2/6 Resultants

The properties of force, moment, and couple were developed in the previous four articles. Now we are ready to describe the resultant action of a group or *system* of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduce the system to its simplest form to describe its action. The *resultant* of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

The most common type of force system occurs when the forces all act in a single plane, say, the x-y plane, as illustrated by the system of three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ in Fig. 2/13*a*. We obtain the magnitude and direction of the resultant force $\mathbf{R}$ by forming the *force polygon* shown in part *b* of the figure, where the forces are added head-to-tail in any sequence. Thus, for *any* system of coplanar forces we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} \qquad (2/9)$$

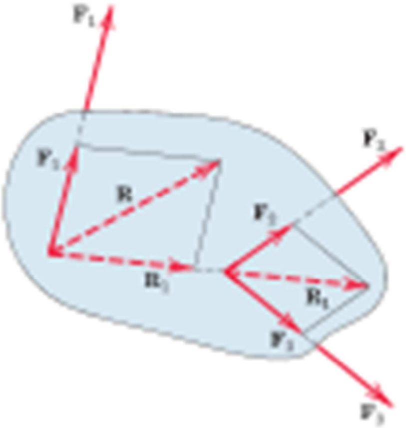


Figure 2/13

Graphically, the correct line of action of $\mathbf{R}$ may be obtained by preserving the correct lines of action of the forces and adding them by the parallelogram law. We see this in part *a* of the figure for the case of three forces where the sum $\mathbf{R}_1$ of $\mathbf{F}_2$ and $\mathbf{F}_3$ is added to $\mathbf{F}_1$ to obtain $\mathbf{R}$. The principle of transmissibility has been used in this process.

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs. 2/14*a* and *b*, where M_1, M_2, and M_3 are the couples resulting from the transfer of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ from their respective original lines of action to lines of action through point *O*.
2. Add all forces at *O* to form the resultant force $\mathbf{R}$, and add all couples to form the resultant couple M_O. We now have the single force–couple system, as shown in Fig. 2/14*c*.
3. In Fig. 2/14*d*, find the line of action of $\mathbf{R}$ by requiring $\mathbf{R}$ to have a moment of M_O about point *O*. Note that the force systems of Figs. 2/14*a* and 2/14*d* are equivalent, and that $\Sigma(Fd)$ in Fig. 2/14*a* is equal to Rd in Fig. 2/14*d*.

