University of Technology

Bio-Medical Engineering department

DESCRIPTIVE GEOMETRY

1st class

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Chapter One

General Concepts

Definition: Descriptive geometry is that branch of Mathematics which has الذي يملك for its object the explanation of the methods of representing توضيح لطريقة التمثيل by drawings:

First. All geometrical magnitudes.

Second. The solution of problems relating to these magnitudes in space.

Therefore, Descriptive geometry is the science of graphic representation and solution of space problems. The fundamentals of descriptive geometry are based on the basic theory of projection الاسقاط, precisely تحديدا orthographic projection الاسقاط المتعامد.

Drafting tools ادوات تخطيطية are used to solve and represent spatial relationships of points, lines, and planes by means of projection.

The principles and concepts of descriptive geometry can be used to solve problems relating to many engineering fields.

Principal topics مواضيع اساسية:

True length of a line

Point view of a line

Edge view of a line

True size and true shape of a plane

General descriptive geometry concepts:

True distance between two points

True distance between a point and a line

True distance between a point and a plane

True distance between parallel lines

True distance between skew lines

True distance between a line and parallel plane

True distance between two planes

True angle between two lines

True angle between two planes

True angle between a line and a plane

Definitions:

Point:

A point represents a location تمثل موقع in space or on a drawing. A point has no height, width, or depth.

Line:

A straight line is the shortest distance between two points.

Parallel Lines:

Straight lines are said to be parallel نقول عنها متوازية if the shortest distance between them remains constant تبقى ثابتة.

Perpendicular Lines:

Two straight lines are said to be perpendicular if the intersection التقاطع of the lines form يشكل a 90 degree angle.

Skew Lines (شماليان) المستقيمان المتخالفان**:**

Skewed lines are two lines that are not in the same plane and do not intersect.

Surface:

A surface is a two-dimensional geometric entity كيان هندسي having no thickness. It has area but not volume.

Normal Surface السطح الاعتيادي**:**

A normal surface is a plane surface that is parallel to a plane of projection. It appease يوافق in true size and true shape on the plane to which it is parallel, and as a vertical or a horizontal line on adjacent planes المستويات المجاورة of projection.

Inclined Surface السطح المائل**:**

An inclined surface is a plane surface that is perpendicular to one plane of projection but inclined to adjacent planes of projection. An inclined surface will project a straight line on the plane to which it is perpendicular and will appear foreshortened سيبدو قصيرا on planes to which it is inclined.

Oblique Surface السطح المنحرف**:**

An oblique surface is a plane that is oblique to all planes of projection. It is not perpendicular to any principle plane of projection. It will always appear as a foreshortened surface in all views. It will never appear as a line or true size and shape plane in any principal view.

Normal Edges الحافات العادية:

A normal edge is a line that is perpendicular to a plane of projection. It will appear as a point in the plane of projection to which it is perpendicular and as a line in true length on adjacent planes of projection.

Inclined Edges:الحافات المائلة

An inclined edge is a line that is parallel to a plane of projection but inclined to adjacent planes. It will appear true length on the plane to which it is parallel and foreshortened on adjacent planes. The true length view of an inclined line is always inclined, while the foreshortened views are either vertical or horizontal lines.

Oblique Edges:

An oblique edge is a line that is oblique to all planes of projection. It is neither parallel nor perpendicular to any principal plane of projection. An oblique edge will appear as a foreshortened line in an inclined position in any view. Because it is not parallel to a plane of projection it will never appear true length.

<u>Descriptive Geometry problem solving concepts:</u>

1. If the direction of sight النظر for a view is parallel to a true length line that line will appear as a point in the resulting view.

2. An inclined line appears true length on the plane to which it is parallel.

3. A true length view of an inclined line is always in the inclined position, while the foreshortened views are either in vertical or horizontal positions.

4. Inclined lines are classified as frontal امامي او راسي, horizontal افقي, or profile جانبي.

5. A true angle between a line and a plane may be measured when the line is true length and the plane is in edge view.

6. An oblique line does not appear true length in a principal view.

7. A plane surface is a surface such that a straight line joining any two points on the surface lies in the surface.

8. Two straight lines in a plane must intersect unless the lines are parallel.

9. A point may be placed in a plane by locating it on a line known to be in the plane.

10. To get the edge view of a plane you must get the point view of a line in the plane.

11. The true angle between a line and a plane of projection is seen in the view in which the given line is true length and the plane in question is in edge view.

12. In most situations tow lines that are parallel in two adjacent views are parallel in space.

13. Planes are parallel to each other if the edge views of the planes are parallel.

14. A line is parallel to a plane if it is parallel to a line in the plane.

15. A plane is parallel to a line if it contains a line that is parallel to the given line.

16. A 90 degree angle formed by perpendicular lines appears true size in any view showing one leg of the angle true length (normal line), provided شريطة ان the other leg does not appear a point in the same view.

17. A line perpendicular to a plane is perpendicular to all lines in the plane that pass through the point of intersection between the line and plane.

18. If a line is perpendicular to a give plane, any plane containing the line is perpendicular to the given line.

19. The shortest distance between any two lines is measured along a line perpendicular to both lines.

<u>Plane Identification:</u>

Horizontal Plane:

The horizontal plane is referred to as a TOP VIEW in Mechanical, PLAN VIEW in Architecture, and HORIZONTAL PLANE in Engineering. The Horizontal plane shows the WIDTH and DEPTH dimensions.

Frontal Plane:

The frontal plane is referred to as a FRONT VIEW in Mechanical, FRONT ELEVATION in Architecture, and FRONTAL PLANE in Engineering. The Frontal Plane shows the WIDTH and HEIGHT dimensions.

Profile Plane:

The Profile Plane is referred to a RIGHT SIDE VIEW in Mechanical, RIGHT ELEVATION in Architecture, and PROFILE PLANE in Engineering. The Profile Plane shows the HEIGHT and DEPTH dimensions.

Chapter Two

PROJECTION

Projection: In order to represent an object such as triangle ABC by line drawing in a plane, imaginary خيالي projectors خطوط الاسقاط emanating المنطلقة from various points on the object are extended until they pierce (تصل) حتى تبلغ plane p or plane of projection as in Figure (2.1). The various piercing points A', B' and C' are then connected with line to form a view of the object. The two common types of projection are the orthographic projection, Figure (2.1), and the perspective المنظور projection Figure (2.2).

Orthographic projection Orthographic projection is a method of representing an object by means of parallel projectors perpendicular to the plane of projection p as shown in Figure (2.1).

Perspective Projection (Central Projection) Figure (2.2) illustrates the basic theory of perspective projections. The projections emanate from points of the object and converge to an observer whose position is called the center of projection. So it is a common projection on which the projector converges to point S.

ORTHOGNAL PROJECTION (Mongean Projection) In orthogonal projection, an object like point, line or plane is represented on two mutual perpendicular planes. So, it called orthogonal (Bi-orthographic) Projection. In this type of projection, objects are referred to three main planes of projection XY, XZ and YZ each two of them are perpendicular to the third. Plane XY is taken horizontal and XZ is taken vertical (called the frontal plane) while YZ is called the profile plane. The three projection planes intersect each other in the three straight lines X, Y and Z and there called coordinate axes. These lines meet at a one point o which called the origin o, Figure (2.3).

To represent these three planes on the paper, plane XZ remains in its place, whereas plane XY rotates about X-axis until positive Y-axis coincides يطابق negative Z-axis and negative Y-axis coincides positive Z-axis. In other hand plane YZ rotates about Z-axis until positive Y-axis coincides negative X-axis, and negative Y-axis coincides positive X-axis. Figure 2.4 shows the representation of three planes on the paper and the coordinate axes that formed.

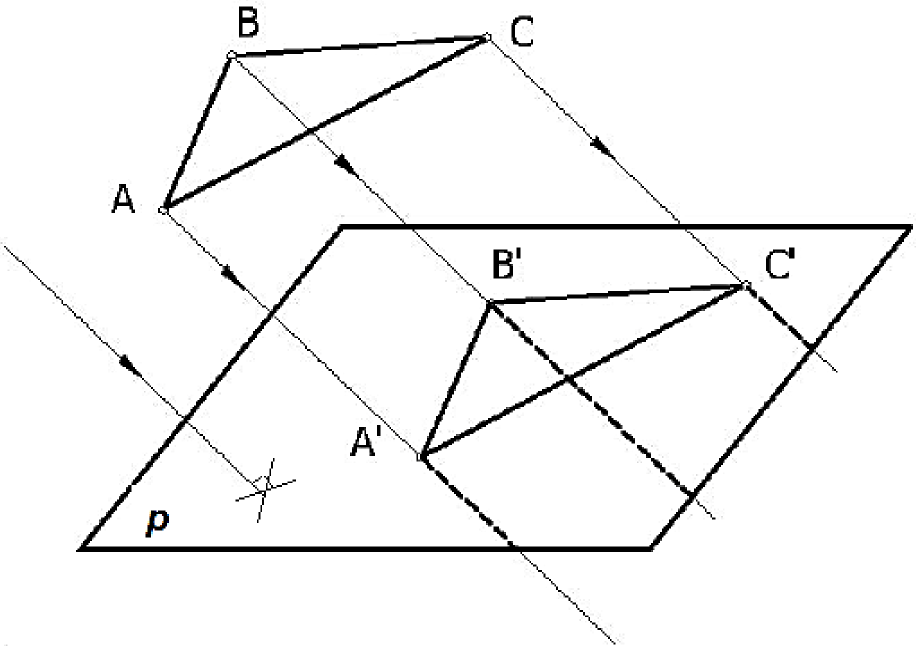

Figure (1)

