coherent Optical System of Modular Imaging Collector (Cosmic). Synthetic apertures have great importance because of its multiple applications in radar operation and radio astronomy. The applications and advantages of the optical systems of synthetic apertures, stimulate the researchers to study this optical systems, but some problems appeared in the optical systems like (apertures distribution methods, aperture manufacturing methods, high materialistic costs, weight), but this problems can be solved by gaving some optical performing losses to this optical systems [61].

The optical systems common interesting with synthetic apertures are divided to three main classes:

1. Astronomy observatory which needed for simple field of visibility and limit.
2. Surveying applications which needed for large field of visibility.
3. Laser transmitter band applications and searching targets by using transmitted telescopes [62].
There are additional applications with requirements which are less importance such as its uses in sun energy concentrations [62].

3.2 Derivation of PSF Equation for Row of Circular Synthetic Apertures

Suppose \( f(x,y) \) entrance signal pass throughout optical system , and \( g(x,y) \) is the exit signal from the same optical system , this arrangement could called (the linear optical system) if it could achieved the two following conditions :

1. When multiplying the entrance signal \( f(x,y) \) by constant such as \( a \) , it will give exit signal multiplying with same constant , e.g. \( ag(x,y) \).
2. When entrance signal be a sum of two or more functions \( \{ af_1(x,y) + bf_2(x,y) \} \), the exit signal will be similar of entrance signal itself \( \{ ag_1(x,y) + bg_2(x,y) \} \), where \( f_1(x,y) \) , \( f_2(x,y) \) are resulting \( g_1(x,y) \) , \( g_2(x,y) \) respectively [28].

That’s mean the linear system is \( \text{(space invariant)} \) if it have the \( \text{(property of stationarity)} \). So the exit signal by this optical system could treated as a linear superposition for generated exit signal from all single point of the body, if the linear operation in the system denoted by \( \mathcal{L} \) \( \{ \} \), the exit and entrance function could written:

\[
g(x,y) = \mathcal{L} \{ f(x,y) \} \quad (3-1)
\]

\[
\therefore \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x',y')\delta(x' - x)\delta(y' - y)dx'dy' = f(x,y) \quad (3-2)
\]

\[
\therefore g(x, y) = \mathcal{L} \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x',y') \delta(x'-x)\delta(y'-y) dx'dy' \right] \quad (3-3)
\]

According to second condition, the effect on system became equally effect on all the element function:

\[
\therefore g(x, y) = \mathcal{L} \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x',y') \delta(x'-x)\delta(y'-y) dx'dy' \right] \quad (3-4)
\]

where

\[
\mathcal{L} \{ \delta(x'-x)\delta(y'-y) \} \quad \text{represented the system to delta function equation (3-1)[28].}
\]

Suppose there is monitor contained \( N \) of similar apertures as in \( \text{figure(3-2a)} \) with apertures centers is \( O_1,O_2,\ldots,O_N \) and coordinates are \( (x_1,y_1),(x_2,y_2),\ldots,(x_N,y_N) \), those apertures when rotation the centers kept fixed at system site coordinates \( (x,y) \), so the point \( (x,y) \) in aperture position \( (j) \) have coordinates of \( (x_j+x',y_j+y') \) that's mean:
Franhopher diffraction function given \( F(x,y) \) at point \( (P) \) in image plane as compositions of generated single field from any single aperture, depended on equation (2-10) as the following:

\[
F(x, y) = \sum_{j=1}^{N} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x', y') e^{i k [x(x_j + x') + y(y_j + y')]/R} dx' dy' \tag{3-5}
\]

or

\[
F(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x', y') e^{i k (xx' + yy')/R} dx' dy' \cdot \sum_{j=1}^{N} e^{i k (xx_j + yy_j)/R} \tag{3-6}
\]

Where

\( f(x',y') \) is a single aperture function.

\[ Kx = K X / R = K\sin\Phi = K\cos\beta \tag{3-7} \]

\[ Ky = K Y / R = K\sin\theta = K\cos\gamma \]

Where \( Ky \) & \( Kx \) are spatial frequency or angular space along \( x \) & \( y \) axis as in figure(3-2b).

---

**Figure (3-2)**

A. Formations Multi supaberture

B. Spatial frequency/28/
From equation (3-7) that:

\[ X = R \frac{Kx}{K} \]  

(3-8)

And by substitution (3-8) in (3-6) gives:

\[
F(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x', y') e^{ik(x'R_{k} + y'R_{k})/R} \ dx' \ dy' \ \sum_{j=1}^{N} e^{ikx_j} \cdot e^{iky_j} 
\]

(3-9)

\[
F(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x', y') e^{i(x'R_{k} + y'R_{k})} \ dx' \ dy' \ \sum_{j=1}^{N} e^{i(kx_j)} \cdot e^{i(ky_j)} 
\]

(3-10)

It is easy to notice that equation (3-10) is the same as equation(3-6), but with (spatial frequency) denotation, the integration is the Fourier transform to single aperture function, while the total is the transformation of any row of delta function set where:

\[ \therefore f(x) = \sum_{j} \delta(x - x_j) \]

\[ \therefore f \{ \delta(x - x_j) \} = e^{ikx_j} \]

\[ \therefore f \{ f(x) \} = \sum_{j} f \{ \delta(x - x_j) \} = \sum_{j} e^{ikx_j} \]

\[ \therefore A_{\delta} = \sum_{j} \delta(x - x_j) \cdot \delta(y - y_j) \]  

(3-11)

Where

\[ f \] referred to Fourier transform.

\[ F(x, y) \] is same Fourier transform.

\[ f \{ f(x,y) \} \] for each aperture function in the row.
The physical aspect of equation (3-12) state that the field distribution in Frenhouspher diffraction type for any row of shape and same conducting symmetrical apertures will equal Fourier transformation for any single aperture function (which means field diffraction distribution) multiplying by resulting diffraction type resultant from the sum of source point which is array by form (formation) one (which means the transformation of $A_\delta$) . The complex amplitude in point $(u',v')$ is on image plane (which faced $(\zeta',\eta')$ in figure (2-1)) which could expressed by using (Fourier transform) of (pupil function), as in equation (3-12).

\[
\therefore f(x, y) = \tau(x, y)e^{ikw(x, y)}
\]

Where $\tau(x, y)$ represented the (real amplitude), often it is equal to unity, the calculation will occur latter containing exit pupil, from that it could predicted by existence intensity in image plane, thus it could expressed on the point spread function by the following integral formula:

\[
F(u, v) = f[f(x, y)]
\]

\[
F(u, v) = n.f\int\int f(x, y)e^{ik(x+vy)}dxdy
\]  

\[
\therefore F(u, v) = n.f\sum_{j=1}^{N}\int f(x', y')e^{i2\pi[(x'+x_j)+(y'+y_j)]}dx'dy'
\]  

\[
\therefore F(u, v) = n.f\sum_{j=1}^{N}\int f(x', y')e^{i2\pi(x'+vy')}.e^{i2\pi(x_j+vy_j)}dx'dy'
\]  

\[
\therefore F(u, v) = n.f\int\int f(x', y')e^{i2\pi(x'+vy')}dx'dy'.\sum_{j=1}^{N} e^{i2\pi(x_j+vy_j)}
\]

Since this calculation concern with circular aperture, by entrancing aberration coefficient in equation (3-17) get:
Chapter three 

PSF for Multi Subapertures

\[ F(u,v) = n \cdot f \int_{x} \int_{y} \tau(x,y) e^{jkw[x',y']} e^{j2\pi[u'x'+y'y']} dx' dy' \sum_{j=1}^{N} e^{j2\pi(ux_j+vy_j)} \]  

(3-18)

suppose that \( \tau(x,y) = 1 \)

\[ F(u,v) = n \cdot f \int_{x} \int_{y} e^{jkw[x',y']} e^{j2\pi[u'x'+y'y']} dx' dy' \sum_{j=1}^{N} e^{j2\pi(ux_j+vy_j)} \]  

(3-19)

And by expression on wave number equal to \( 2\pi/\lambda \) and take aberration coefficient by wavelength, equation will be:

\[ F(u,v) = n \cdot f \int_{x} \int_{y} e^{j2\pi[w(x',y')+(ux'+vy')]} dx' dy' \sum_{j=1}^{N} e^{j2\pi(ux_j+vy_j)} \]  

(3-20)

Use the integration limits to include synthetic aperture area as **figure** (3-3):

\[ F(u,v) = n \cdot f \int_{x} \int_{y} e^{j2\pi[w(x',y')+(ux'+vy')]} dx' dy' \sum_{j=1}^{N} e^{j2\pi(ux_j+vy_j)} \]  

(3-21)