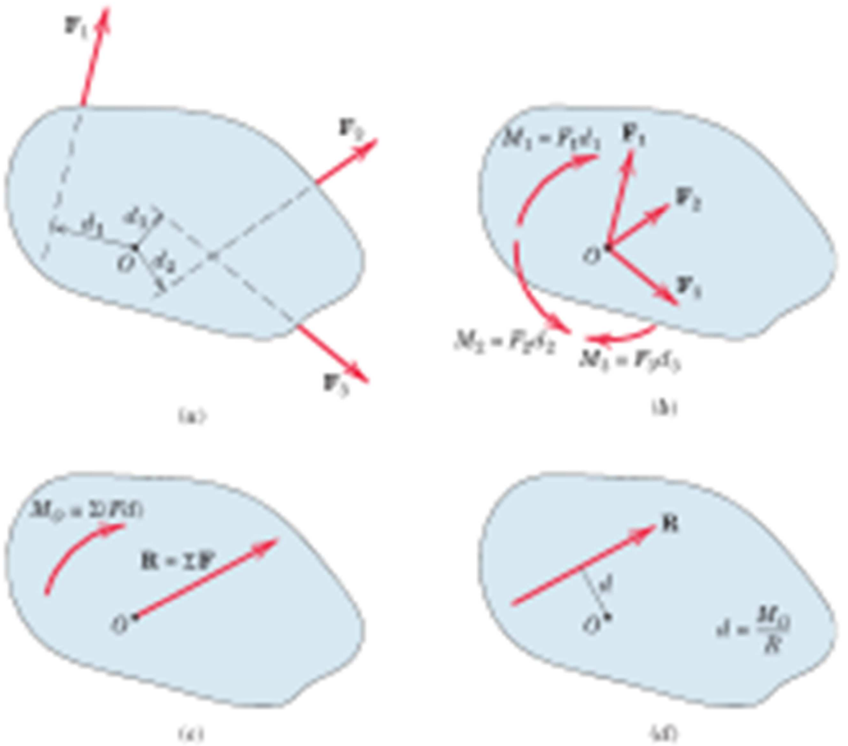


Figure 2/14

Principle of Moments

This process is summarized in equation form by

$$\boxed{\begin{aligned} \mathbf{R} &= \Sigma \mathbf{F} \\ M_O &= \Sigma M = \Sigma(Fd) \\ Rd &= M_O \end{aligned}} \qquad (2/10)$$

The first two of Eqs. 2/10 reduce a given system of forces to a force–couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of $\mathbf{R}$, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of *nonconcurrent force systems*; we call this extension the *principle of moments*.

For a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum ΣM_O about that point is zero. Thus, the line of action of the resultant $\mathbf{R} = \Sigma\mathbf{F}$, determined by the first of Eqs. 2/10, passes through point O. For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force $\mathbf{R}$ for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple. The three forces in Fig. 2/15, for instance, have a zero resultant force but have a resultant clockwise couple $M = F_3d$.

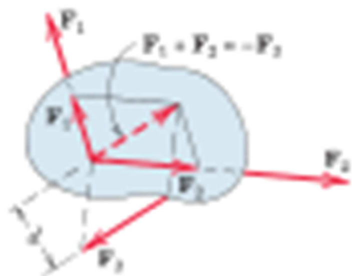
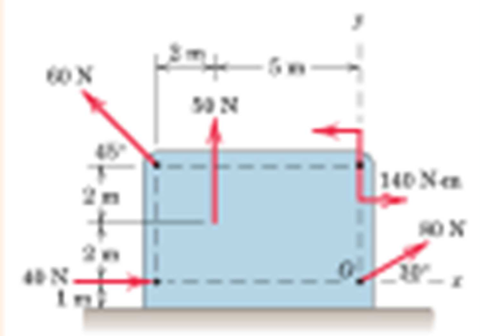


Figure 2/15

Sample Problem 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ *Ans.*

$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$$= -237 \text{ N·m}$$

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|]$ $\quad 148.3d = 237 \quad d = 1.600 \text{ m}$ *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the *x*-axis and is tangent at point *A* to a circle of 1.600-m radius with center *O*, as shown in part *b* of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. *a*, dictate the final placement of **R**. Had M_O been counterclockwise, the correct line of action of **R** would have been the tangent at point *B*.

The resultant **R** may also be located by determining its intercept distance *b* to point *C* on the *x*-axis, Fig. *c*. With R_x and R_y acting through point *C*, only R_y exerts a moment about *O* so that

$$R_y b = |M_O| \quad \text{and} \quad b = \frac{237}{132.4} = 1.792 \text{ m}$$

Alternatively, the *y*-intercept could have been obtained by noting that the moment about *O* would be due to R_x only.

A more formal approach in determining the final line of action of **R** is to use the vector expression

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is a position vector running from point *O* to any point on the line of action of **R**. Substituting the vector expressions for **r**, **R**, and $\mathbf{M}_O$ and carrying out the cross product result in

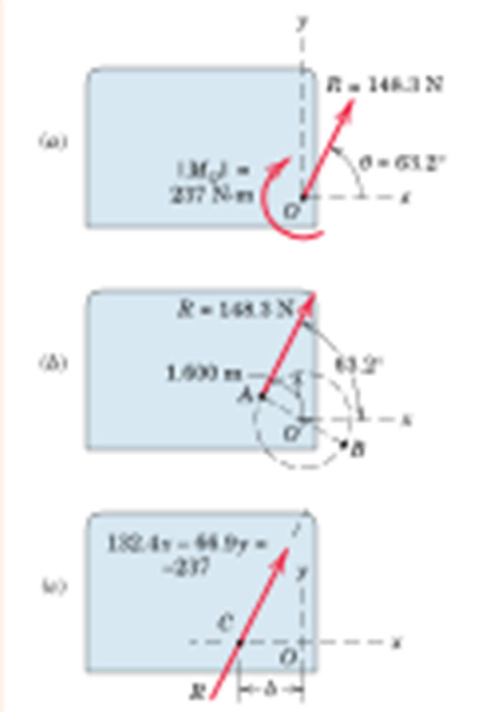
$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

