# Electrical Engineering Fundamentals

EE1

## 1 Basic Concepts & Basic Laws

### 1.1 Basic Concepts

#### 1.1.1 System of Units

The basic SI units

| Quantity | Basic unit | Symbol |
|---|---|---|
| Length | meter | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Luminous intensity | candela | cd |

The SI prefixes

| Multiplier | Prefix | Symbol |
|---|---|---|
| $10^{18}$ | exa | E |
| $10^{15}$ | peta | P |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10$ | deca | da |
| $10^{-1}$ | deci | d |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

Examples

$10\ MHz \Rightarrow 10 \times 10^{6}\ Hz$

$2\ mA = 2 \times 10^{-3} = 0.002\ A$

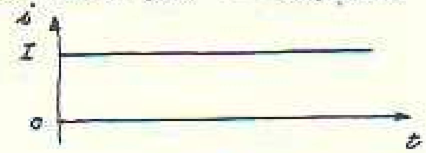
$5\ \mu s = 5 \times 10^{-6}\ s$

## 1.1.2 Charge and Current

The electric charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). The charge of an electron is ($-1.602 \times 10^{-19}$ C).

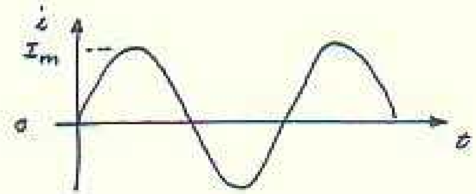
**Electric Current**: is the rate of change of charge, measured in amperes (A). The current (I) is defined mathematically as:

$$i = \frac{dq}{dt}$$



d.c current

$$\therefore \; q = \int_{t_1}^{t_2} i \, dt$$



a.c current

* A direct current (dc) is current that remains constant with time. The symbol (I) is usually used to represent such a constant current.

* An alternating current (ac) is a current that is varying sinusoidally with time. A time varying current is represented by the symbol ($i$).

**Example**: Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s, if the current passing the terminal is $i = (3t^2 - t)$ A.

**Solution**:

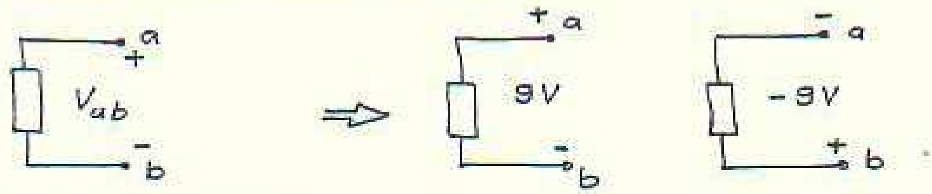
$$q = \int_{t_1}^{t_2} i \, dt$$

$$= \int_{1}^{2} (3t^2 - t) \, dt = \left( t^3 - \frac{t^2}{2} \right) \Big|_1^2$$

$$= (8 - 2) - \left(1 - \frac{1}{2}\right) = \underline{\underline{5.5 \text{ C}}}$$

### 1.1.3 Voltage

: The voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).




Polarity of Voltage $V_{ab}$

* For the voltage $V_{ab}$ ⇒ This means that the potential of point a is higher than that of point b

$$V_{ab} = V_a - V_b$$

### 1.1.4 Power and Energy :

* Power : is the time rate of expending or absorbing energy, measured in watts (W)

$$\Rightarrow \quad p = \frac{dw}{dt}$$

where p is the power in watts (W), w is the energy in joules (J), and t is the time in seconds (s)

We have; $p = \frac{dw}{dt} \implies p = \frac{dw}{dt} \cdot \frac{dq}{dq}$

$$\Rightarrow p = \frac{dw}{dq} \cdot \frac{dq}{dt} = v.i$$

$$\therefore \; p = vi$$

* The energy absorbed or supplied by an element

from time $\underline{t_o}$ to time $\underline{t}$ is :

$$\boxed{w = \int_{t_o}^{t} p\,dt = \int_{t_o}^{t} vi\,dt}$$

Energy is the capacity to do work, measured in joules (J)

* The electric power utility companies measure energy in watt-hour (Wh), where

$$\boxed{1 \text{ Wh} = 3{,}600 \text{ J}}$$

__Example__ : How much energy does a 100 W electric bulb consume in 2 hours ?

__Solution__ :

$$w = pt = 100 \times 2 = 200 \text{ Wh}$$

or

$$w = pt = (100 \text{ W})(2 \times 60 \times 60) = 720000 \text{ J} = 720 \text{ kJ}$$

which is the same result (if you convert from joules to watts or vise-versa).

## 1.1.5 Circuit Elements

: An electric circuit is an interconnection of electrical elementic.

* Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

Circuit Elements
- → Passive Elements (can not generate energy) ⇒ Resistors, capacitors, and inductors
- → Active Elements (can generate energy) ⇒ Generators, batteries, operational amplifiers

* The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. 551

Voltage (or current) sources < Independent sources / Dependent sources

* An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

* Dependent sources (or controlled sources) are active elements in which the source quantity is controlled by another voltage or current. (It will be discussed later).