**Figure (3-3)**

A. One circular aperture  
B. Subapertures where \( N = 4 \)

Also it may write **equation (3-20)** by triangular functions by using this relationship:

\[ e^{j\theta} = \cos\theta + j\sin\theta \]
We get:

\[ F(u, v) = n \int \int [\cos \{2\pi[w(x', y') + ux' + vy']\} + i \sin \{2\pi[w(x', y') + ux' + vy']\}] dx' dy' \]

\[ \sideset{\sum}{N}_{j=1} \cos \{2\pi(ux_j + vy_j)\} + i \sin \{2\pi(ux_j + vy_j)\} \] 

(3.22)

suppose that: \( m' = 2\pi v \)

\( z' = 2\pi u \)

So that equation will be:

\[ F(z', m') = n \int \int [\cos \{2\pi[w(x', y') + z'x' + m'y'] + i \sin \{2\pi[w(x', y') + z'x' + m'y']\}] dx' dy' \]

\[ \sideset{\sum}{N}_{j=1} \cos \{z'x_j + m'y_j\} + i \sin \{z'x_j + m'y_j\} \] 

(3.23)

It may be enough with one axis in image plane because the similarity intensity distribution on both axis \( m', z' (m' = 0) \) so that the above equation will be:

\[ F(z') = n \int \int [\cos \{2\pi[w(x', y') + z'x'] + i \sin \{2\pi[w(x', y') + z'x']\}] dx' dy'. \]

\[ \sideset{\sum}{n}_{j=1} \cos (z'x_j) + i \sin (z'x_j) \] 

(3.24)

When intensity of fallen light of no coherent, then point spread function PSF is given by function multiplication of \( F(z') \) by its complex conjugate which is:

\[ G(z') = |F(z')|^2 \] 

(3.25)
The sin term is disappeared because function of sin is \textit{(odd function)}.

The physical meaning of \textit{equation (3-27)} state that intensity distribution for row of circular apertures similar in shape equal to intensity distribution of single aperture function multiplied by the resultant of the sum of intensity for points source formed in one kind.

\textbf{3.2.1 PSF for Diffraction - Limited System for Row of Synthetic Apertures:}

Diffraction-limited system means that aberration function equal to zero thin pupil function equal one which is:

\[
f(x,y) = \tau(x,y) = 1
\]  

\text{(3-28)}

By substitution the value \(f(x,y)\) in equation of point spread function, in order to find the value of normalization factor which make function \(G(z')=1\), when \(z' \rightarrow 0 \ [59]\), and \(G(0)=1\), then the intensity function is as follows:

\[
\therefore G(z') = n.f \left[ \int_{-1/N}^{1/N} \int_{-1/N}^{1/N} e^{i\xi' x} d\xi' d\eta' \sum_{j=1}^{N} e^{i\xi' x_j} \right]^2
\]  

\text{(3-29)}
\[ G(\omega) = 1 = n.f \left[ \int_{-\frac{1}{\sqrt{N}}}^{+\frac{1}{\sqrt{N}}} \int_{-\frac{1}{\sqrt{N}}}^{+\frac{1}{\sqrt{N}}} dx' dy' \right]^2 \Rightarrow 1 = n.f \left[ 2 \int_{-\frac{1}{\sqrt{N}}}^{+\frac{1}{\sqrt{N}}} \sqrt{\frac{1}{N} - y^2} dy' \right]^2 \]

\[ 1 = n.f \left[ 4 \int_{0}^{\frac{1}{\sqrt{N}}} \sqrt{\frac{1}{N} - y^2} dy' \right]^2 \]

(3 - 30)

suppose that:

\[ Y = \frac{1}{\sqrt{N}} \sin \theta \Rightarrow dy = \frac{1}{\sqrt{N}} \cos \theta \, d\theta \]

when

\[ Y = 0 \Rightarrow 0 = \frac{1}{\sqrt{N}} \sin \theta \Rightarrow \theta = 0 \]

Also when

\[ Y = \frac{1}{\sqrt{N}} \Rightarrow \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{N}} \sin \theta \Rightarrow \theta = \frac{\pi}{2} \]

By substitution \( Y \) values in the integration limits of equation (3-30), yield:

\[ 1 = 16 \cdot n.f \left[ \int_{0}^{\frac{\pi}{2}} \sqrt{\frac{1}{N} - \frac{1}{N} \sin^2 \theta} \cdot \frac{1}{\sqrt{N}} \cos \theta \, d\theta \right]^2 \]

\[ 1 = \frac{16}{N} \cdot n.f \left[ \int_{0}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta \right]^2 \Rightarrow 1 = \frac{4}{N} \cdot n.f \left[ \int_{0}^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta \right]^2 \]

\[ 1 = \frac{4}{N} \cdot n.f \left[ \theta + \frac{1}{2} \sin^2 \theta \right]_{0}^{\frac{\pi}{2}} \Rightarrow 1 = \frac{4}{N} \cdot n.f \left[ \frac{\pi}{2} \right]^2 \]

\[ \therefore 1 = \frac{\pi^2}{N} \cdot n.f \Rightarrow n.f = \frac{N}{\pi^2} \]

(3 - 31)

In case of the one circular aperture (\( N=1 \)) with area \( \pi \) then the normalization factor will be equal to:

\[ n.f = \frac{1}{\pi^2} \]
3.2.2 PSF with Focus Error for Row of Synthetic Apertures:

Focus error is given in equation (1-41) so point spread function of a row of subapertures in presence of focus error will be as follow:

\[
F(u,v) = n f \left\{ \mathbf{\int} \mathbf{\int} e^{2\pi j (u x + v y)} \left[ \mathbf{\sum}_{j=1}^{N} e^{2\pi j (u x_j + v y_j)} \right] \right\} \mathbf{dxdy} \mathbf{\sum}_{j=1}^{N} e^{2\pi j (u x_j + v y_j)} .
\]

Or

\[
PSF = G(z') = n f \left\{ \mathbf{\int} \mathbf{\int} \cos \{ 2\pi j (x^2 + y^2) + z' x \} \mathbf{dxdy} \mathbf{\sum}_{j=1}^{N} \cos \xi x_j \right\}.
\]

3.2.3 Optimum Balance for Multi Aperture System:

The more important problems in diffraction theory is aberration balancing with different orders. The idea of balancing containing selection of maximum aberration orders set to object or invert the low aberration orders effect, to reduce aberration \[24\], then obtaining on best quantity of images.

The used method was by (Marechal) \[24\], that to find the minimum value of mean square deviation in wavefront, so we gate on maximum value of central intensity according to Strehl criterion.

The wavefront square deviation average (distinctness) could counted by the following relation \[63\]:

\[
V = \frac{1}{A} \mathbf{\int} W^2 \mathbf{dA} \mathbf{\sum}_{\text{area}} W^2 \mathbf{dA} - \left[ \frac{1}{A} \mathbf{\int} W \mathbf{dA} \mathbf{\sum}_{\text{area}} W \mathbf{dA} \right]^2.
\]

Where

- \(A\) pupil area.
- \(W\) aberration function.
- \(dA\) differentiation area element.
At polarity axis used the equation (3.34) will write on the following shape:

\[ V = \frac{1}{A_r} \int_0^\infty \left[ \int_0^{2\pi} W(r,\phi)^2 \, rdrd\phi \right] \, d\phi - \int_0^\infty \left[ \frac{1}{A_r} \int_0^{2\pi} W(r,\phi) \, rdrd\phi \right] \, d\phi \]  \quad (3.35)

Marechal has explained the maximum intensity which light distributes criterion in point source image of system contained low aberration at circular exit pupil using:

\[ I = (1 - 2\pi^2 V / \lambda) \]  \quad (3.36)

### 3.2.4 Optimum Balance for Multi Aperture System with Focus Error

Focus error is given by following relationship:

\[ W = w_{20}r^2 \]  \quad (3.37)

Equation (3.37) is substituted in equation (3.35) give the following:

\[ V = \frac{N}{\pi} \int_0^{2\pi} \left( \int_0^{\sqrt{N}} (w_{20}r^2)^2 \, rdrd\phi \right) \, d\phi - \int_0^{2\pi} \left[ \frac{N}{\pi} \int_0^{\sqrt{N}} (w_{20}r^2) \, rdrd\phi \right] \, d\phi \]  \quad (3.38)

By solving the integrations get:

\[ V = \frac{4NW_{20}^2 - 3W_{20}^2}{12N^4} \]  \quad (3.39)

In case of using Rayleigh condition of greatest intensity \( I = 0.8 \) given in equation (3.35) get:

\[ V \leq \frac{\lambda^2}{180} \]  \quad (3.40)

By substitution V value from equation (3.39) in equation (3.40) give:

\[ \frac{4NW_{20}^2 - 3W_{20}^2}{12N^4} \leq \frac{\lambda^2}{180} \Rightarrow W_{20} \leq \frac{\lambda N^2}{\sqrt{15(4N - 3)}} \]  \quad (3.41)
Which is representing the value of accepted focus error in optical system by means of the number of subapertures \((N)\). In order to check the expression suppose that the number of subapertures equal to one \((N=1)\) so:

\[
W_{20} = \lambda / \sqrt{15} = 0.25 \lambda
\]  (3-42)
2.1 Introduction

For the importance of the point spread function (PSF) in testing and evaluating optical systems, it becomes the dependent measurement to know the optical systems efficiency \[58\]. It has been studied for optical systems with circular aperture, where the (PSF) could evaluate the image efficiency for point object in image plane.