Thus, the desired line of action, Fig. *c*, is given by

$$132.4x - 66.9y = -237$$

By setting $y = 0$, we obtain $x = -1.792$ m, which agrees with our earlier calculation of the distance *b*.



Helpful Hints

① We note that the choice of point *O* as a moment center eliminates any moments due to the two forces which pass through *O*. Had the clockwise sign convention been adopted, M_O would have been +237 N·m, with the plus sign indicating a sense which agrees with the sign convention. Either sign convention, of course, leads to the conclusion of a clockwise moment M_O.

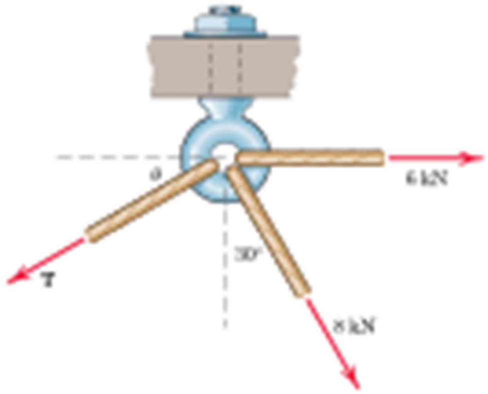
② Note that the vector approach yields sign information automatically, whereas the scalar approach is more physically oriented. You should master both methods.

PROBLEMS

Introductory Problems

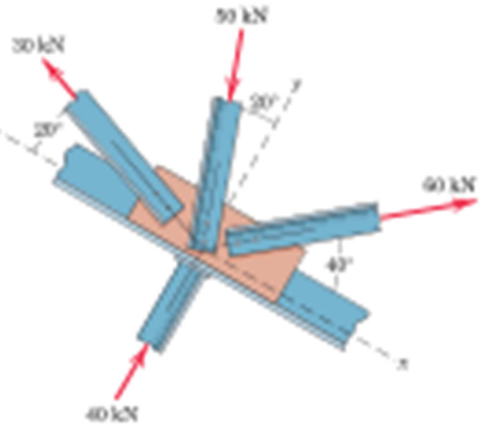
2/77 Calculate the magnitude of the tension **T** and the angle θ for which the eye bolt will be under a resultant downward force of 15 kN.

Ans. $T = 12.85$ kN, $\theta = 38.9°$



Problem 2/77

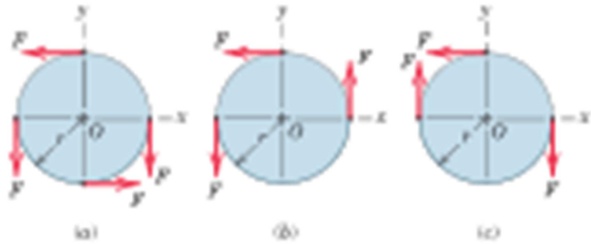
2/78 Determine the resultant **R** of the four forces acting on the gusset plate. Also find the magnitude of **R** and the angle θ_x which the resultant makes with the x-axis.



Problem 2/78

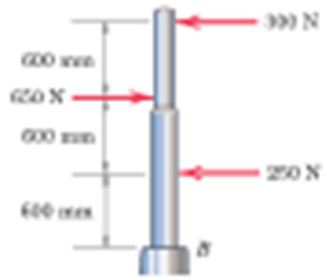
2/79 Determine the *equivalent force–couple system* at the origin O for each of the three cases of forces being applied to the edge of a circular disk. If the resultant can be so expressed, replace this force–couple system with a stand-alone force.