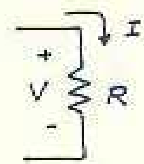
## 1.2 Basic Laws:

### 1.2.1 Ohm's Law (1826):

Ohm's Law states that the voltage $V$ across a resistor is directly proportional to the current $I$ flowing through the resistor.

$$V \propto I$$

$$\Rightarrow \boxed{V = IR}$$ قانون اوم

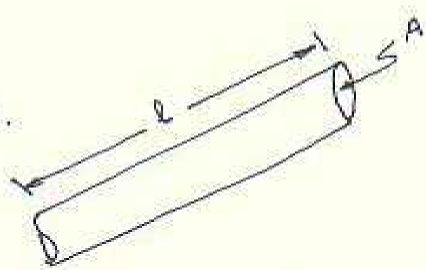



where $R$ is the resistance. The resistance $R$ denotes the ability of an element to resist the flow of electric current, it is measured in ohms ($\Omega$).

For any material, the resistance $R$ depends on its physical dimensions as follows:

$$R = \rho \frac{\ell}{A}$$

where $\rho$ is the resistivity of the material.




⇒ Good conductors have low resistivities (such copper, aluminum, etc..)

⇒ Insulators have high resistivities (such as mica, paper, etc...)

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## Resistivities of Common Materials

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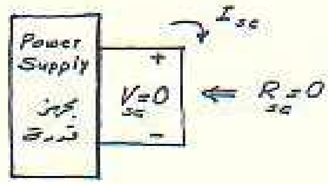
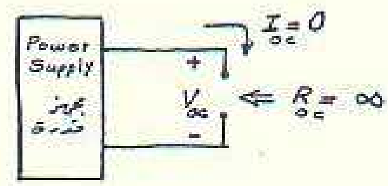
| Material | Resistivity ($\Omega . m$) | Usage |
|---|---|---|
| Silver | $1.64 \times 10^{-8}$ | Conductor |
| Copper | $1.72 \times 10^{-8}$ | " |
| Aluminum | $2.80 \times 10^{-8}$ | " |
| Gold | $2.45 \times 10^{-8}$ | " |
| Carbon | $4.00 \times 10^{-5}$ | Semiconductor |
| Germanium | $47.0 \times 10^{-2}$ | " |
| Silicon | $6.40 \times 10^{-2}$ | " |
| Paper | $10^{10}$ | Insulator |
| Mica | $5 \times 10^{11}$ | " |
| Glass | $10^{12}$ | " |
| Teflon | $3 \times 10^{12}$ | " |

* The resistance of a short circuit element is approaching zero
* The resistance of an open circuit is approaching infinity.




Short circuit with $R_{sc}=0$
$V_{sc}=0$




Open circuit with $R_{oc}=\infty$
$I_{oc}=0$

* Conductance (G)

A useful quantity in circuit analysis is the reciprocal of resistance (R), is called the conductance (G);

$$G = \frac{1}{R} = \frac{i}{v}$$

The conductance can be explained as the ability of an element to conduct electric current, it is measured in mhos (℧) or in siemens (S).

$$\therefore \quad i = Gv$$

and :

$$p = vi = i^2 R = \frac{v^2}{R} \qquad \text{watts (W)}$$

OR

$$p = vi = v^2 G = \frac{i^2}{G}$$

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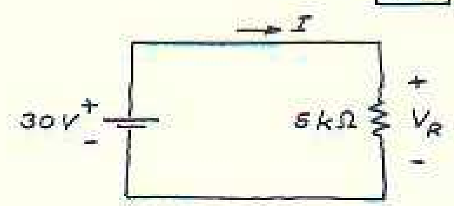
**Example**: In the circuit shown, calculate the current $I$, the conductance $G$, and the power $P$




**Solution**: $V_R = V$

- the current $\quad I = \dfrac{V_R}{R} = \dfrac{30}{5\times10^3} = 6\times10^{-3} = \underline{6\ mA}$

- the conductance $\quad G = \dfrac{1}{R} = \dfrac{1}{5\times10^3} = 0.2\times10^{-3} = 0.2\ mS$

- the power $\quad P = V_R I = 30\,(6\times10^{-3}) = \underline{180\ mW}$

  or $\quad P = I^2R = (6\times10^{-3})^2(5\times10^{3}) = \underline{180\ mW}$

  or $\quad P = V_R^2 G = (30)^2(0.2\times10^{-3}) = \underline{180\ mW}$

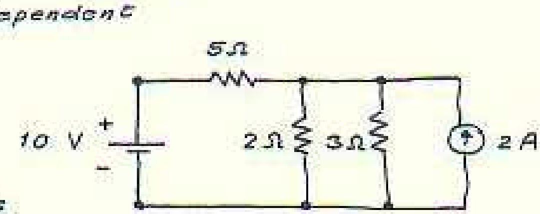
## 1.2.2 Nodes, branches and loops

* A branch represents a single element in the electric circuit, such as a voltage source or a resistor etc...