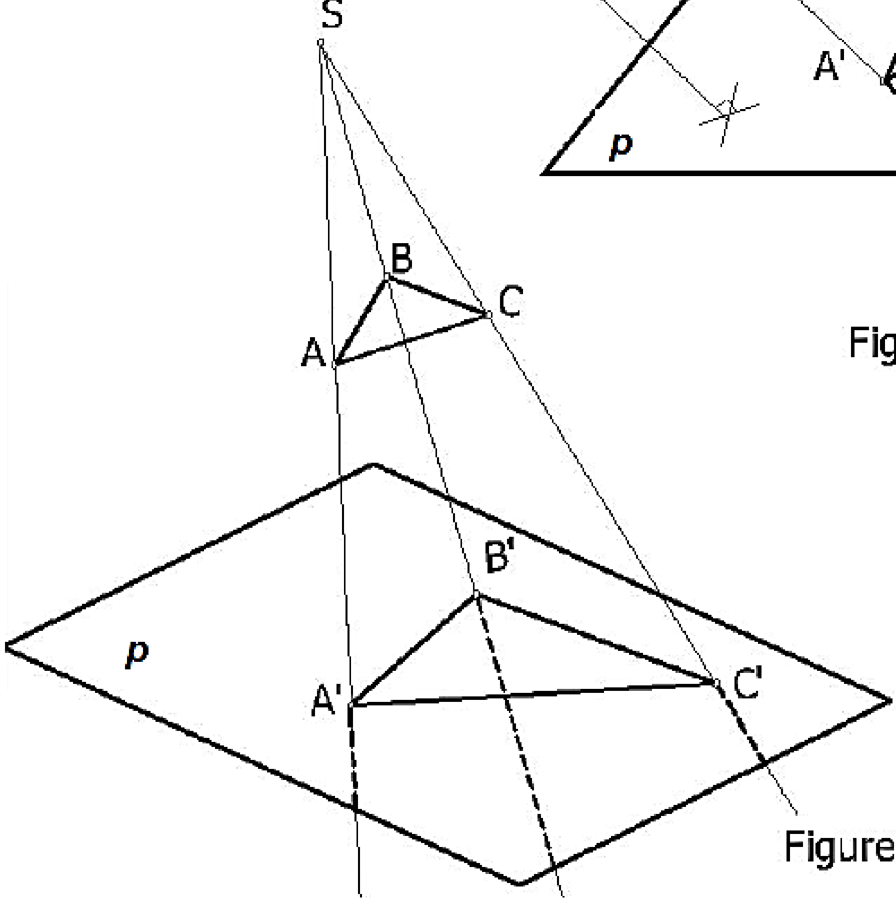

Figure (2)

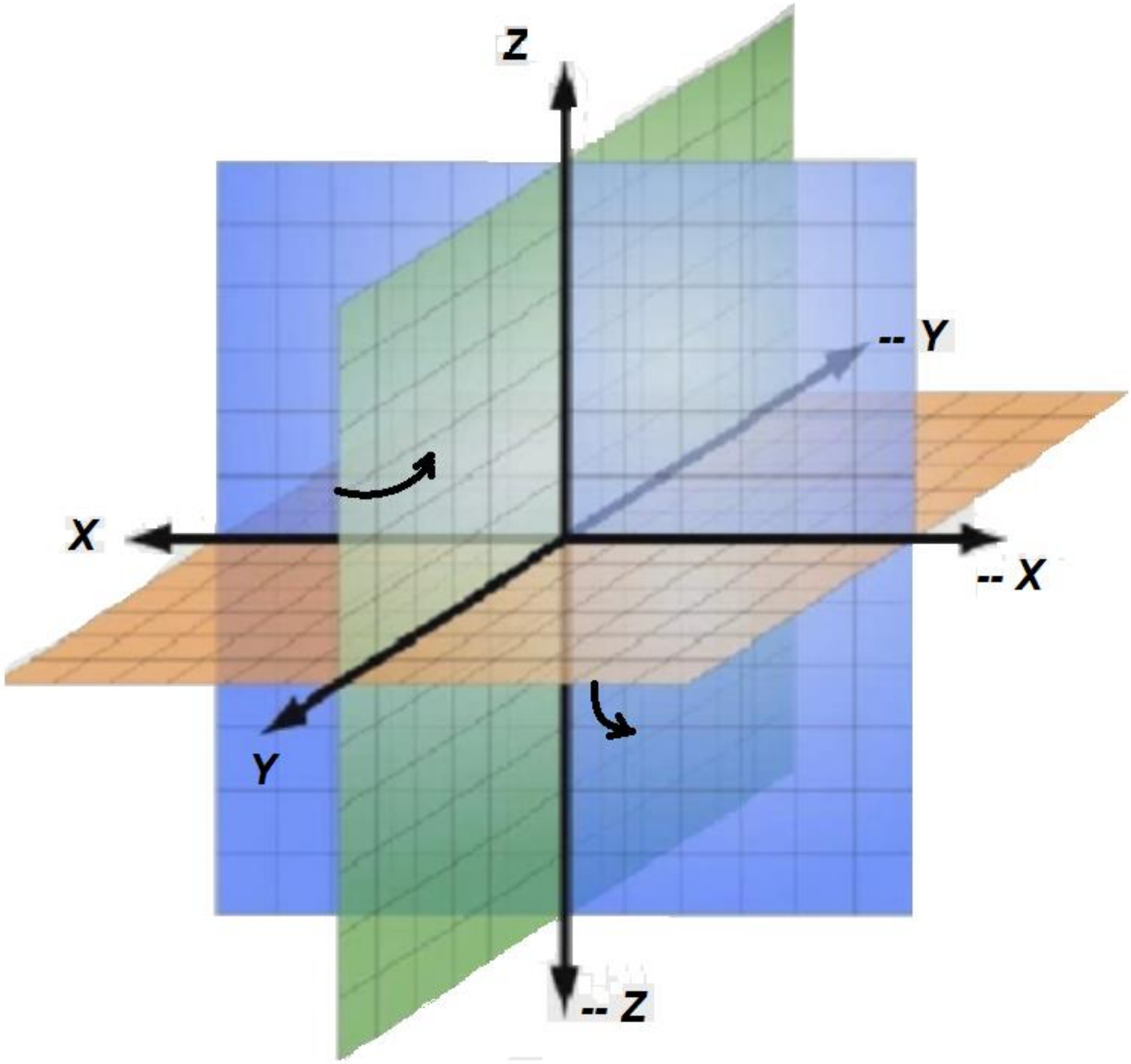

Figure (2.3) The main planes

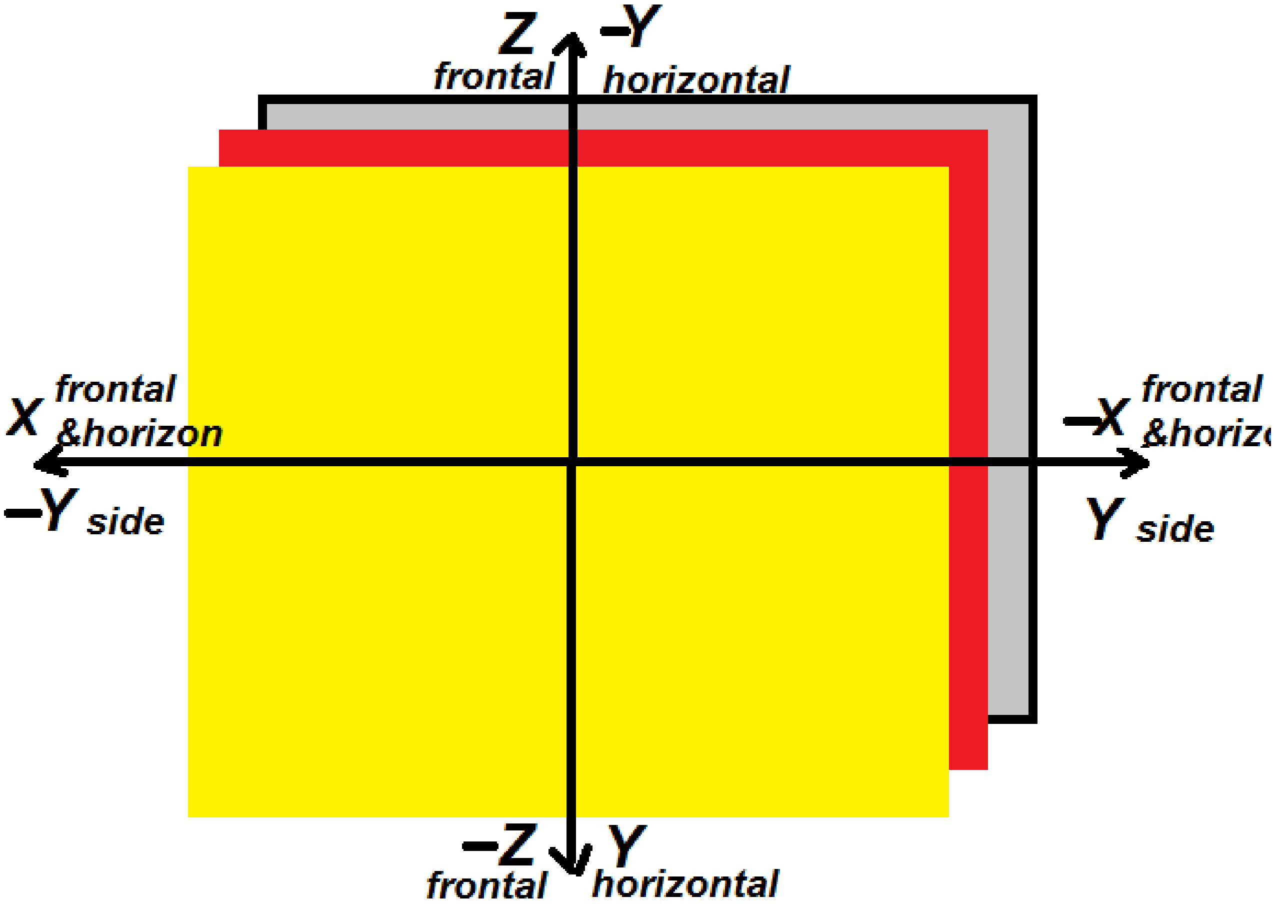

Figure (2.4) Representation of the main planes on the paper and the coordinate axes that formed

Projection of point: Point P is usually specified by its coordinate's x_P, y_P and z_P measured from π**1, π2** and π**3** Planes respectively. P', P" and P"' are the projections of the point p on the projection planes π**1**, π**2** and π**3**. Point P' , (x_P, y_P, z_P=0) lies in the horizontal plane π**1** (i.e. z=0), while P" (x_P, y_P = 0, z_P) lies in the frontal plane π**2** (i.e. Y=0), P' and P" may be written as P'(x_P, y_P) and P" (x_P, z_P). (x_P=0, y_P , z_P) lies in the profile plane π**3** (i.e. X=0). P"' may be written as P"' (y_P, z_P).