2.2 Point Spread Function (PSF)

The image of point source formed by a lens system is known as the point spread function (PSF) of the lens. Other names for the PSF include (impulse response) , (Green's function) , and (Fraunhofer diffraction pattern) , it is one of the most completed functions for describing the performance of an optical system and can be extended to include the effects of an obstructed aperture, apodization and any factor external to the optical system \[4,58\] . The most fundamental type of objects used in the testing of optical components is point source of light which is effectively of negligibly small dimensions. The PSF is a characteristic of the system under test. When the system contains less aberration, the image of a bright point is a finite disk of light surrounded by a series of weak diffraction rings; only the first is usually bright enough to be visible to the eye. This phenomenon was early expanded by Airy [5]. To calculate the intensity in the image plane for monochromatic point source, suppose that an optical system contains a lens forming an image for a point source, and it is known that all optical systems have aperture in some place in the optical system and it form an image in object space called \(\text{Entrance pupil}\) and that in image space called \(\text{Exit pupil}\) as in figure (2-1) [10].
Chapter two
Point Spread Function

Figure (2-1)
Optical axes, object plane, entrance pupil, exit pupil, image plane /10/

The ray which comes from the point source is spherical and of less aberration with its spherical center in the point source like in figure (2-2).

Figure (2-2)
Diffraction image of a point source /10/

The wave $(E)$ passes through Entrance pupil and the wave $(E')$ passes through exit pupil. The complex amplitude function distributed on the front $(EB)$ is $u_0(x, y)$ and The complex amplitude function distributed on the front spherical reference $(E'B')$ for the center $(o')$ in the image.
space is \( u_0(x', y') \), therefore it could define the pupil function for the optical system as [28].

\[
f(x, y) = \frac{u_0(x', y')}{u_0(x, y)}
\]  

(2-1)

The coordinates \((X, Y), (X', Y')\) are real space coordinates for entrance and exit pupil respectively. The ray heights that pass through the edges of entrance pupil and exit pupil are \((h, h')\) respectively, for less aberration system:

\[ h = y_{\text{max}}, \quad h' = y'_{\text{max}} \]

It could the reduced coordinate to describe the pupil function [28]

\[
x = \frac{X}{h}, \quad y = \frac{Y}{h}
\]

\[
x' = \frac{X'}{h}, \quad y' = \frac{Y'}{h}
\]  

(2-2)

The spherical wave \((EB)\) is empty of aberration and diffraction, it is an ideal and it could be \(u_0(x, y) = 1\) on the other side the wave \((E'D')\) represents the real wave that suffers from the aberrations and diffraction. The wave \((E'B')\) will be represented as spherical reference and the phase difference between the two waves \((E'D', E'B')\) in the aperture \(B'\) is \(k(B'D')\) where \(k = \text{the wave number}\). Where the path difference \((B'D')\) between the two waves represents the aberration function for coordinates \((x', y')\) at the exit pupil therefore [28].

\[ W(x', y') = B'D' \]  

(2-3)

The pupil function can be written as:-

\[
f(x, y) = \tau(x, y)e^{ikW(x, y)}
\]  

(2-4)

Using the polar coordinates forms :-

\[ r = \sqrt{x'^2 + y'^2}, \quad \phi = \tan^{-1}\left(\frac{y'}{x'}\right) \]
Then the equation (2-4) will take the new form as:

\[ f(x,y) = \tau(r',\phi') e^{i\kappa(r',\phi')} \]

(2-5)

\( \tau(r',\phi') \) and \( \tau(x',y') \) represents the real amplitude function distributed in exit pupil and it is called \emph{(pupil transparency)} or called \emph{(Transmission function)} and it taken different values according to its use, using the reduced coordinates of the object, the reduced coordinates is defined as \[28\].

\[ u = \left\{ \frac{n \sin \alpha}{\lambda} \right\} \xi, v = \left\{ \frac{n \sin \alpha}{\lambda} \right\} \eta \]

(2-6)

and the reduced coordinates of the image:

\[ u' = \left\{ \frac{n \sin \alpha}{\lambda} \right\} \xi', v' = \left\{ \frac{n \sin \alpha}{\lambda} \right\} \eta' \]

(2-7)

Where \( \eta', \xi', \eta, \xi \) represent the coordinates in the image space and object space respectively where:

\[ \sin \alpha = \frac{h}{R} \text{ and } \sin \alpha' = \frac{h'}{R'} \]

\( R, R' \) represent the radii of spherical waves in the entrance and exit pupil respectively, therefore it can express PSF in its integral form as \[28\].

\[ F(u',v') = \frac{1}{A'} \iint f(x,y) e^{i2\pi(u'x + v'y)}dxdy \]

(2-8)

Where

\( A' \) represents the area of exit pupil .

\( F(u',v') \) represents the complex amplitude function .

The equation's limit can be defined from the circular aperture of normalized area \((\pi)\) and radii equal \((1)\) as:-
Chapter two

Point Spread Function

\[ f(x, y) = \begin{cases} \tau(x, y)e^{jW(x, y)} & x^2 + y^2 \leq 1 \\ 0 & x^2 + y^2 > 1 \end{cases} \quad (2-9) \]

Let \( m' = 2\pi v' \), \( z' = 2\pi u' \) equation (2-8) takes the new form [28].

\[ F(z', m') = \frac{1}{A'} \int \int f(x, y)e^{j(z'x + m'y)} \, dx \, dy \quad (2-10) \]

The intensity \( G'(u', v') \) can be expressed as:

\[ G'(u', v') = F(u', v')(u', v') \quad (2-11) \]

where

\[ F^*(u', v') \quad \text{the complex conjugate.} \]

\[ G(u', v') = n.f \left| \int \int f(x, y)e^{2\pi j(u'x + v'y)} \, dx \, dy \right|^2 \quad (2-12) \]

or

\[ G(z', m') = n.f \left| \int \int f(x, y)e^{j(z'x + m'y)} \, dx \, dy \right|^2 \quad (2-13) \]

The intensity distribution on the two axis \((z', m')\) are symmetric, so it can reduce to one axis only.

Let \( m' = 0 \) therefore equation (2-13) will take the form:-

\[ G(z') = n.f \left| \int \int f(x, y)e^{jz'x} \, dx \, dy \right|^2 \quad (2-14) \]
2.2.1 PSF for Circular Aperture.

2.2.1.1 PSF for a Diffraction-Limited System

If the optical system is of less aberration it is called "diffraction-limited system". For more simplification let \( \tau(x, y) = 1 \), therefore \( f(x, y) = 1 \), then the intensity function is as follows:

\[
G(z') = n.f \left| \int_{-1}^{1} \int_{-1}^{1} e^{iz'x} \ dx \ dy \right|^2
\]  
(2-15)

To find the normalization factor that made \( G(z')=1 \) when \( z' \to 0 \), i.e. equation (2-15) equal (1).

\[
1 = n.f \left| \int_{-1}^{1} \int_{-1}^{1} e^{iz'x} \ dx \ dy \right|^2
\]  
(2-16)

When \( z' \to 0 \), \( e^{iz'x} = 1 \)

\[
1 = n.f \left| \int_{-1}^{1} \int_{-1}^{1} dx \ dy \right|^2
\]  
(2-17)

\[ n.f = \frac{1}{\pi^2} \]

Substituting the normalization factor \( (n.f) \) in equation (2-14) the equation will take the new form:

\[
G(z') = \frac{1}{\pi^2} \left| \int_{-1}^{1} \int_{-1}^{1} \tau(x, y) e^{iz'x} \ dx \ dy \right|^2
\]

For a diffraction-limited system:

\[
G(z') = \frac{1}{\pi^2} \left| \int_{-1}^{1} \int_{-1}^{1} \tau(x, y) e^{iz'x} \ dx \ dy \right|^2
\]  
(2-18)
\[
G(z') = \frac{1}{\pi^2} \left| \int_{-1}^{1} \int_{-1}^{1} \tau(x, y) \left[ \cos(z'x) + j \sin(z'x) \right] dxdy \right|^2 \tag{2-19}
\]

\[
G(z') = \frac{1}{\pi^2} \left[ \int_{-1}^{1} \int_{-1}^{1} \tau(x, y) \cos(z'x) dxdy \right]^2 + \frac{1}{\pi^2} \left[ \int_{-1}^{1} \int_{-1}^{1} \tau(x, y) j \sin(z'x) dxdy \right]^2 \tag{2-20}
\]

\[
\sin(z'x) \text{ is odd function, therefore (second) part of the integral will be negligible, and the equation will be:}
\]

\[
G(z') = \frac{1}{\pi^2} \left[ \int_{-1}^{1} \int_{-1}^{1} \tau(x, y) \cos(z'x) dxdy \right]^2 \tag{2-21}
\]

**Equation (2-21)** represents PSF for a diffraction-limited system.

By using the polar coordinates where:

\[
v = p \cos \psi \quad , \quad u = p \sin \psi
\]

\[
p = \sqrt{u^2 + v^2} \quad , \quad \psi = \tan^{-1} \left( \frac{v}{u} \right)
\]

Where

- \( \Psi \) the angle in polar coordinates.
- \( P \) polar distance from the plane center.