Ans. (a) $\mathbf{R} = -2F\mathbf{j}$ along $x = -r$
(b) $\mathbf{R} = -F\mathbf{i}$ along $y = 2r$
(c) $\mathbf{R} = -F\mathbf{i}$ along $y = -r$



Problem 2/79

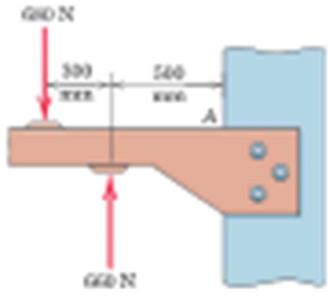
2/80 Determine the height h above the base B at which the resultant of the three forces acts.



Problem 2/80

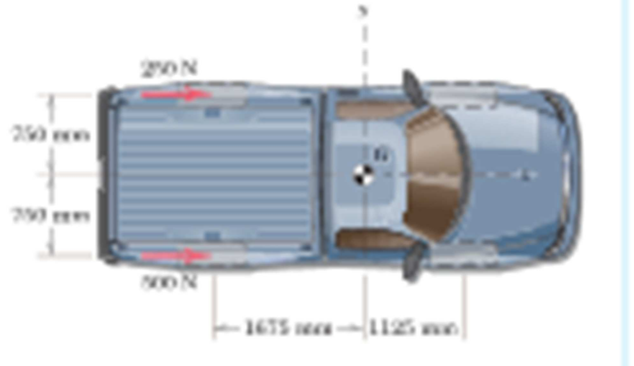
2/81 Where does the resultant of the two forces act?

Ans. 10.70 m to the left of A



Problem 2/81

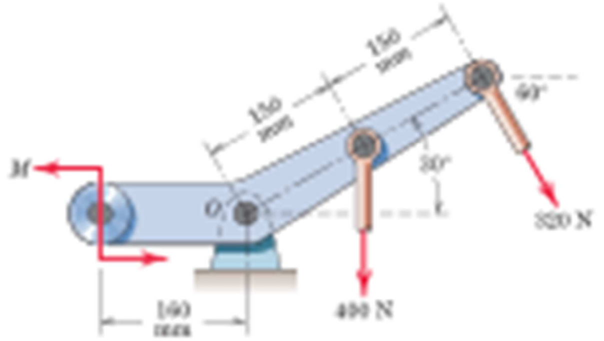
2/82 Under nonuniform and slippery road conditions, the two forces shown are exerted on the two rear-drive wheels of the pickup truck, which has a limited-slip rear differential. Determine the y-intercept of the resultant of this force system.



Problem 2/82

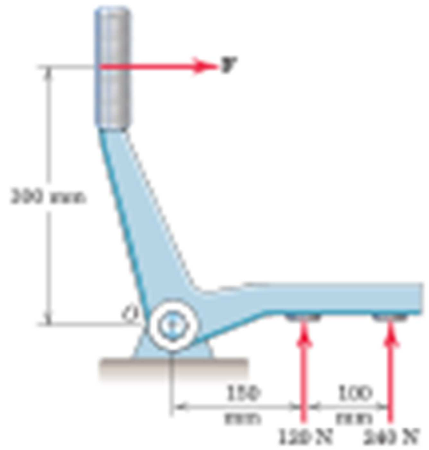
2/83 If the resultant of the two forces and couple M passes through point O, determine M.

Ans. $M = 148.0$ N·m CCW



Problem 2/83

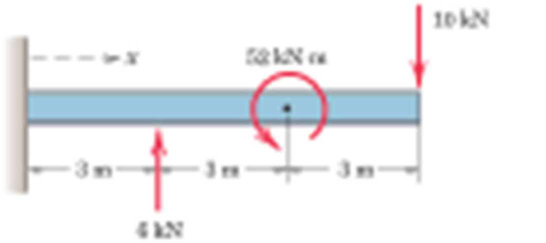
2/84 Determine the magnitude of the force **F** applied to the handle which will make the resultant of the three forces pass through O.