If we rotate the horizontal plane π**1** about x-axis till it coincide with the frontal (vertical) plane π**2,** the ground line خط الارض (x-axis) is drawn, here in the working surface of our drawing paper, horizontally while y and z axes are drawn perpendicular on it from the origin point o. Now, we can easily represent P', P" and P"' on the drawing paper by rotating the horizontal and side planes as represented in figure (2.4) above, the representation of P', P" and P"' are shown in Figures 2.5 and 2.6. On the drawing paper, we can conclude that the line connecting the projections P' and P" is perpendicular to the x-axis and designated as the ground line.

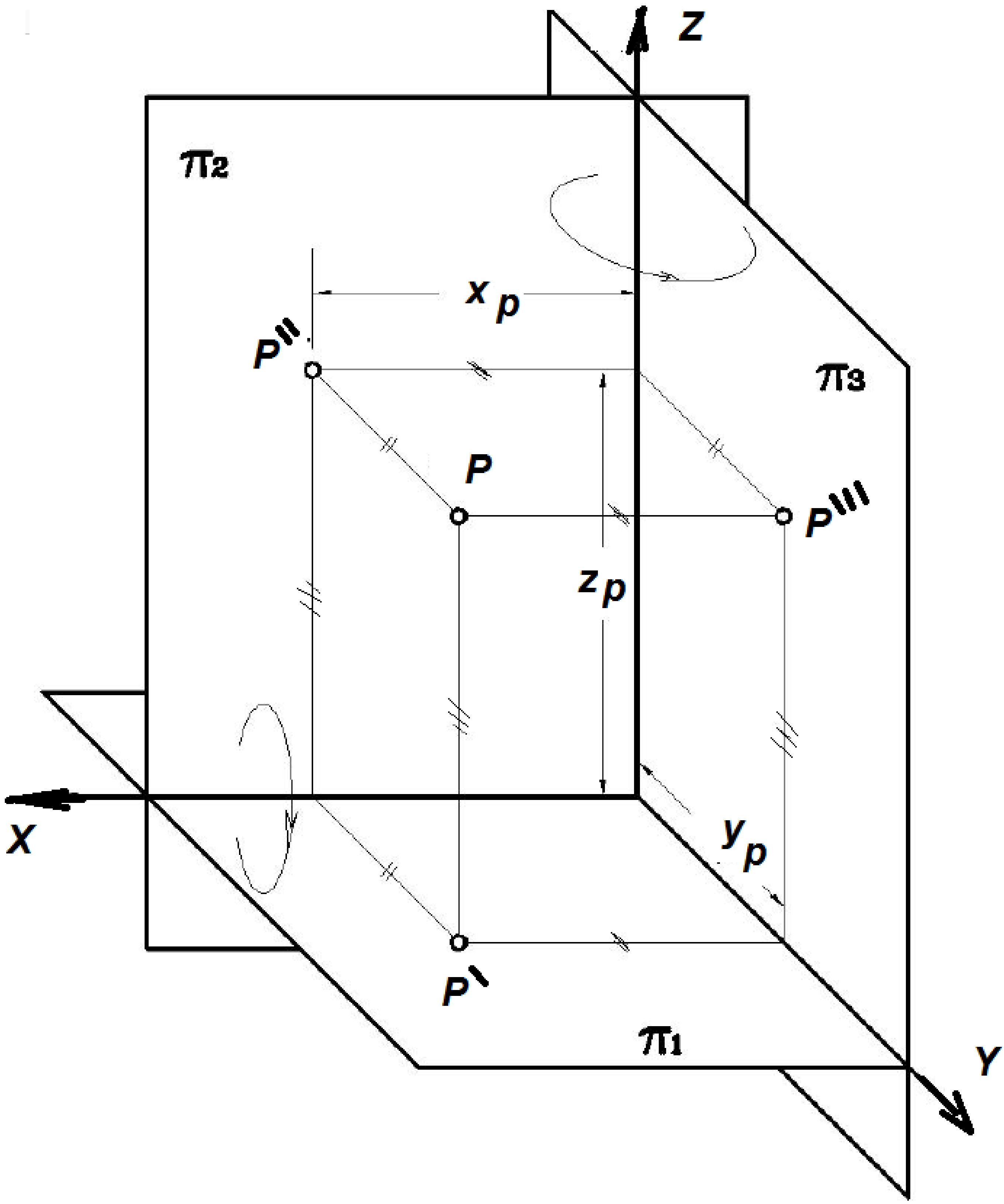

Figure (2.5)

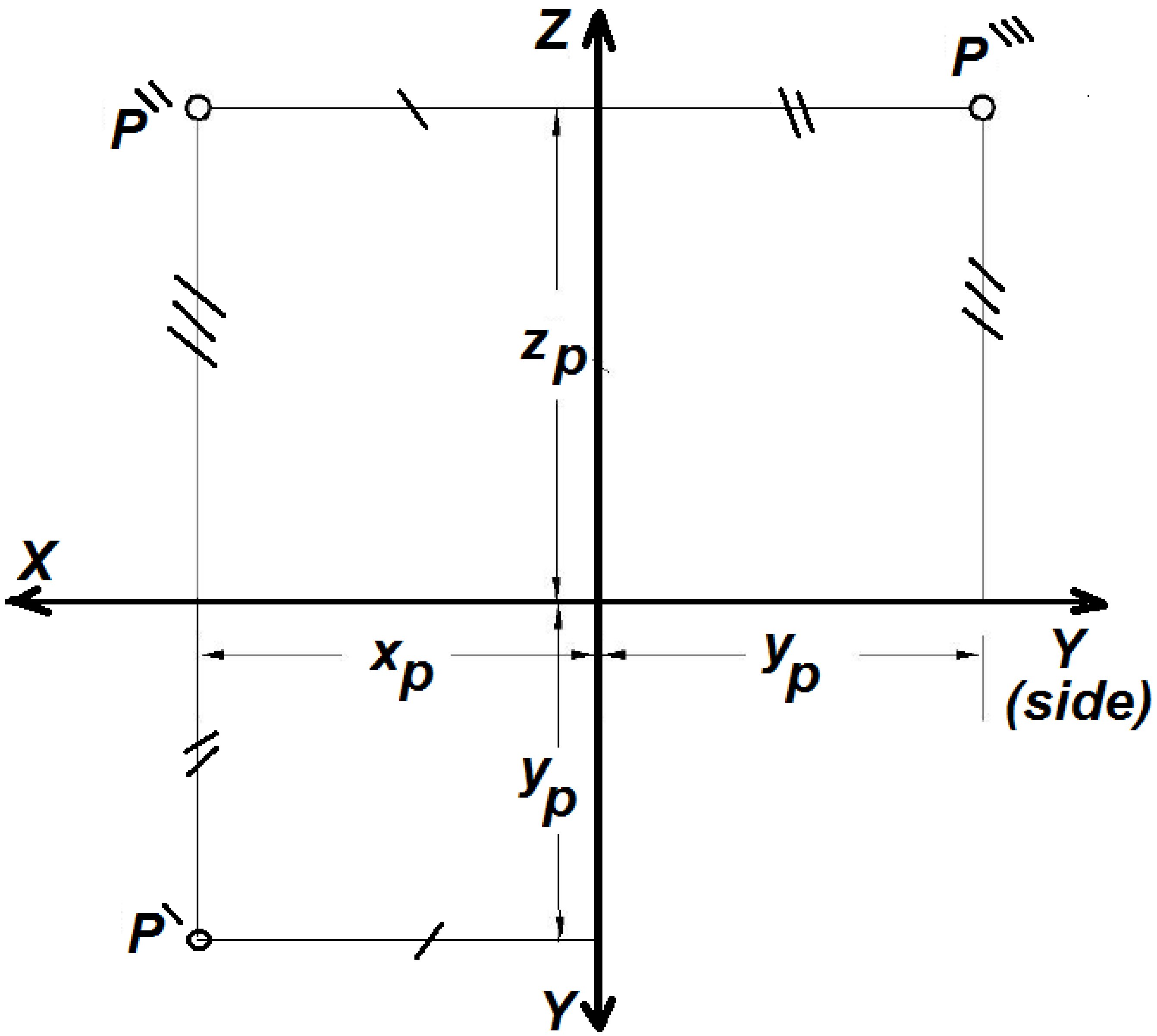

Figure (2.6)

Figure 2.7 shows the locus المحل الهندسي of the possible frontal, horizontal, and side projections of the different point that represented.

Example 1 Represent the following points (construct the horizontal and frontal projections on the drawing paper), A(2,3,1) , B(4,-3,1) , C(6,4,0) , D(7,0,2) , E(5,3,-3), F(-1,2,3) and G(-1.5,-2,-2.5).

Solution**:** the projections for the given points are illustrated on the figure (2.8).