**Equation (2-8)** takes the new form:-

\[
F(p, \psi) = n.f \int_{0}^{2\pi} \int_{0}^{1} \tau(r, \phi) e^{jkW(r, \phi)} e^{i2\pi pr \cos(\psi - \phi)} rdrd\phi \tag{2-22}
\]

For a less aberration system and \( \tau(x, y) = 1 \) **equation (2-22)** is solved as:-

\[
F(\ p \ , \ \psi \ ) = \frac{2J_1(2\pi p)}{2\pi p} \tag{2-23}
\]

where \( J_1(2\pi p) = 1st \) order Bessel function and the intensity is given as:

\[
G(p, \psi) = \left| F(p, \psi) \right|^2 = \left[ \frac{2J_1(2\pi p)}{2\pi p} \right]^2 \tag{2-24}
\]

**Equation (2-24)** represents the intensity in Airy diffraction.
2.2.1.2 PSF with Focus Error

The spherical aberrations polynomial is:

\[ W(x, y) = W_{20}(x^2 + y^2) + W_{40}(x^2 + y^2)^2 + W_{60}(x^2 + y^2)^3 + \ldots \]  \hspace{1cm} (2-25)

If the optical system does not contain spherical aberration and when focus axial is moved from focus plane to another therefore the equation of spherical aberration gives inductively the focus error \( W_{20}(x^2 + y^2) \). When pupil function \( f(x, y) = \tau(x, y)e^{ikW(x, y)} \) is substituted in equation (2-14) gives:

\[
G(z') = \frac{1}{\pi^2} \left| \int_{\frac{-1}{\sqrt{1-y^2}}}^{\frac{1}{\sqrt{1-y^2}}} \int_{\frac{-1}{\sqrt{1-x^2}}}^{\frac{1}{\sqrt{1-x^2}}} \tau(x, y)e^{ikW(x, y)} e^{i\varphi'x} dxdy \right|^2 \]  \hspace{1cm} (2-26)

\[
G(z') = \frac{1}{\pi^2} \left| \int_{\frac{-1}{\sqrt{1-y^2}}}^{\frac{1}{\sqrt{1-y^2}}} \int_{\frac{-1}{\sqrt{1-x^2}}}^{\frac{1}{\sqrt{1-x^2}}} \tau(x, y) \left[ \cos(kW(x, y) + z'x) + j \sin(kW(x, y) + z'x) \right] dxdy \right|^2 \]  \hspace{1cm} (2-27)

where \( \sin(kW(x, y) + z'x) \) is an odd function therefore

\[
G(z') = \frac{1}{\pi^2} \left| \int_{\frac{-1}{\sqrt{1-y^2}}}^{\frac{1}{\sqrt{1-y^2}}} \int_{\frac{-1}{\sqrt{1-x^2}}}^{\frac{1}{\sqrt{1-x^2}}} \tau(x, y) \cos(kW(x, y) + z'x) dxdy \right|^2 \]  \hspace{1cm} (2-28)

In substitution the value of the wave number \( (k = \frac{2\pi}{\lambda}) \) and regarding the aberrations function measured in unit of wave number we shall get:

\[
G(z') = \frac{1}{\pi^2} \left| \int_{\frac{-1}{\sqrt{1-y^2}}}^{\frac{1}{\sqrt{1-y^2}}} \int_{\frac{-1}{\sqrt{1-x^2}}}^{\frac{1}{\sqrt{1-x^2}}} \tau(x, y) \cos(2\pi W_{20}(x^2 + y^2) + z'x) dxdy \right|^2 \]  \hspace{1cm} (2-29)

By using equation (2-22) and substituting the value of focus error \( W_{20}r^2 \) would get:-

\[
F(p, \psi) = \frac{1}{\pi} \int_{0}^{\frac{2\pi}{r}} \int_{0}^{\frac{2\pi}{r}} \tau(r, \phi)e^{ikW_{20}r^2} e^{i2\pi r \cos(\psi - \phi)} rdrd\phi \]  \hspace{1cm} (2-30)
Let \( p = 0 \) along the optical axis then :-

\[
F(0,0) = \frac{1}{\pi} \int_{r=0}^{1} \int_{\phi=0}^{2\pi} \tau(r, \phi)e^{ikW_{20}r^2} r\,dr\,d\phi
\]

\[
\text{(2-31)}
\]

For more simplification \( \tau(r, \phi) = 1 \)

\[
F(0,0) = e^{i\pi W_{20} / \lambda} \sin \left( \frac{\pi W_{20}}{\lambda} \right) \left( \frac{\pi W_{20}}{\lambda} \right)
\]

\[
\text{(2-32)}
\]

the intensity will be:

\[
G(0,0) = \left[ \sin \left( \frac{\pi W_{20}}{\lambda} \right) \right]^2
\]

\[
G(0,0) = \text{sinc}^2 \left( \frac{\pi W_{20}}{\lambda} \right)
\]

\[
\text{(2-33)}
\]

The axial intensity go to zero when the focus error takes the values \( (W_{20} = \pm \lambda, \pm 2\lambda, \pm 3\lambda) [44]. \)
Chapter Four                   Numerical Calculation Results & Discussion

Chapter four
Numerical Calculation, Results and Discussion

4.1 Introduction

The equations which containing on triangle functions and aberration, it couldn't be solved by using the normal methods, thus a numerical integration methods is used to solve this equations, these methods are (Trapezoidal rule), (Simpson rule) which has the distribution of points will be homogeneous along the integration limits, (Gauss Quadrature method) which considered as the numerical methods more positively, which has the distribution of points will be inhomogeneous on integration limits.

The numerical integration methods are divided by points type distribution to two parts:

First: The numerical integration methods with equal distances between points, the points distribution are homogeneous along the integration interval, some kinds of these methods are Trapezoidal, Simpson, Filon and others.

Second: Integration methods with Inhomogeneous distribution of point’s, which often having different weights factors. The values of those points and weights listed in special tables for number of points across integration interval, kinds of these method are Gauss, Filon and others.

Although of the complex which contained Gauss method comparing with integration methods of points homogenous distribution, it considered the suitable method to solve equation which containing oscillation function like (cos, sin) function, which gave high accuracy in counting those function [64].
4.2 Field Angle

During this research, it was notice clearly that the most factor which effected the equations concern with the present calculation and its results which were compared with the previous results, this factor was field angle ($\theta$) which effected on (focus error) and entered in most of equations as the following scheme.

$$\text{PSF} \quad \theta \Rightarrow W_{20} \uparrow \Rightarrow \text{Strehl ratio} \downarrow \Rightarrow I(x,y)$$

The aberration effect in all optical systems except of (typical) is appeared clearly throughout focus error which generated as result of the variation in field angle.

This research explained the effect of field angle on (PSF) equations for [(one circuler aperture), (different numbers of circuler apertures), (free aberration systems) and (systems contained of focus error)].

It is notice that changing set of effected elements in field angle like [(number of subapertures, magnification, radial distance)] led to change in value of PSF.