Problem 2/84

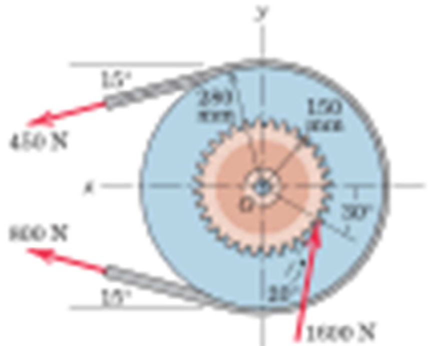
2/85 Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.

Ans. $R = 4$ kN down at $x = 5$ m



Problem 2/85

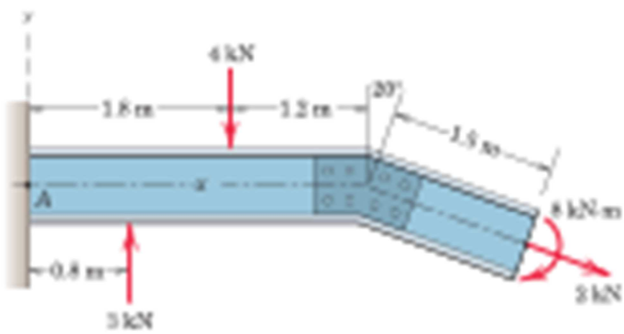
2/90 The gear and attached V-belt pulley are turning counterclockwise and are subjected to the tooth load of 1600 N and the 800-N and 450-N tensions in the V-belt. Represent the action of these three forces by a resultant force **R** at O and a couple of magnitude M. Is the unit slowing down or speeding up?



Problem 2/90

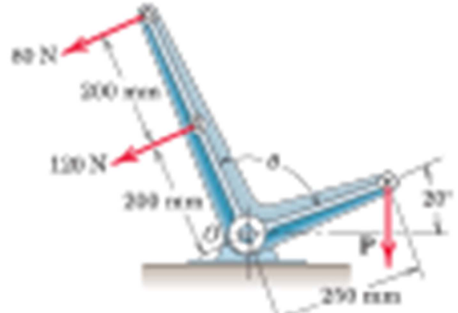
2/91 The design specifications for the attachment at A for this beam depend on the magnitude and location of the applied loads. Represent the resultant of the three forces and couple by a single force **R** at A and a couple M. Specify the magnitude of **R**.

Ans. $\mathbf{R} = 1.879\mathbf{i} - 1.684\mathbf{j}$ kN, $R = 2.52$ kN
$M = 14.85$ kN·m CW



Problem 2/91

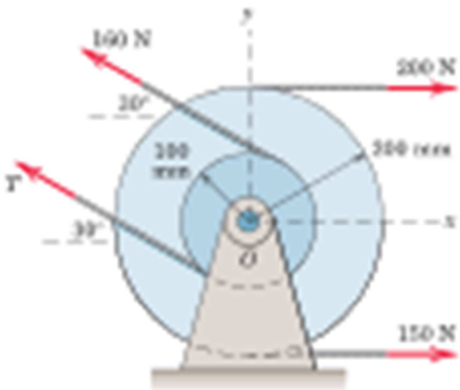
2/92 In the equilibrium position shown, the resultant of the three forces acting on the bell crank passes through the bearing O. Determine the vertical force **P**. Does the result depend on θ?



Problem 2/92

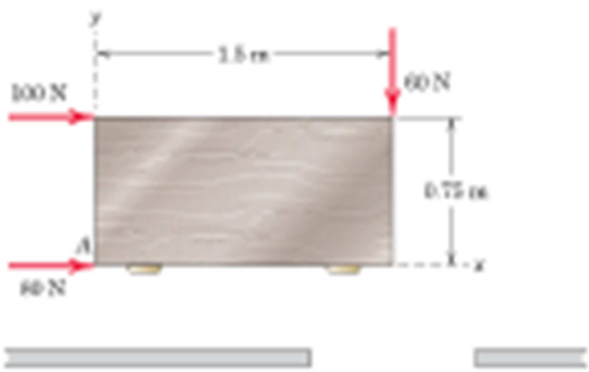
2/93 Two integral pulleys are subjected to the belt tensions shown. If the resultant **R** of these forces passes through the center O, determine T and the magnitude of **R** and the counterclockwise angle θ it makes with the x-axis.