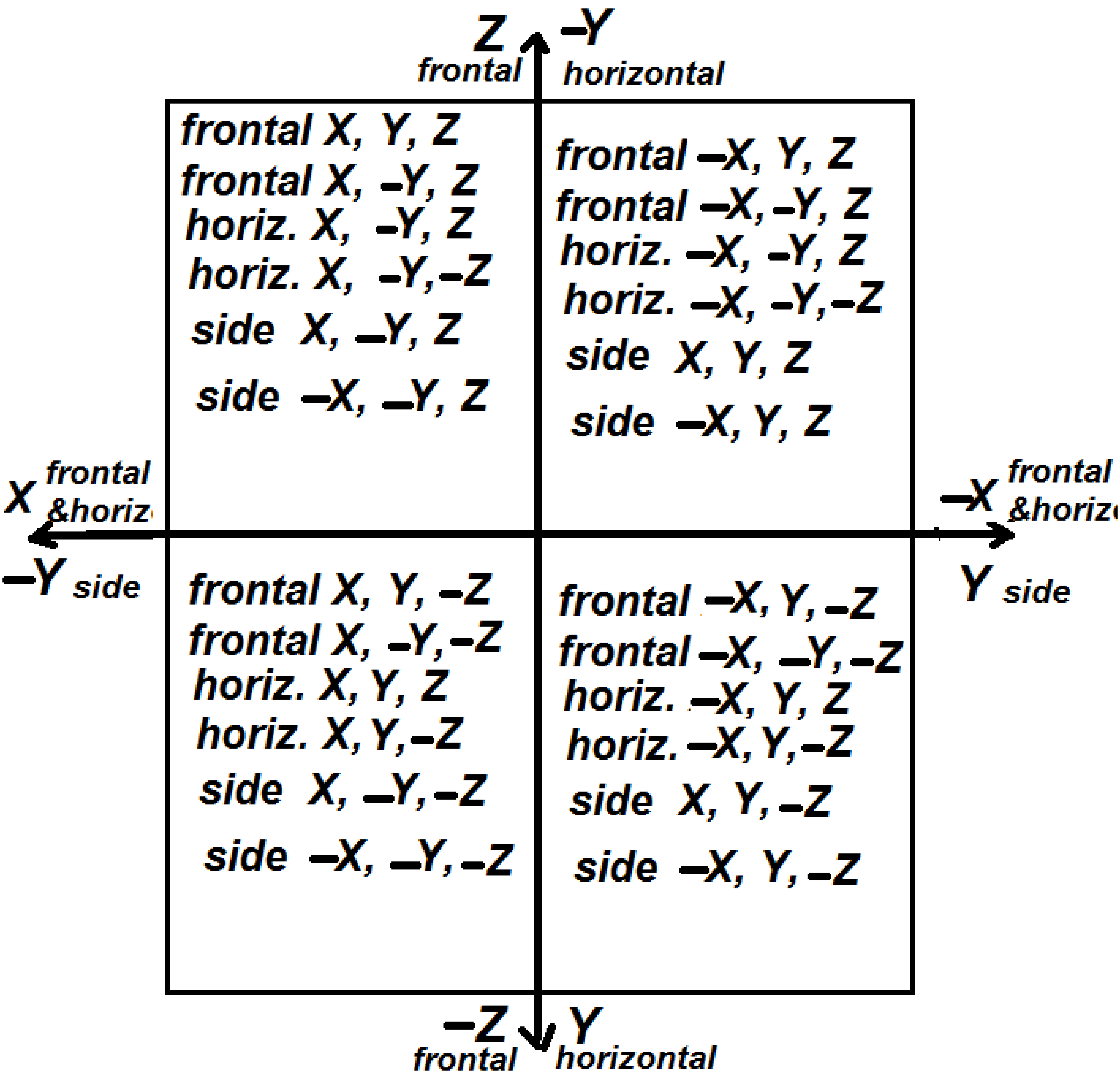

Figure (2.7) the possibility of point projections

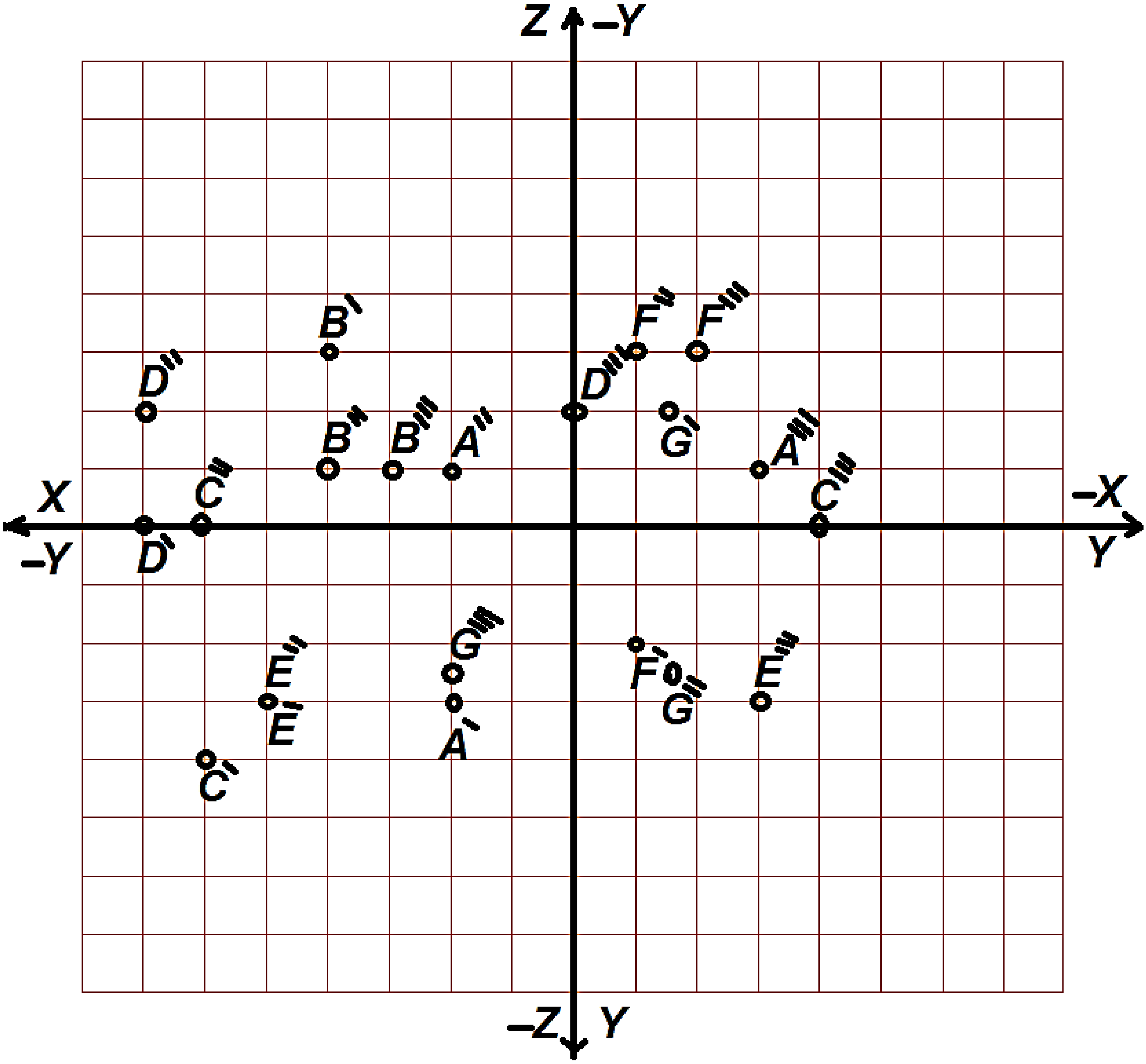

Figure (2.8)

Projection of a straight line

In geometry, the position of a straight line m is established ينشأ by locating any two non-coincident غير متطابقتين points A and B. The horizontal, frontal and side projections m', m'' and m''' of a line m are constructed by joining توصيل the like projection المساقط المتشابهة of the two points A and B, figure (2.9). Therefore m'= A'B', m''= A''B'', and m'''=A'''B''' as shown in Figure (2.10). To represent the projection of a line m; Determine the horizontal, vertical and side projections of the two points A and B (i.e. A', A'', A''' B', B'', and B''' are obtained); to specify m' join وصل A' with B' and join A'' with B'' gives m''. Also join A''' with B''' to

obtain m'''. In addition ***the straight line may be defined by two intersecting planes, and by a point and direction.***

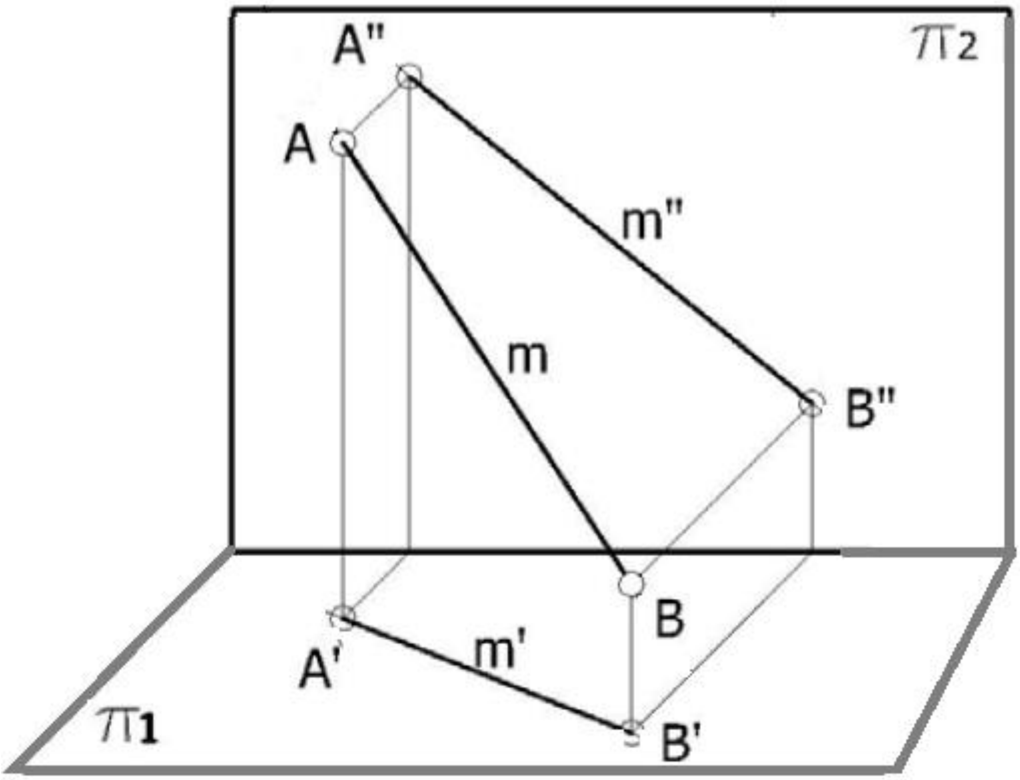

Figure (2.9)