4.3 Mathematical Relation Between Field Angle & PSF

This relation is considered as important one, which is studied in this research because it could control throughout the main equation (2-10) PSF which could adding or deleting aberration to it.

$$F(u', v') = \frac{1}{A'} \int \int f(x, y) \cdot e^{j2\pi(u'x + v'y)} \, dx \, dy$$

The singel circular aperture with limited boundaries (idealistic) could be written with its area ($\pi$) by equation (2-18):
\[ G(z') = \frac{1}{\pi} \left[ \int_{-1}^{1} \int_{-\sqrt{1-z'^2}}^{\sqrt{1-z'^2}} \tau(x, y) e^{i z' x} \, dx \, dy \right]^2 \]

There is no aberration in this equation e.g. there is no relation to connect between field angle & PSF at idealist case it infrequent happened, at normal case it contained aberration which been whither of first degree third or fifth, all cases containing focus error \((W_{20})\). To explain the aberration rewrite the aberration function equation:

\[ W(x,y) = W_{20}(x^2+y^2)+W_{40}(x^2+y^2)^2+W_{60}(x^2+y^2)^3 \]  

(4-1)

Thus the equation of PSF of single circular aperture of system could wrote contained focus error:

\[
G(z') = \frac{1}{\pi^2} \left[ \int_{-1}^{1} \int_{-\sqrt{1-z'^2}}^{\sqrt{1-z'^2}} \tau(x,y) \cos(2\pi W_{20}(x^2+y^2)+z'x) \, dx \, dy \right]^2 \\
+ \left[ \int_{-1}^{1} \int_{-\sqrt{1-z'^2}}^{\sqrt{1-z'^2}} \tau(x,y) \sin(2\pi W_{20}(x^2+y^2)+z'x) \, dx \, dy \right]^2
\]  

(4–2)

Since focus error is existing in this equation, thus it may write in following shape:

\[ W_{20} = \left[ D / (16 f / No.) \right] \left[ (1 - m) / m \right] \theta^2 \]  

(4–3)

Where

- \( D \) Primary mirror diameter
- \( f / No. \) Focal number
- \( m \) Telescope magnification
- \( \theta \) Field angle
4.4 Gauss method

Gauss method which is considered as application of numerical integration of Legendre polynomial $P_N^{-}(x)$, where Gauss was proved that if function $f(t)$ represented limitation of $(2N-1)$ degree or less, for that:

$$\int_{-1}^{1} f(t) dt = \sum_{i=1}^{N} W_i f(t_i) \quad (4-4)$$

Where

$W_i$ Gauss weights factors which are set of numerical values depended on number of points $N$ used, it could counted them throughout equation containing Legendre sequence.

$t_i$ Represents Legendre sequence root of $N$ degree that $P_N^{-}(t_i)$.

In reference [65] a weight values are listed, special points in this method is arrived to $N^{-96}$.

Dr.Al-Hamdani [66] has referred to perfect number in Gauss points (distributing of twenty of selected Gauss points on two axis $x,y$ in exit aperture) which gave accurate results of Simpson method, also he referred that the effect of exit aperture points distribution on results accuracy, he suggested perfect distribution of points in circular aperture, figure (4-1) explained the circular aperture integration limits with perfect distribution of Gauss points on exit aperture, where short of distance on axis $x$ contained less number of points, long distance contained on more number of points, that to reduce time of counts which increased by Gauss points increasing. Since the accuracy is also increasing with increasing number of the Gauss points, thus the balance operation has performed between accuracy and accounts time. The perfect distribution and perfect number in Gauss points were selected as what the reference [67] has referred to.
Where the integration limits of function $f(x)$ is $[a,b]$ , it could enter the compensation:

$$x_i = \frac{b - a}{2} t_i + \frac{b + a}{2}$$

For that, the equation (4-4) could written by the following formula:

$$\int_{a}^{b} f(x) dx = \frac{b - a}{2} \sum_{i=1}^{\tilde{N}} W_i f(x_i) \quad (4 - 5)$$

### 4.5 Numerical Evaluation of PSF for a Diffraction-Limited System (One Circular Aperture)

Single circular aperture of optical system equation in Gauss term could be written:

$$PSF = G(z', m') = n.f \left[ \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \exp(i(z'x + m'y)) dx dy \right]^2 \quad (4 - 6)$$

Now separate the real and imaginary parts, the above equation can be written as:
It is possible to take only one coordinate in the image plane because of the similarity in the intensity distribution in the two coordinates \(u \& v\) (i.e. put \(v=0\)) then equation (4-7) becomes:

\[
G(z') = n.f \left[ \int \int \cos(z'x + m'y) dx dy + \int \int j \sin(z'x + m'y) dx dy \right]^2
\]  

(4 – 7)

In terms of Gauss's equation:

\[
G(z') = n.f \left[ \sum_{i=1}^{20} H_i \sqrt{1 - y^2} \sum_{j=1}^{20} H_j \cos m \right]^2 + \left[ \sum_{i=1}^{20} H_i \sqrt{1 - y^2} \sum_{j=1}^{20} H_j \sin m \right]^2
\]  

(4 – 8)

Where

\[
m = k(uxi + vyi) \\
x_i = \sqrt{1-y^2} \\
t_i y_j = t_j
\]

Here, the integration was carried out over the exit aperture circular of the form \(xy = \pi\), and twenty points of Gauss points were chosen along the \(x \& y\) axes, the increase in the number of points in the integral necessitates a considerable increase in computation time for a computer program, so for most compromise between accuracy and computation time for most cases.

### 4.6 Numerical Evaluation of PSF with Focus Error

(One Circular Aperture)

If the system is empty of any aberration, when multiply focus plane displacement to another focus plane, then the aberration equation is given by the focus error i.e. \(W_{20}(x^2+y^2)\), as equation (2-13):
\[ PSF = G(z') = n.\int \int f(x, y).e^{i\omega x} \, dx \, dy \]

\[ PSF = G(z') = n.\int \int \tau(x, y).e^{ikw(x, y)}.e^{i\omega x} \, dx \, dy \]

\[ G(z') = n.\int \int \tau(x, y).[\cos(kW(x, y) + z'x) + i \sin(kW(x, y) + z'x)] \, dx \, dy \]

By substitution of wave number value \( k = \frac{2\pi}{\lambda} \) and considering that:

\[ G(z') = n.\int \int \tau(x, y).\cos(2\pi W_{20}(x^2 + y^2) + z'x) \, dx \, dy \]

This equation representing insertion of focus error factor which could be written with Gauss term as following:

\[ G(z') = n.\int \int \tau(x, y).\cos(2\pi W_{20}(x^2 + y^2) + z'x) \, dx \, dy \]
4.7 Effect of The Field Angle on Resolving Power for (One Circular Aperture)

There are many variables in field angle equation, basically the main interest of the present study to find the effect of the change in the apertures number and comparing with effect of the single circular aperture. Also showed the variance effect of other factors on resolving power, thus interested with studying of all variables in equations (4-8) & (4-10) which effected in resolving power, which are:

**Effect of radial distance (D) on Resolving Power:**

For studying the (radial distance) effect on resolving power, substitute radial distance values which are (0.25,0.5,2,4,8) for each value of magnification values in PSF equation for one circular apertures. We executed to plot the figures (4-28), (4-29), (4-30), and (4-31) throughout values obtained by special program.

In these figures it is notice that there are variances in resolving power comes from change radial distance values with constant of other variable values.

In figure (4-28) the best resolving power with D=8, so it is best image clearance in this case, whenever decreasing in radial distance value there is decreasing in resolving power. As in figure (4-29) when D=0.5 the resolving power being better than D=0.25. Figures (4-30) & (4-31) explained that, So it may conclude that whenever decreasing in D value from 8 to 0.25, the resolving power will decrease, best state of resolving power being at D=8.
(b) Effect of magnification(M) on Resolving power :

The effect of the magnification on resolving power has been studied through substituting of magnification values by (0.05,0.1,0.2,0.4) for each of radial distance values that be used in PSF equation of one circular aperture of the same program. *Figures (4-48) (4-49) (4-50) (4-51) & (4-52)* show the values obtained by the programs.

In these figures there is change in resolving power with radial distance constant value for one circular aperture came as result of the variance of magnification.

When $M=0.4$ the resolving power would have large decease as in *figure (4-48)*, and increasing at $M=0.1,0.2$ as in *figure (4-49) & (4-50)* the best resolving power could obtained at $M=0.05$ in *figure(4-51)&(4-52)*, which is best clarity of image, so each increasing of $M$ there is decreasing in resolving power. So the resolving power for optical system with single circular power change if we change in values the radial distance and magnification.