Ans. $T = 60$ N, $R = 193.7$ N, $\theta = 34.6°$



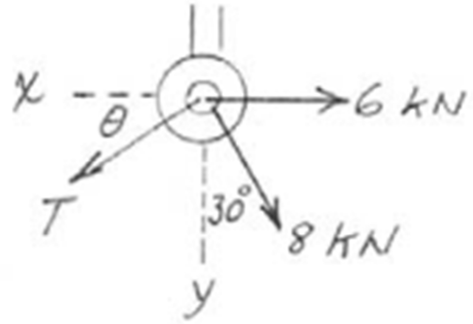
Problem 2/93

2/94 While sliding a desk toward the doorway, three students exert the forces shown in the overhead view. Determine the equivalent force–couple system at point A. Then determine the equation of the line of action of the resultant force.



Problem 2/94

2/77



$R = R_y = 15 = T\sin\theta + 8\cos 30°$

$R_x = 0 = T\cos\theta - 6 - 8\sin 30°$

So $T\sin\theta = 8.07$

$T\cos\theta = 10$

Divide & set $\theta = \tan \frac{8.07}{10}$

$\underline{\theta = 38.9°}$

$T = \frac{10}{\cos 38.9°} = \underline{12.85 \text{ kN}}$

2/78

$R_x = \Sigma F_x = 60\cos 40° + 50\sin 20° - 30\cos 20°$

$= 34.87 \text{ kN}$

$R_y = \Sigma F_y = 60\sin 40° + 40 - 50\cos 20° + 30\sin 20°$

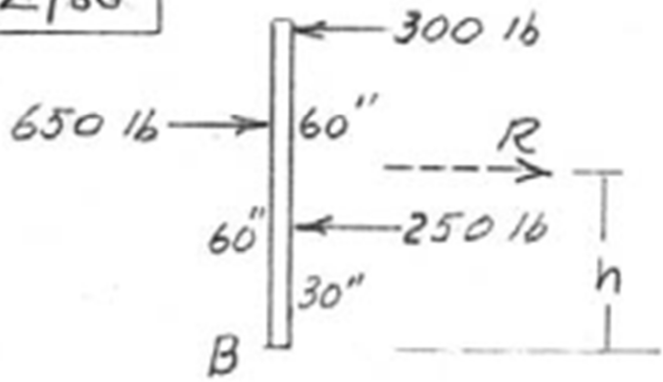
$= 41.84 \text{ kN}$

$R = \sqrt{34.87^2 + 41.84^2} = \underline{54.5 \text{ kN}}$

$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{41.84}{34.87} = \underline{50.2°}$