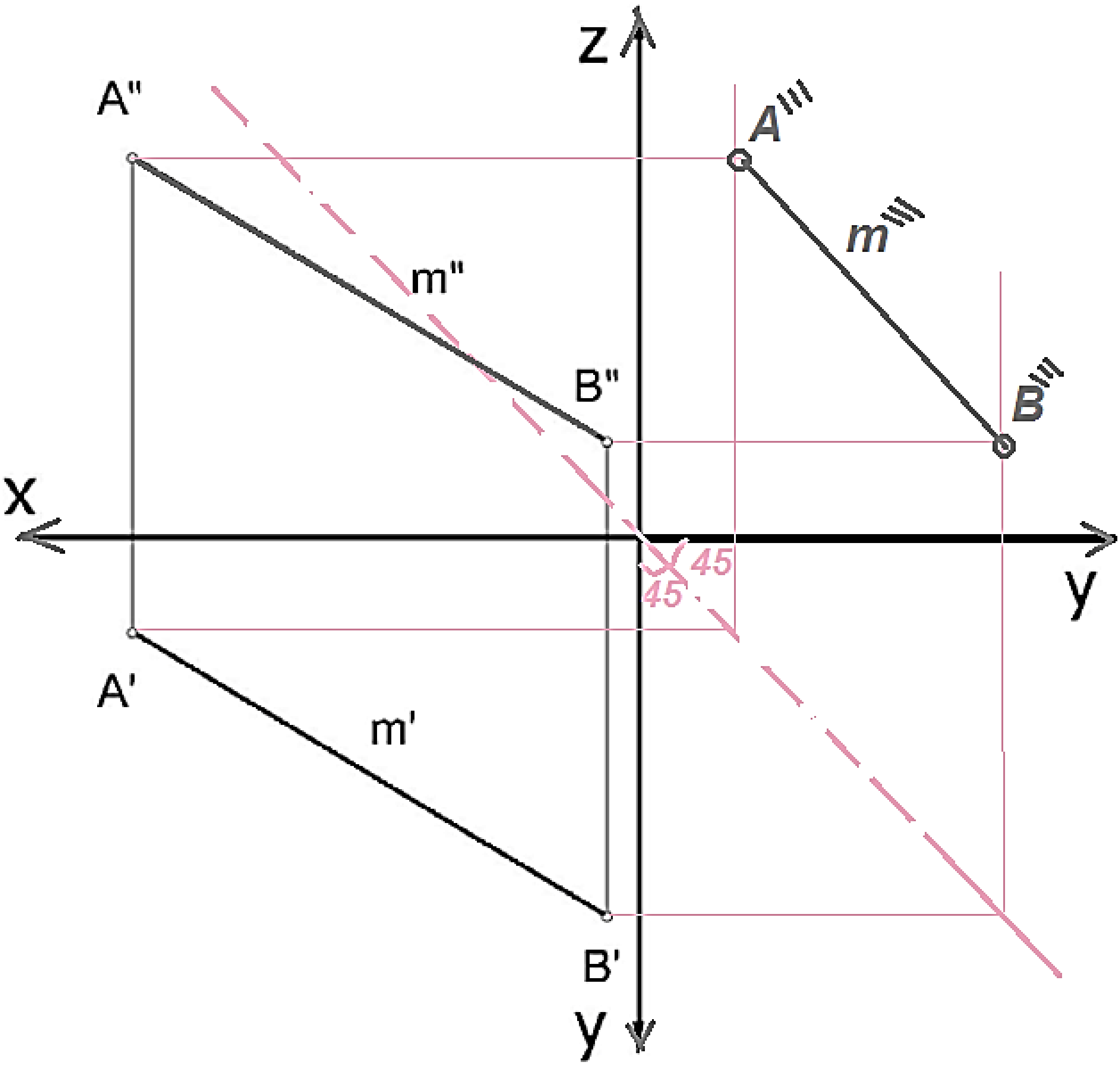

Figure (2.10)

Piercing Points (Traces) الاثر of a Line m

The trace of a line is the point in which the line intersects the projection plane. As shown in Figure (2.11), the traces of a line m are the points at which m intersects تقطع the two planes of projections π**1** and π**2** at T_1 and T_2 respectively. $T_1(x,y,0)$ lies تقع in the horizontal plane π1 (i.e. $T_1=T_1'$ and T_1'' lies on the ground line) and $T_2(x,0,z)$ lies in the frontal plane π**2** (i.e. $T_2=T_2''$ and T_2' lies on the ground line) thus $T_1(x,y)$ is called the horizontal trace while $T_2(x,z)$is called the frontal (vertical) trace.

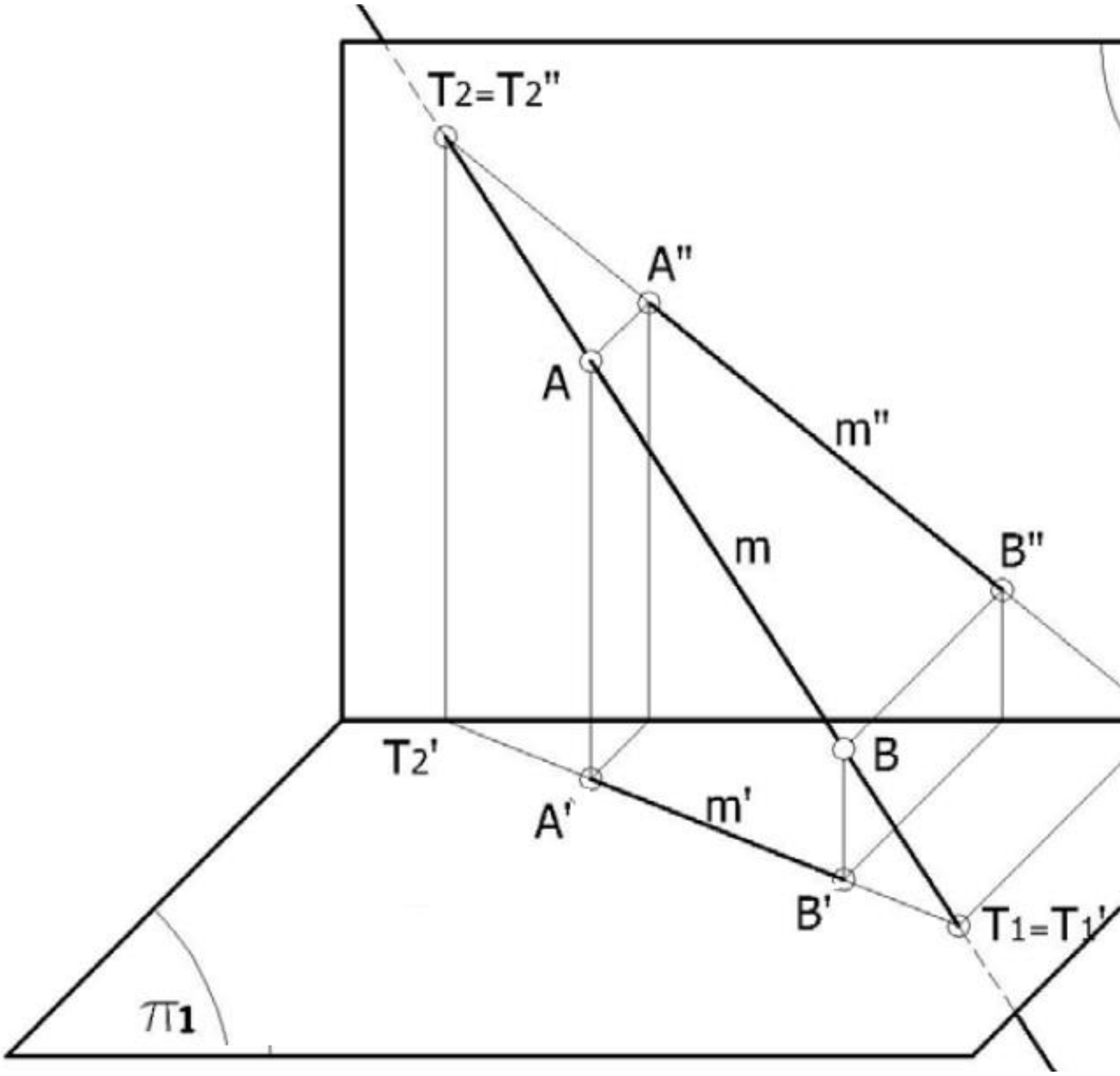

Figure (2.11)

Example 2

Represent a line m [A (2, 1, 4), B (7, 4, 1)] and find its traces.

Solution: [Figure (2.12)] First; Represent the horizontal and frontal projections (A’, B’, A” and B”) for the two given points. The horizontal and frontal projections of the line m are m’=A’B’ and m”=A”B”. to determine the trace T_1 and T_2 of the line m, extend both m’ and m” until they meet the x-axis at T_1’ and T_2” respectively. Erect أقم a perpendicular line to x-axis through T_1’ intersecting m’ at T_1, and another line through T_2” meeting m” at T_2. In general, if a line m is represented by it projections m’ and m” then, its traces T_1 and T_2 are easily obtained as following; Let the two projections m’ and m” meet the ground line at the two distinct points T_2’ and T_1”, Erect two perpendicular lines through T_2’ and T_1” to meet m” and m’ at T_2 and T_1 respectively. In other hand, if the traces (T_1 and T_2) of a line m are available then the two orthogonal projections m’ and m” are easily determined.