4.8 The Numerical Evaluation of PSF for a Diffraction-Limited System (Multi circular apertures)

The *equation (3-29)* may rewrite with Gauss term for synthetic apertures optical system as following:
Chapter Four
Numerical Calculation Results & Discussion

\[ PSF = G(z') = n \cdot f \left[ \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} e^{i\xi x'} \, dx' \, dy' \cdot \sum_{j=1}^{N} e^{i\xi x_j} \right]^2 \]

Since

\[ Y_j = \frac{b - a}{2} t_i + \frac{b + a}{2} \]
\[ = \frac{1}{\sqrt{N}} t_i \]

Therefore \( Y_j \) value equal :

\[ Y_j = \frac{1}{\sqrt{N}} t_i \quad \text{(4 - 11)} \]

Since :

\[ x_i = \frac{b - a}{2} t_i + \frac{b + a}{2} \]
\[ = \frac{1}{\sqrt{N}} t_i + \frac{1}{\sqrt{N}} y_j^2 t_i \]

So \( x_i \) value equal :

\[ x_i = \frac{1}{\sqrt{N}} - y_j^2 t_i \quad \text{(4 - 12)} \]

By using \textit{equation}(4-4) obtaining :

\[ \therefore F(u, v) = n \cdot f \left[ \left( \frac{1}{\sqrt{N}} \right) + \left( \frac{1}{\sqrt{N}} \right) \sum_{j=1}^{20} W_j \right] \left( \frac{1}{\sqrt{N}} - y_j^2 \right) \sum_{i=1}^{20} W_i \cos 2\pi(ux_i + vy_j). \]
\[ \sum_{j=1}^{20} \text{Exp} \{2\pi(ux_j + vy_j)\} \right] \]

\[ \text{Or by other term :} \]

\[ \therefore F(u, v) = n \cdot f \left[ \left( \frac{1}{\sqrt{N}} \right) \sum_{j=1}^{20} W_j \left( \frac{1}{\sqrt{N}} - y_j^2 \right) \sum_{i=1}^{20} W_i \cos 2\pi(ux_i + vy_j). \]
\[ \sum_{j=1}^{20} \cos 2\pi(ux_j + vy_j) + i \sin 2\pi(ux_j + vy_j) \right] \quad \text{(4 - 14)} \]
4.9 Numerical Evaluation of PSF with Focus Error
(Multi Circular Apertures)

The equation (3-32) is written with Gauss term as following:

\[
F(u, v) = n.f \left[ \frac{1}{N} \sum_{j=1}^{20} W_j \left( \frac{1}{N} - y_j^2 \right) + \frac{1}{N} \sum_{i=1}^{20} W_i \cos \right] [2\pi(x_i^2 + y_i^2) + (ux_i + vy_i)]^2
\]

Or by other shape:

\[
F(u, v) = n.f \left[ \sum_{j=1}^{20} W_j \left( \frac{1}{N} - y_j^2 \right) \sum_{i=1}^{20} W_i \cos \left[ 2\pi(x_i^2 + y_i^2) + (ux_i + vy_i) \right] \right]^2
\]

Because of symmetry, we satisfying with only one axes by letting \( V=0 \), thus the equation (4-16) be:

\[
F(u, v) = n.f \left[ \sum_{j=1}^{20} W_j \left( \frac{1}{N} - y_j^2 \right) \sum_{i=1}^{20} W_i \cos \left[ 2\pi(x_i^2 + y_i^2) + (ux_i + vy_i) \right] \right]^2
\]

By supposing that \( Z' = 2\pi u \), the equation (4-17) became:

\[
F(z') = n.f \left[ \sum_{j=1}^{20} W_j \left( \frac{1}{N} - y_j^2 \right) \sum_{i=1}^{20} W_i \cos \left[ 2\pi(x_i^2 + y_i^2) + z'x_i^2 \right] \right]^2
\]
4.10 Effect the field angle on Resolving Power for  
(Multi-Circular apertures)

To design optical system of multi synthetic circular apertures with high resolving power. The relationship between PSF and field angle and it effect on image enhancement must be studied, which is show that any change in field angle value lead to change in resolving power of optical system.

In the present study, all variables in equations (4-14)&(4-18) effect on field angle such as aperture number radial distance and magnification were studied deeply.

A program in Q.Basic language has been written to calculate this equations and finding best value of these variable which can be used in designing an optical system of high resolving power.

Another effect on resolving power must be explain, it is aperture distribution, the configuration from separation apertures to another by equal distance to double diameter synthetic apertures [the distance from synthetic aperture center to coordinate axis center (the origin main aperture center divided to synthetic aperture) equal to double diameter of synthetic aperture] and figures from (4-2) to (4-22) show the distribution of synthetic aperture.

The following is the discussion of the figures:

(a) Effect the number of subaperture (N) on Resolving Power:

To obtain suitable apertures numbers for optical system, an even apertures numbers (4,6,8,10) are used for each radial distance value to get enhancement image. Figures (4-23) (4-24) (4-25) (4-26) & (4-27) represented resolving power for different aperture number and constant radial distance. Figure (4-23) shows the radial distance fixed at (0.25), the resolving power is increased when the number of subaperture increased compared to single circular aperture N=1, Figure (4-24) shows
same think when radial distance = 0.5. From these two figures the resolving power increasing when increasing subaperture numbers and the radial distance is increasing, at **Figures (4-25)&(4-26)** when D=2,4 respectively, we will find closely values of resolving power for each N=1,2,4, and N=6,8, but N=10 stayed as best value. **Figure (4-27)** we noticed low clearance, whenever the apertures number is increasing there is increasing in resolving power, thus obtaining best optical system image be at increasing N.

(b) **Effect of radial distance (D) on Resolving Power:**

To obtain best image of the optical system, by increasing aperture number, image quality will be increasing, the present study considered aperture number of N (4,6,8,10)λ, with better radial distance value of D=0.25, 0.5, 2, 4, 8 to get best value of D.

At N=4 of **figures (4-32) (4-33) (4-34) & (4-35)** the value of magnification is fixed with changing the value of D. It is noticed that whenever D value increased the resolving power will increase. The best resolving power in D=8, to be sure of that let N=6,8 as in **figures (4-36) (4-37) (4-38) (4-39) & (4-40) (4-41) (4-42) (4-43)** respectively. the large value obtained in the radial distance equal N=8, but **Figures (4-44) (4-45) (4-46) & (4-47)** for N=10, show the resolving power increased if the radial distance increased.

(c) **Effect of magnification (M) on resolving power:**

By studied the effect of apertures numbers and radial distance, it was obtained high resolving power, but to increasing that resolving power it must study the third effect factor which is magnification by values M=0.05, 0.1, 0.2, 0.4, in **figures (4-53) (4-54) (4-55) (4-56) & (4-57)** the aperture numbers N=4 and fixed value of D with all M value. It is noticed that the best value of resolving power is being at M=0.05, then M= 0.1, 0.2 and minimum value at M=0.4. and N=6 the procedure is
Chapter Four

Numerical Calculation Results & Discussion

repeated, it is noticed that high resolving power at $M=0.05$ but is equal to resolving power for $N=4$, as in figures (4-58) (4-59) (4-60) (4-61) & (4-62) for all values of $D$. At $N=8,10$ as in figures (4-63) (4-64) (4-65) (4-66) & (4-67) for $N=8$ and (4-68) (4-69) (4-70) (4-71) & (4-72) for $N=10$ the best resolving power being at $M=0.05$ then $0.1$, began start to increasing all value of $D$ is increased to obtain on high resolving power.

Throughout these figures which are obtained by the mathematical program and comparison with numerical evaluation, it could be obtained on best optical system image at increasing the apertures numbers, increasing the radial distance value and decreasing magnification value. Best resolving power at $N=4,8,10$, $D=8,6,2$ and $M=0.05,0.1$. But it should notice that resolving power being less at $N=8,10$, $D=6,2$, $M=0.1$, because the apertures distribution being at best $N=4$, $D=8$, $M=0.05$.

4.11 Effect of Apertures Number on Focus Error

Throughout the mathematic relation between field angle and focus error, shows the effect of apertures numbers on focus error. and the figures which have mentioned represented changing of point spread function PSF with field angle values and different numbers of synthetic apertures ($N=4,6,8,10$), also single aperture ($N=1$), we notice that the subapertures number increasing led to large and clear improving in point spread function, such as PSF value be for single aperture at focus error existing with value of $W_{20}=0.25\lambda$ is $0.810647$, also noticing the focus error being in $W_{20}=\lambda$ for fourth subapertures gave same value of intensity $(0.810647)$, also obtaining on same intensity at focus error by $W_{20}=1.5\lambda$ at six subapertures obtaining on same intensity at focus error $(W_{20}=2\lambda)$ for eighth state of subapertures and same intensity the focus error be $(W_{20}=2.5\lambda)$ at ten subaperture.
It could conclude that the increasing in apertures numbers by half led to obtain same intensity at increasing of aberration by one, thus it could found the relation between focal error and apertures number which is:

\[ W = \frac{N}{2} \lambda \]

Throughout same mentioned figures, the distance from coordinates centre \((\text{original point})\) and subaperture centre \((R)\), should be equal to subaperture double radius, if it decreased the ten subapertures distribution would be interfering and the shape of aperture being variance to subject of thesis.

That is clear in figures which showed aperture distribution for that notice in this thesis, that changing in distance between subapertures to took on best results, which showed distance between subapertures numbers to obtain on best image in the multi apertures optical system.