$\underline{\underline{R} = 34.9\underline{i} + 41.8\underline{j} \text{ kN}}$

2/80



$R = \Sigma F = 650 - 250 - 300$

$= 100 \text{ lb}$

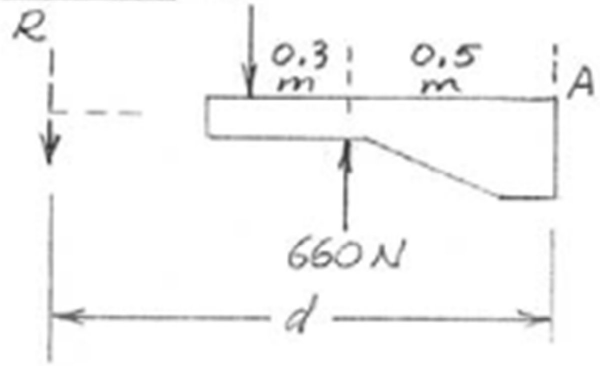
$Rh = \Sigma M_B;$

$100h = 650(60) - 300(90)$

$- 250(30)$

$\underline{h = 45 \text{ in.}}$

2/81



$R = \Sigma F = 680 - 660 = 20\ N$

$Rd = \Sigma M_A$

$20d = 680(0.8) - 660(0.5)$

$\underline{d = 10.70\ m \quad \text{to the left of } A}$

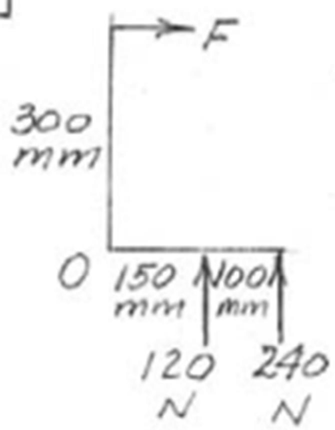
2/83

$M_o = 0$, so

$\circlearrowleft_{+} M - 400(0.150 \cos 30°) - 320(0.300) = 0$

$\underline{M = 148.0\ N \cdot m}$

2/84



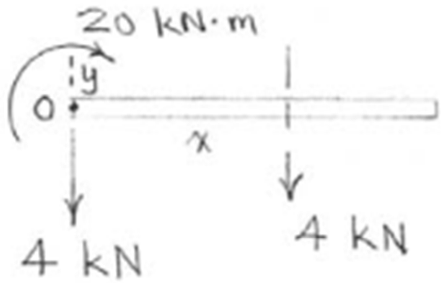
$\Sigma M_O = 0$ for R to pass through O

$300F - 120(150) - 240(250) = 0$

$\underline{F = 260\ N}$

2/85 Force-Couple system at point O:

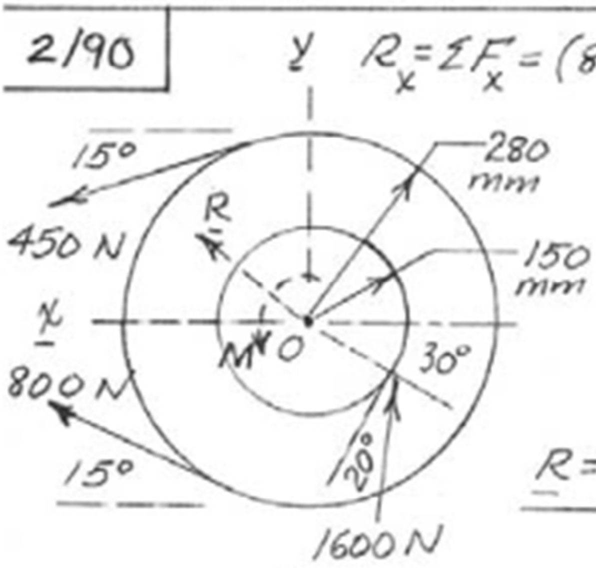
$$\left.\begin{array}{l} \underline{R} = \Sigma \underline{F} = (6-10)\underline{j} = -4\underline{j} \quad kN \\ \circlearrowleft_{+} M_o = 6(3) - 10(9) + 52 = -20\ kN \cdot m \end{array}\right\}$$



$x = \dfrac{M_o}{R} = \dfrac{20}{4}$

$= \underline{5\ m}$ (on beam!)