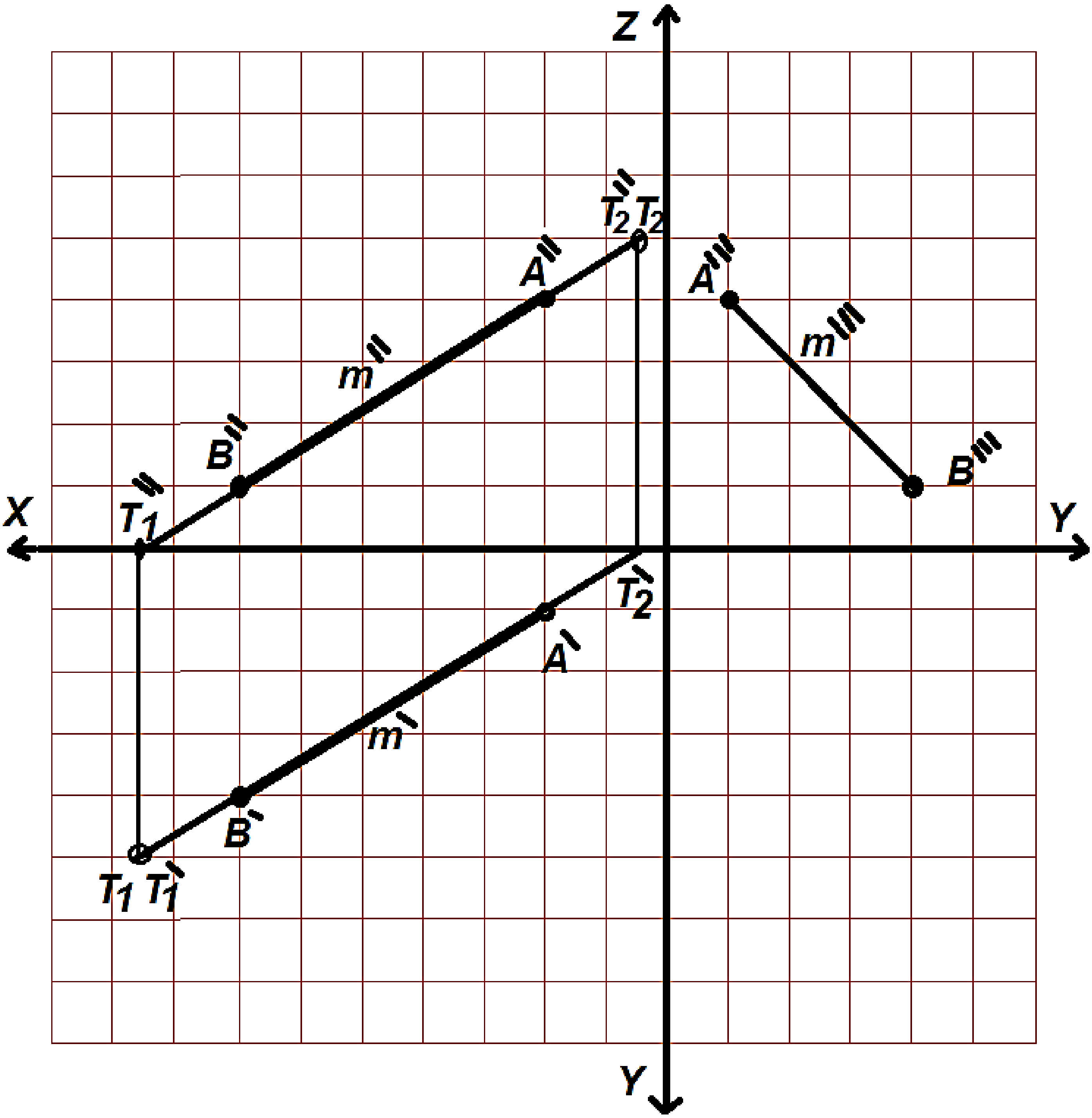

Figure (2.12)

Example 3: find the trace T3 of line m in example 2 on the side plane.

Solution: figure (2.13) below illustrate the trace T3 of line m in example 2 on the side plane.

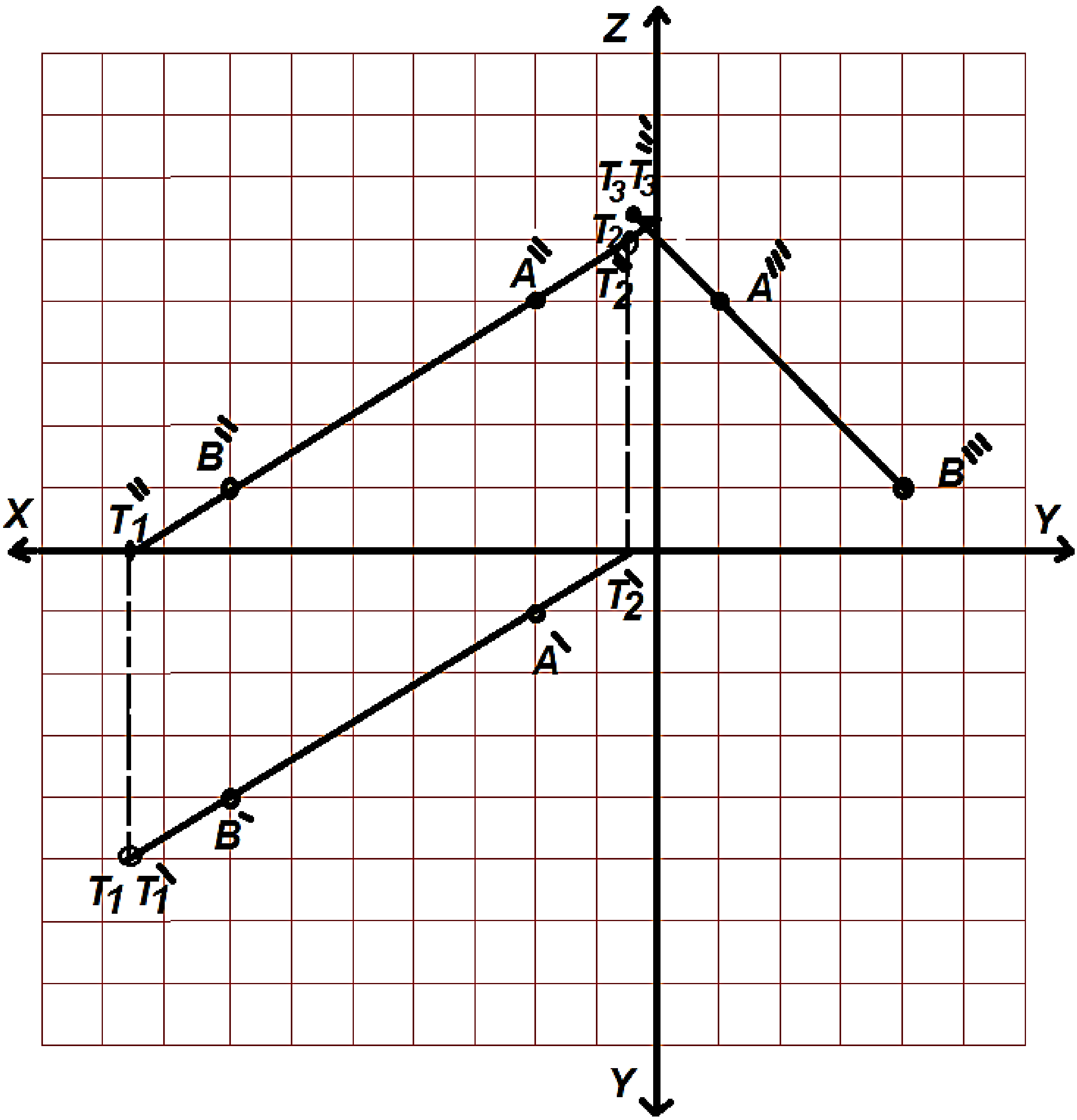

Figure (2.13)

<u>Example 4</u>: Determine the two orthogonal projections of a line k given its traces T_1 (1,-4, 0) and T_2 (6, 0, 3).

<u>Solution</u>: (Figure 2.14) represent the orthogonal projections for the two given traces points, keeping in mind the coordinates of T_1 (1,-4, 0), T_2 (6, 0, 3), then T_1', T_1'', T_2' and T_2'' are determined. To construct k', join T_1' with T_2' while joining T_1'' with T_2'' gives k''.

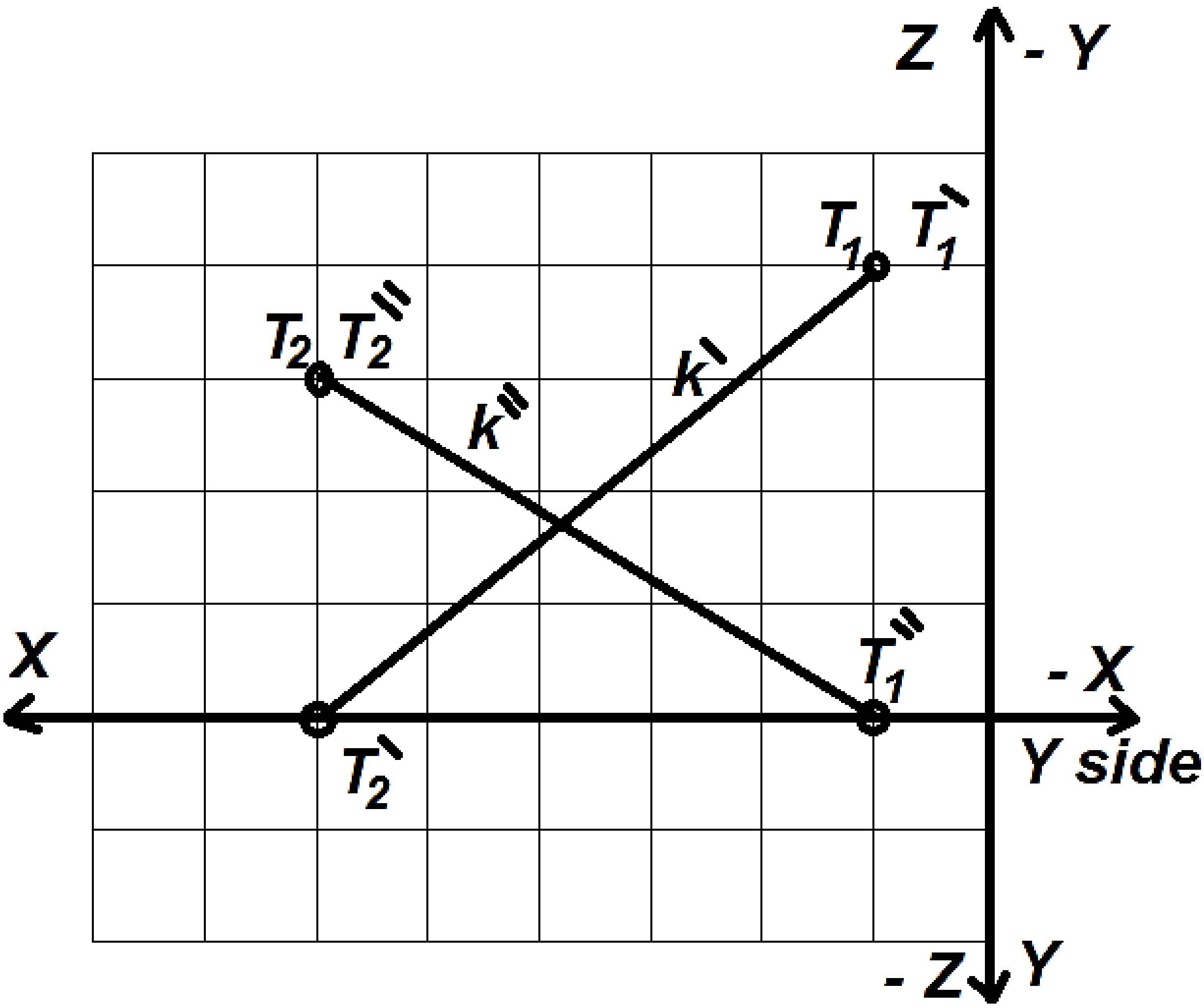

Figure 2.14

Particular Positions (معينة) مواقع خاصة of Straight Lines:

In some cases, a straight line may be either parallel or perpendicular to one, or more, of the principal planes of projection. In this case, the line is designated according to the plane which it parallel or perpendicular.