So it must choose a suitable distance between subapertures to avoid the apertures interference, changing tap and subapertures form.
Figure (4.2)
Illustrates the position of a single aperture
(N=1 & R=1)
Figure (4-3)
Illustrates the position of a single aperture
(N=1 & R=2)
Figure (4 - 4)
Illustrates the position of a single aperture
(N=1 & R=4)
Figure (4-5)
Illustrates the position of a single aperture
(N=1 & R=6)
Figure (4-6)
Illustrates the position of a single aperture
(N=1 & R=8)
Figure (4-7)
Illustrates the distribution of multi-subaperture (N=4 & R=2)
Figure (4-8)
Illustrates the distribution of multi-subaperture (N=4 & R=4)
Figure (4.9)
Illustrates the distribution of multi-subaperture
(N=4 & R=6)
Figure 4-10
Illustrates the distribution of multi-subaperture
(N=4 & R=8)
Figure (4-11)
Illustrates the distribution of multi-subaperture 
(N=6, R=2)
Figure (4-12) Illustrates the distribution of multi-subaperture (N=6 & R=4)
Figure (4-13)
Illustrates the distribution of multi-subaperture
(N=6 & R=6)
Figure (4-14)
Illustrates the distribution of multi-subaperture
(N=6 & R=8)
Figure 4.15
The distribution of multi-subaperture
(N=8 & R=2)
Figure 4-16
The distribution of multi-subaperture
(N=8 & R=4)
Figure (4.17)
The distribution of multi-subaperture
(N=8 & R=6)
Figure (4 - 18)
The distribution of multi – subaperture
(N=8 & R=8)
Figure (4-19)
The distribution of multi-subaperture
(N=10 & R=2)
Figure (4-20)
The distribution of multi-subaperture
(N=10 & R=4)
Figure (4.21)
The distribution of multi-subaperture
(N=10 & R=6)
Figure (4-22)
The distribution of multi-subaperture
(N=10 & R=8)
Chapter Four  
Numerical Calculation Results & Discussion

Figure (4-23)
PSF vs. FIELD ANGLE for different No. of Subaperture
D=0.25

Figure (4-24)
PSF vs. FIELD ANGLE for different No. of Subaperture
D=0.5
Figure (4-25)
PSF vs. FIELD ANGLE for different No. of Subaperture 
\( D = 2 \)

Figure (4-26)
PSF vs. FIELD ANGLE for different No. of Subaperture 
\( D = 4 \)
Figure (4-27)
PSF vs. FIELD ANGLE for different No. of Subaperture
D=8
Chapter Four
Numerical Calculation Results & Discussion

Figure (4-28)
PSF vs. FIELD ANGLE for different Radial distance
N=1  M=0.05

Figure (4-29)
PSF vs. FIELD ANGLE for different Radial distance
N=1  M=0.1
Chapter Four
Numerical Calculation Results & Discussion

Figure (4-30)
PSF vs. FIELD ANGLE for different Radial distance
N=1  M=0.2

Figure (4-31)
PSF vs. FIELD ANGLE for different Radial distance
N=1  M=0.4
Chapter Four  
Numerical Calculation Results & Discussion

Figure (4-32)
PSF vs. FIELD ANGLE for different Radial distance
N=4  M=0.05

Figure (4-33)
PSF vs. FIELD ANGLE for different Radial distance
N=4  M=0.1
Chapter Four  
Numerical Calculation Results & Discussion

Figure (4-34)
PSF vs. FIELD ANGLE for different Radial distance
N=4  M= 0.2

Figure (4-35)
PSF vs. FIELD ANGLE for different Radial distance
N=4  M=0.4
Figure (4-36)
PSF vs. FIELD ANGLE for different Radial distance
N=6  M=0.05

Figure (4-37)
PSF vs. FIELD ANGLE for different Radial distance
N=6  M=0.1
Figure (4-38)
PSF vs. FIELD ANGLE for different Radial distance
N=6  M=0.2

Figure (4-39)
PSF vs. FIELD ANGLE for different Radial distance
N=6  M=0.4
Figure (4-40)
PSF vs. FIELD ANGLE for different Radial distance
N=8 M=0.05

Figure (4-41)
PSF vs. FIELD ANGLE for different Radial distance
N=8 M=0.1
Chapter Four  
Numerical Calculation Results & Discussion

Figure (4-42)
PSF vs. FIELD ANGLE for different Radial distance
N=8  M= 0.2

Figure (4-43)
PSF vs. FIELD ANGLE for different Radial distance
N=8  M=0.4
Figure (4-44)
PSF vs. FIELD ANGLE for different Radial distance
N=10  M=0.05

Figure (4-45)
PSF vs. FIELD ANGLE for different Radial distance
N=10  M=0.1
Figure (4-46)
PSF vs. FIELD ANGLE for different Radial distance
$N=10 \quad M=0.2$

Figure (4-47)
PSF vs. FIELD ANGLE for different Radial distance
$N=10 \quad M=0.4$
Chapter Four  
Numerical Calculation Results & Discussion

**Figure (4-48)**  
PSF vs. FIELD ANGLE for different magnification  
*N=1  D=0.25*

**Figure (4-49)**  
PSF vs. FIELD ANGLE for different magnification  
*N=1  D=0.5*
Figure (4-50)
PSF vs. FIELD ANGLE for different magnification
N=1  D=2

Figure (4-51)
PSF vs. FIELD ANGLE for different magnification
N=1  D=4
Figure (4-52)
PSF vs. FIELD ANGLE for different magnification
N=1  D=8
Figure (4-53)
PSF vs. FIELD ANGLE for different magnification
N=4  D=0.25

Figure (4-54)
PSF vs. FIELD ANGLE for different magnification
N=4  D=0.5
PSF vs. FIELD ANGLE for different magnification

Figure (4-55)

N=4  D=2

Figure (4-56)

PSF vs. FIELD ANGLE for different magnification

N=4  D=4
Figure (4-57)
PSF vs. FIELD ANGLE for different magnification
N=4  D=8
Chapter Four: Numerical Calculation Results & Discussion

Figure (4-58)
PSF vs. FIELD ANGLE for different magnification

D=0.2
N=6
Figure (4-59)
PSF vs. FIELD ANGLE for different magnification
N=6   D=0.5

Figure (4-60)
PSF vs. FIELD ANGLE for different magnification
N=6   D=2
Figure (4-61)
PSF vs. FIELD ANGLE for different magnification

\[ N = 6 \quad D = 4 \]

Figure (4-62)
PSF vs. FIELD ANGLE for different magnification
\[ N = 6 \quad D = 8 \]
Figure (4-63)
PSF vs. FIELD ANGLE for different magnification
N=8  D=0.25

N=8  F=2.5  D=0.25
Magnification =0.1
Magnification =0.2
Magnification =0.4
Magnification =0.05

N=8  F=2.5  D=0.5
Magnification =0.1
Magnification =0.2
Magnification =0.4
Magnification =0.05
Figure (4-64)
PSF vs. FIELD ANGLE for different magnification
N=8  D = 0.5

Figure (4-65)
PSF vs. FIELD ANGLE for different magnification
N=8  D=2
Figure (4-66)
PSF vs. FIELD ANGLE for different magnification
N=8  D=4

Figure (4-67)
PSF vs. FIELD ANGLE for different magnification
N=8  D=8
Figure (4-68)

PSF vs. FIELD ANGLE for different magnification

N=10  D=0.25
Figure (4-69)
PSF vs. FIELD ANGLE for different magnification
N=10  D=0.5

Figure (4-70)
PSF vs. FIELD ANGLE for different magnification
N=10  D=2
Chapter Four  
Numerical Calculation Results & Discussion

**Figure (4-71)**  
PSF vs. FIELD ANGLE for different magnification  
N=10 D=4

**Figure (4-72)**  
PSF vs. FIELD ANGLE for different magnification  
N=10 D=8
4.12 Conclusions

1. The equations and prepared programs of the present work are correct with high accuracy because of closely special state results (one aperture) with previous analysis results.

2. The subapertures gave resolving power higher than single aperture power.

3. The radial distance increasing between single aperture centre and subaperture center leaded to increase the resolving power.

4. The fourth subapertures are best of all other subapertures that gave best resolving power.

5. The displacement of single aperture from monitor centre toward all directions is not effect on point spread function.

6. Because the shapes which treated with having symmetrical around the Cartesian axis that original is origin of single aperture, so there is no change in resolving power when is inverting.

7. The increasing of subapertures number will increase the intensity at presence of equal aberration value comparing with single aperture (the aberration effect will decrease by increasing aperture number).

8. There is no focus error value effect at separator distances increasing between the subapertures on intensity value (PSF isn't effect).

9. The relation between focus error value and apertures number to obtaining the same intensity was found by: \( w_{20} = \frac{N}{2} \lambda \)
10. The point spread function value by use prepared programs was performed in this research that gave accurate results which are compared with the theoretical value shown in equation (2-24), the results were with high accuracy in this research.