2/90



$R_x = \Sigma F_x = (800 + 450)\cos 15° - 1600 \sin(30° - 20°)$

$= 1207 - 278 = 929.6 \text{ N}$

$R_y = \Sigma F_y = 1600 \cos 10°$

$+ (800 - 450) \sin 15°$

$= 1576 + 90.6 = 1666 \text{ N}$

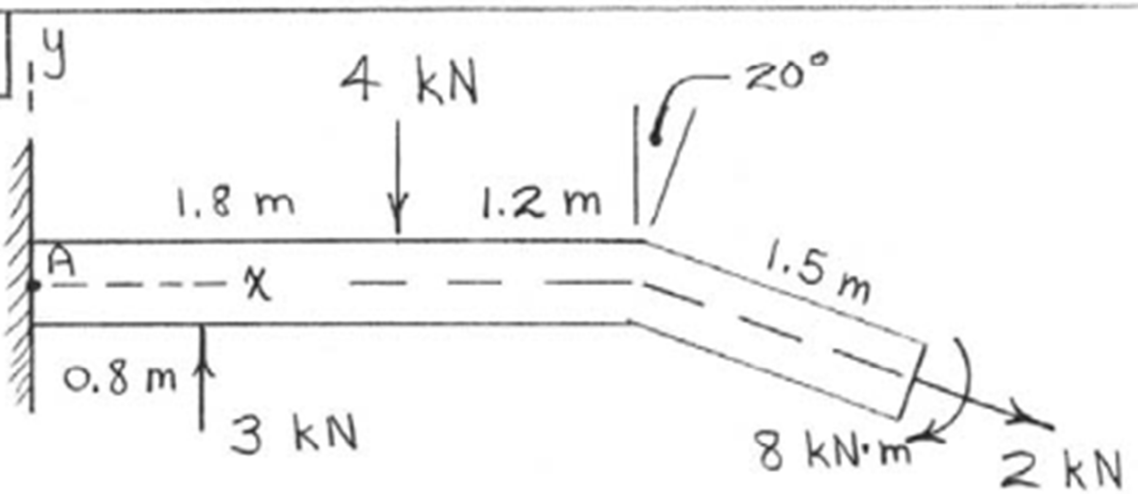
$\underline{R} = 930\underline{i} + 1666\underline{j} \text{ N}$

$M = \Sigma M_O$ (+↺); $M = 1600 \cos 20° (0.150) + (450 - 800) 0.280$

$= 225.5 - 98.0 = 127.5 \text{ N·m CCW}$

so unit is speeding up in CCW dir.

2/91



$R_x = \Sigma F_x = 2 \cos 20° = 1.879 \text{ kN}$

$R_y = \Sigma F_y = 3 - 4 - 2 \sin 20° = -1.684 \text{ kN}$

$\underline{R} = 1.879\underline{i} - 1.684\underline{j} \text{ kN}, \quad R = \sqrt{1.879^2 + 1.684^2} = 2.52 \text{ kN}$

(+↻) $M = \Sigma M_A = 4(1.8) - 3(0.8) + 2 \sin 20° (3.0) + 8$

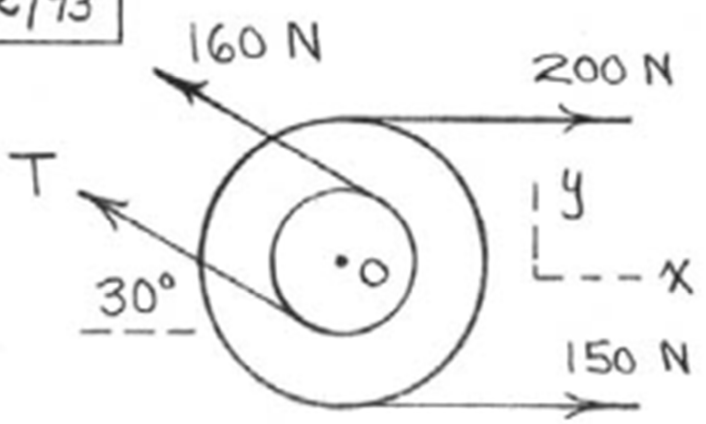
$= 14.85 \text{ kN·m CW}$

2/92 $\Sigma M_O = 0$ since $\underline{R}$ passes through O.

$40(8) + 60(4) - 5P\cos 20° = 0$, $\underline{P = 119.2 \text{ lb}}$

Moment of 40-lb & 60-lb forces unaffected by θ so result for P is not dependent on θ.

2/93



$\begin{cases} r_1 = 0.1 \text{ m} \\ r_2 = 0.2 \text{ m} \end{cases}$

$+\circlearrowleft M_O = 0 : 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$

$\underline{T = 60 \text{ N}}$

$R_x = \Sigma F_x = 200 + 150 - (160 + 60)\cos 30° = 159.5 \text{ N}$

$R_y = \Sigma F_y = (160 + 30)\sin 30° = 110 \text{ N}$

$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \text{ N}}$

$\theta = \tan^{-1}(R_y / R_x) = \underline{34.6°}$