1. **Horizontal Line**

The horizontal line h is a line that is parallel to the horizontal plane of projection π_1 and makes an angle β with the vertical plane as shown in Figure (2.15). The figure shows the following:

a- **The frontal projection h" is drawn parallel to x-axis.**
b- **The horizontal projection h' makes an angle β with x-axis and equal to the considered the line h (true length of h).**

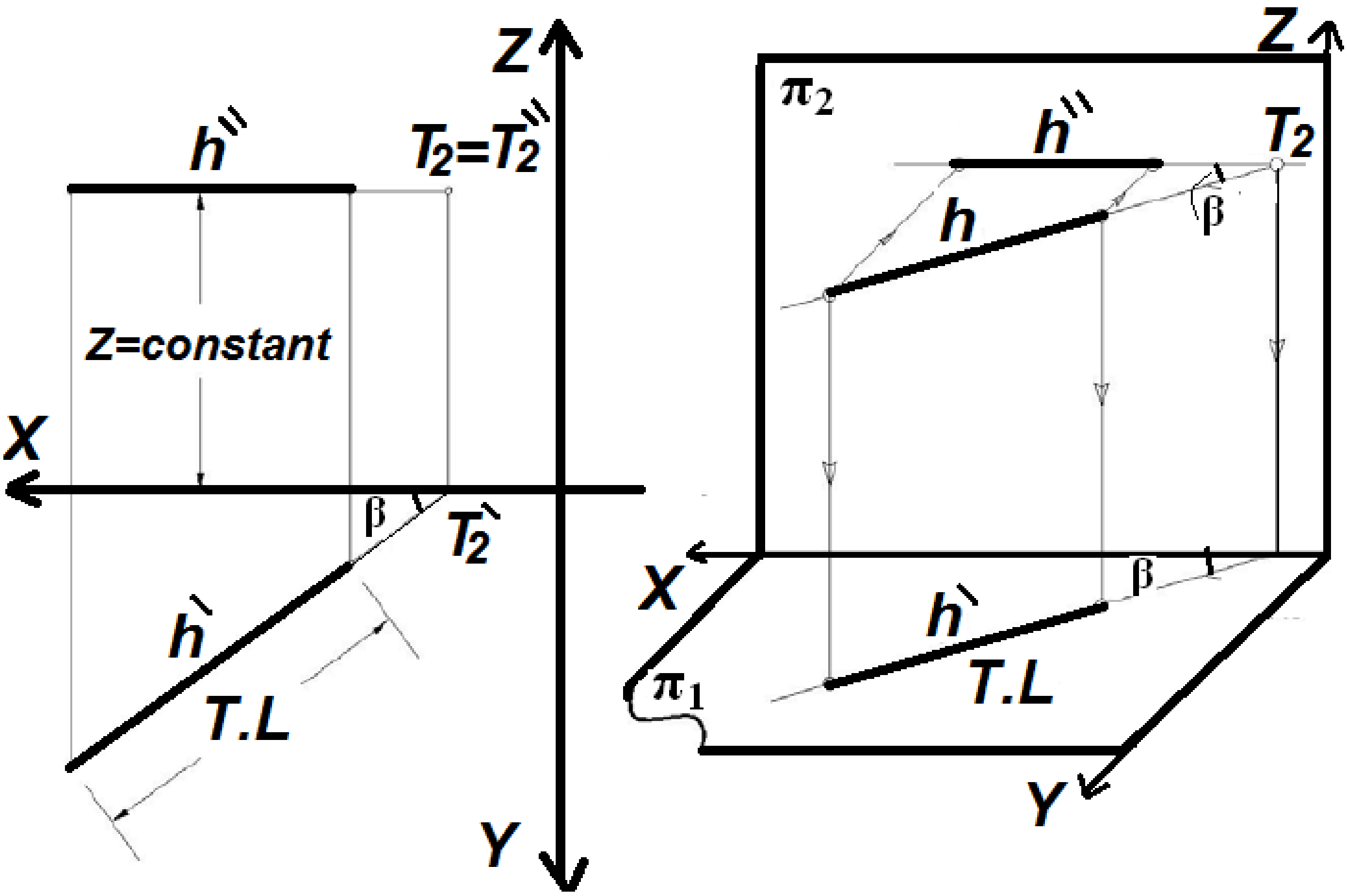

Figure 2.15

2. **Frontal line**
The frontal line f is a line that parallel to the frontal plane of projection π_2 and makes an angle α with the horizontal plane as shown in Figure (2.16). The figure shows the following:
1- The horizontal projection f' is drawn parallel to x-axis.

2- The frontal projection f" makes an angle α with x-axis and equals to the considered the line f (i.e. true length of f).

3- This line exhibits يظهر only a horizontal trace T_1 , and the frontal trace T_2 is at infinity في ما لانهاية.

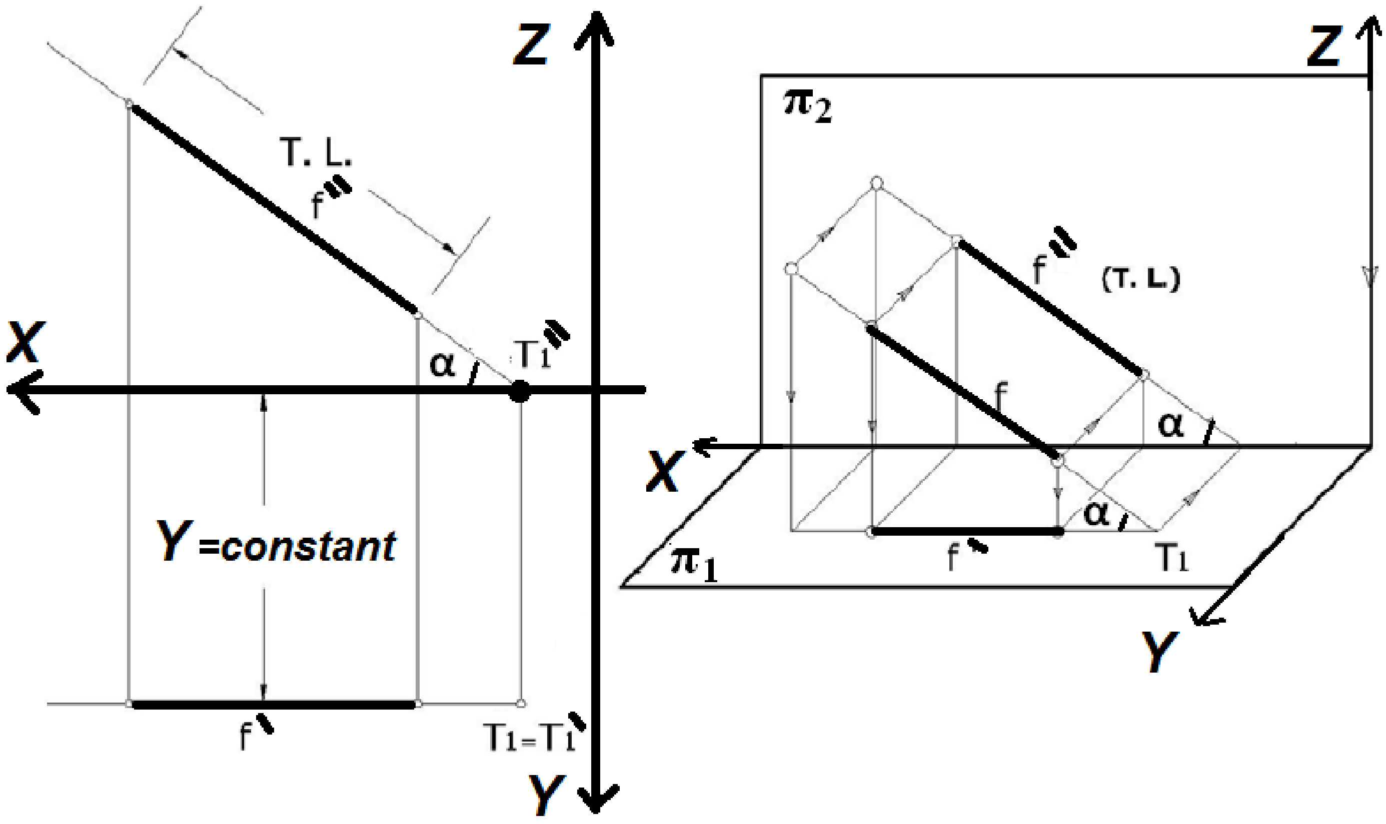

Figure 2.16

3. **Vertical Line v**

The vertical line v, Figure (2.17), is a line that parallel to the frontal plane of projection π_2 and perpendicular to the horizontal plane π_1. Figure shows the following:

1- The horizontal projection v' is a point.

2- The frontal projection v" is true length perpendicular to x-axis.

3- The horizontal trace T_1 coincides with the horizontal projection v' and the frontal trace T_2 is at infinity.