11. The separator distances between synthetic apertures should be of doubles synthetic apertures radius, in case of synthetic apertures number increasing on two, it should increased the separator distances starting by Cartesian coordinates centre (centre of single aperture) till synthetic aperture centre, reverse that it would be interfuse between synthetic apertures, at last made unregulated form (like annular aperture), in that it did not applied what had been to synthetic aperture. as , letting $N=4$, the value of $R$ should be greater than 1, because that $R=1$ make interfusing in four apertures, when $N=8$ the value of $R=2$, also make interfusing between eight apertures, the same situation is applied with $N=10$, for this reason the value of $R$ should consider carefully during calculation.
4.13 **Future Work**

1. The calculation of accumulated energy in image by using synthetic apertures.

2. Studying the intensity distribution by using body like edge of shape (circular-square-spherical) subapertures.

3. Calculation of the point spread function for abstracted various subapertures with various abstraction ratios.
References

23. L. Rayliegh, Scientific papers 1,432-435 (1899).
References

57. R. R. Shannon Spie V.531, Geometrical optics p.27,(1985) .
References


Supervisor Certification

I certify that this thesis entitled *(Design Study of Various Synthetic Aperture Configuration)* was prepared by *(Kamal Hussein Kazem Al-lamy)* under my supervision at the Laser and Optoelectronics Engineering Department, University of Technology in a partial fulfillment of the requirements for the degree of Master of Science in Optoelectronics Engineering

Signature:

Supervisor: *Assist Professor*

**Dr. Ali H. Al-Hamdani**

Date: / 5 /2008

In view of the available recommendation, I forward this thesis for debate the examination committee.

Signature:

Name: *Dr. Sabah A. Dhahir*

Title: Lecturer

Date: / 5 /2008

Department of Laser and Optoelectronics Engineering
Certification of the Linguistic Supervisor

I certify that this thesis entitled *(Design Study of Various Synthetic Aperture Configuration)* was prepared under my linguistic supervision.

Its language was amended to meet the style of the English Language

Signature:

Name:

Title:

Date: / 5 /2008
<table>
<thead>
<tr>
<th>symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A'</td>
<td>Area of the Exit Pupil</td>
</tr>
<tr>
<td>A(u,v)</td>
<td>Amplitude Point Spread Function</td>
</tr>
<tr>
<td>Bang</td>
<td>Angular Diameter of Airy Disk</td>
</tr>
<tr>
<td>Bdiff</td>
<td>Airy Disk Diameter</td>
</tr>
<tr>
<td>CTF</td>
<td>Contrast Transfer Function</td>
</tr>
<tr>
<td>DSF</td>
<td>Disk Spread Function</td>
</tr>
<tr>
<td>D</td>
<td>Radial Distance</td>
</tr>
<tr>
<td>EX.P.</td>
<td>Exit Pupil</td>
</tr>
<tr>
<td>EN.P.</td>
<td>Entrance Pupil</td>
</tr>
<tr>
<td>f</td>
<td>Focal Length</td>
</tr>
<tr>
<td>(f/No.)</td>
<td>Focal Number</td>
</tr>
<tr>
<td>f(x',y')</td>
<td>Pupil Function</td>
</tr>
<tr>
<td>I_{max}</td>
<td>Maximum Intensity</td>
</tr>
<tr>
<td>I_{min}</td>
<td>Minimum Intensity</td>
</tr>
<tr>
<td>I(z)</td>
<td>Misfocus Intensity</td>
</tr>
<tr>
<td>J_{1(·)}</td>
<td>Bessel Function of the First Kind</td>
</tr>
<tr>
<td>K</td>
<td>Wave Number</td>
</tr>
<tr>
<td>Symbol</td>
<td>Mathematical Expression</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>Kx, Ky</td>
<td>Spatial Frequency or Angular Space Along x &amp; y Axis</td>
</tr>
<tr>
<td>LSF</td>
<td>Line Spread Function</td>
</tr>
<tr>
<td>M</td>
<td>Mirror</td>
</tr>
<tr>
<td>m</td>
<td>Magnification</td>
</tr>
<tr>
<td>MTF</td>
<td>Modulation Transfer Function</td>
</tr>
<tr>
<td>MMT</td>
<td>Multiple Mirror Telescope</td>
</tr>
<tr>
<td>n.f</td>
<td>Normalization Factor</td>
</tr>
<tr>
<td>N</td>
<td>Refraction Index</td>
</tr>
<tr>
<td>OTF</td>
<td>Optical Transfer Function</td>
</tr>
<tr>
<td>OPD</td>
<td>Optical Path Difference</td>
</tr>
<tr>
<td>P</td>
<td>Polar Distance from the Plane Center</td>
</tr>
<tr>
<td>P_n(x)</td>
<td>Legendre Polynomial</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>r</td>
<td>The Radius Distance B'E' in Exit Plane</td>
</tr>
<tr>
<td>R,R'</td>
<td>Radii of Spherical Wave in the Entrance and Exit Pupil Respectively</td>
</tr>
<tr>
<td>S</td>
<td>Resolution (Separation of Two Points in the image Plane)</td>
</tr>
<tr>
<td>S.R</td>
<td>Strehl Ratio</td>
</tr>
<tr>
<td>ti</td>
<td>Legendre Sequence Root</td>
</tr>
<tr>
<td>(u,v)</td>
<td>Reduced Coordinates in the Object Plane</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>(u',v')</td>
<td>Reduced Coordinates in the Image Plane</td>
</tr>
<tr>
<td>(V)</td>
<td>Variance</td>
</tr>
<tr>
<td>(W(x,y))</td>
<td>Aberration Function</td>
</tr>
<tr>
<td>(W(x',y'))</td>
<td>Aberration Polynomial</td>
</tr>
<tr>
<td>(w_i)</td>
<td>Gaussian Weigh Values</td>
</tr>
<tr>
<td>(W_a)</td>
<td>Wave Aberrations</td>
</tr>
<tr>
<td>(W_{20})</td>
<td>Focus Error</td>
</tr>
<tr>
<td>(x',y')</td>
<td>The Coordinates of the Exit Pupil Function</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Wavelength</td>
</tr>
<tr>
<td>(\phi)</td>
<td>The Angle Between The Two Variable (x,r)</td>
</tr>
<tr>
<td>((\xi,\eta))</td>
<td>Cartesian Coordinates in the Object Plane</td>
</tr>
<tr>
<td>((\xi',\eta'))</td>
<td>Cartesian Coordinates in the Image Plane</td>
</tr>
<tr>
<td>(\beta r)</td>
<td>Depth of Modulation</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Field Angle</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Obscuration Ratio</td>
</tr>
<tr>
<td>(\tau(x,y))</td>
<td>Transmission Function</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>Angle in Polar Coordinates</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Depth of Focus</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>The Amount of Principle Ray High on the Optical Axis in the Image Plane</td>
</tr>
</tbody>
</table>
الخلاصة

أن هذا البحث هو دراسة نظرية لتوزيع الشدة في صورة جسم نقطي لمنظومة بصرية متعددة الفتحات الدائرية المركبة، حيث تم اشتقاق علاقات رياضية ووضع برامج حسابية لهذا الغرض. وتم عمل ومقارنه بين النتائج الحالية وبحث سابقه لفتحه دائرية مفردة.

هناك الكثير من الدوال لها تأثير فعال على توزيع الشدة في المنظومة البصرية، تم الاعتماد في هذا البحث بشكل أساسي على دراسة تأثير دالة الانتشار النقطي (PSF) في هذا العمل لأن الأنواع الأخرى قد تسبب خطأ في الحساب.

أن أهم ثلاثة عوامل رئيسية تؤثر في دالة الانتشار النقطي هي (عدد الفتحات، المسافة الفاصلة، التكبير). وقد تم دراسة تأثير هذه العوامل لمنظومات بصرية محددة الحيود أو تحتوي على خطا بؤري، وتم اشتقاق جميع المعادلات الضرورية لهذا الغرض وحساب العلاقات الرياضية لها باستخدام طريقة التكامل العددي لكاوس ثم برمجتها باستخدام Q.Basic ومقارنة النتائج التي تم الحصول عليها مع النتائج التحليلية لباحث سابقة لتأكد من صحة العلاقات وبرنامج .
في هذا البحث تم دراسة التوازن الأمثل (Optimum Balance) بشكل مختصر وشاقع العلاقات الخاصة بها لمنظومة محددة الحيويد وأخرى تحتوي على خطاً بوري ونمجمة من الفتحات الائنية المركبة، لتأكيد اختلاف قيم التوازن بين مهولة بصرية متحدة الفتحات الائنية المركبة ومنظومة بصرية ذات فتحة دائمة مفردة.

 أهم استنتاج في هذا البحث أن الفتحات الائنية المركبة قدرة تحليل عالية مقارنةً مع الفتحة الائنية المفردة، وأن تأثير الزيونغ على الشدة المركزية يقل عند زيادة عدد الفتحات الائنية المركبة بالمقارنة مع الفتحة الائنية المفردة لنفس المنظومة البصرية.