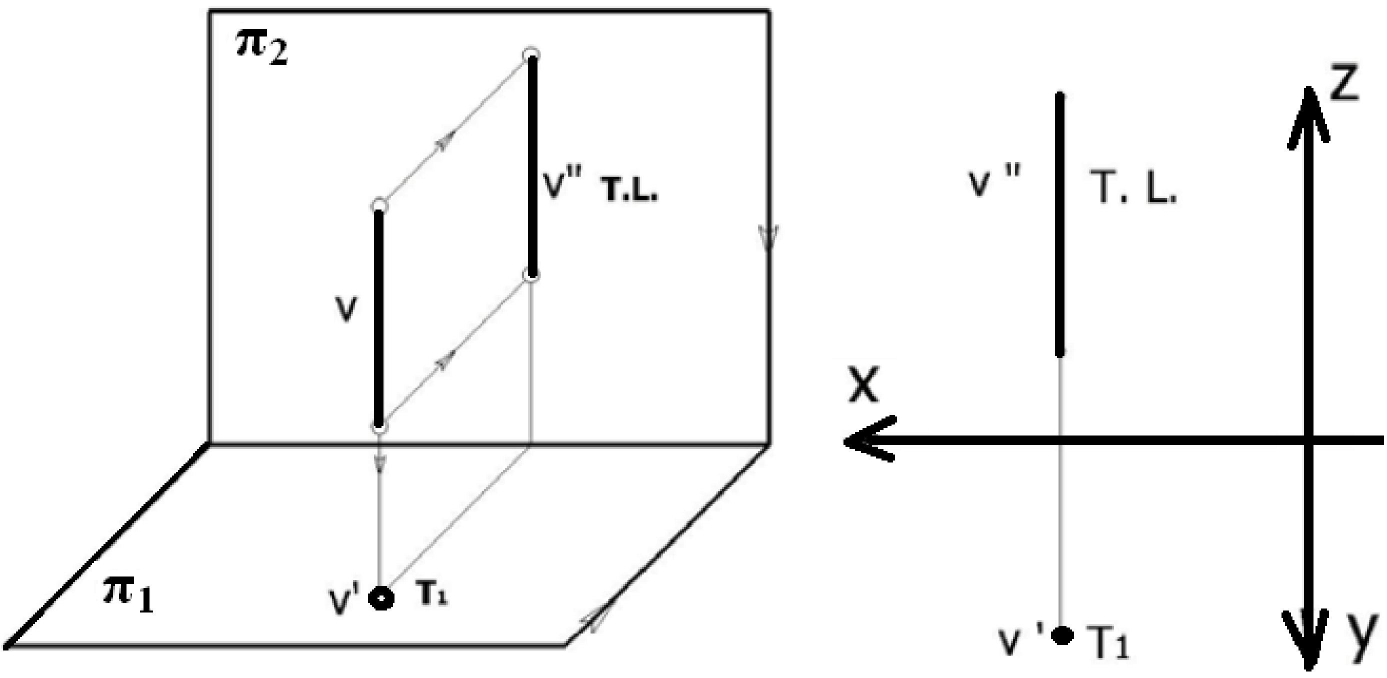

Figure 2.17

4. **A line u perpendicular to $\pi 2$**

The line u is a line which perpendicular to the frontal plane π_2 as in Figure (2.18). The figure shows the following:

1-The frontal projection u" is a point.

2-The horizontal projection u' is true length and perpendicular to x-axis.

3-The frontal trace T_2 coincides with u" and the horizontal trace T_1 is at infinity.

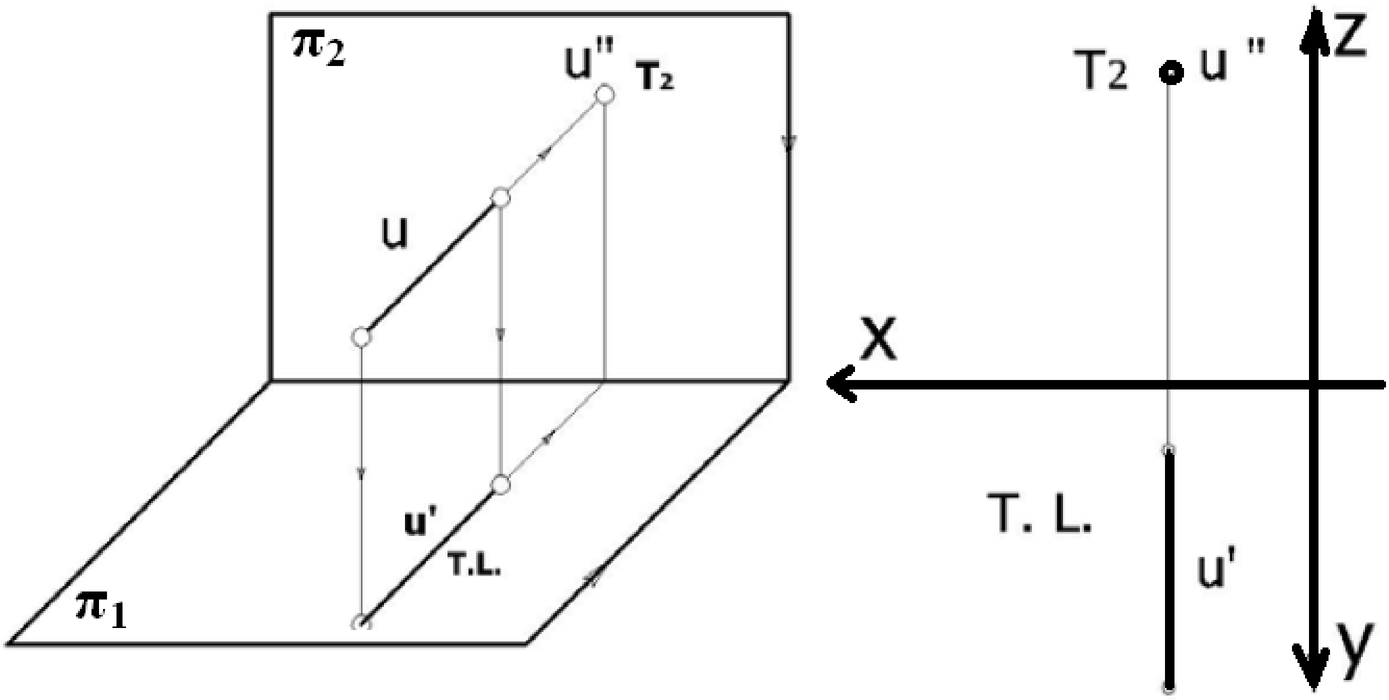

Figure 2.18

Example 5

Represent the triangle A(2,3,4) B(4,1,Z) C(8,Y,Z) on which AB is horizontal and BC is frontal of 5 units length.

Solution: **(Figure 2.19) Represent the projections A', A", B' and vertical line at x=8 on which C' and C" are belonged. Coordinate *z*=4 for point B because AB is a horizontal line and B" is drawn corresponding to B' (on the same vertical line). To establish C" and C', from B" and with length=5 intersect the vertical line, (x=8), determine C" [i.e. the true length of BC, BC is frontal]. Since BC is frontal, coordinate y=1 for point C (as point B), then C' is obtained (we have two positions for point C") and we can represent A"B"C" and A'B'C'.**

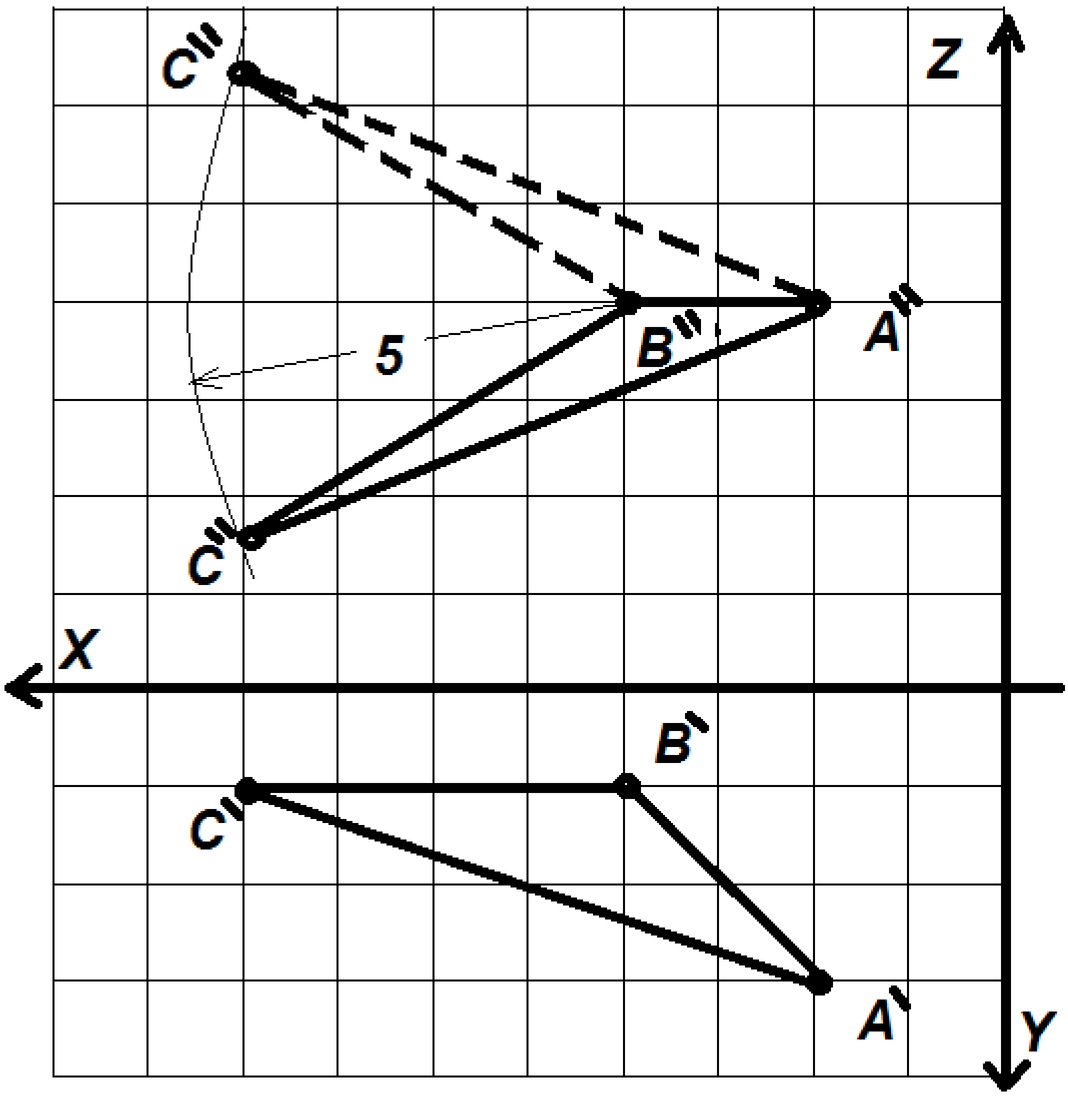

Figure 2.19