* A node represents the point of connection between two or more branches.

* A loop is any closed path in a circuit.

**Example**: For the circuit shown, determine the number of branches, nodes and the independent loops.




**Solution**: Since there are 5 elements

$\Rightarrow$ Number of branches = $\underline{5}$ $\qquad$ 10V, 5Ω, 2Ω, 3Ω, and 2A

Number of nodes = $\underline{3}$ (as shown in the figure).

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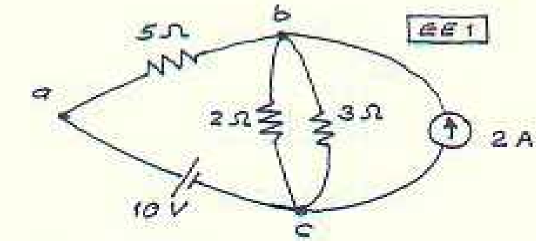
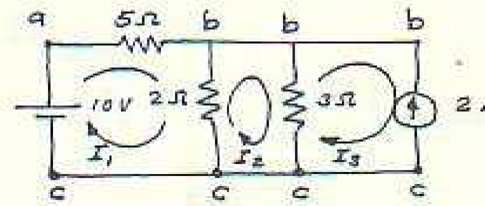
⇒ There are 3 nodes :
a , b and c




* The number of the independent loops = 3




⇒ loop 1 or loop abc :
contains (10V, 5Ω, 2Ω)

⇒ loop 2 or loop bcb :
contains (2Ω , 3Ω)

& ⇒ loop 3 or loop bcb :
contains ( 3Ω , 2A)

Notes : There are more than 3 (dependent) loops in this example , we had only calculated the INDEPENDENT loops which are only 3.

IN GENERAL ; Any circuit with b branches , n nodes and ℓ independent loops , the following fundamental theorem of network topology :

$$b = \ell + n - 1$$

* Two or more elements are in SERIES if they are cascaded sequentially and consequently carry the SAME current.

* Two or more elements are in PARALLEL if they are connected to the same two nodes and have consequently the same VOLTAGE across them.

## 1.2.3 Kirchoff's Laws (1847) :

قانون كيرشوف للتيار * Kirchoff's current law (KCL) ; states that the algebraic sum of all current entering a node is zero or : The sum of currents entering a node is equal to the sum of curents leaving that node.

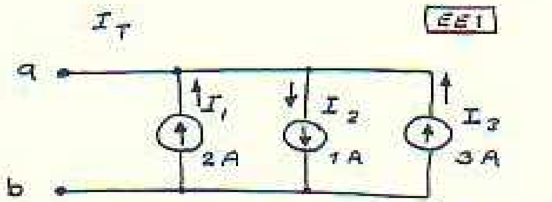
$$\sum_{n=1}^{N} I_n = 0$$

or

$$\sum_{m=1}^{M} I_{mi} = \sum_{n=1}^{N} I_{no}$$

where $I_{mi}$ are the currents entering the node and $I_{no}$ are the currents leaving the node.

**Example**: For the network shown, calculate the total current $I_T$



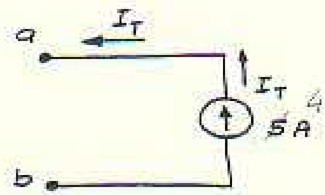

Current Sources in Parallel

**Solution**

According to KCL;

$$I_T = I_1 - I_2 + I_3$$
$$= 2 - 1 + 3 = \underline{\underline{5\ A}}$$

∴ The equivalent circuit for the network can be as shown ⇒




* Kirchoff's Voltage Law (KVL); states that the algebraic sum of all voltages around a closed path (or loop) is zero.

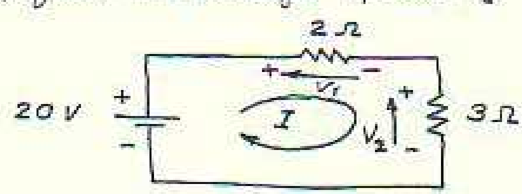
قانون كرشوف للفولتية

∴ Mathematically KVL states that:

$$\boxed{\sum_{m=1}^{M} V_m = 0}$$

where $M$ is the number of voltages in the loop (or the number of branches in the loop), and $V_m$ is the $\underline{m\text{th}}$ voltage.

**Example**: For the circuit shown, find the voltages $V_1$ and $V_2$




**Solution**

$$V_1 = 2I$$
$$V_2 = 3I$$

From KVL:

$$\sum V = 0 \Rightarrow 20 - V_1 - V_2 = 0 \Rightarrow 20 = 3I + 2I$$

$$\Rightarrow 5I = 20 \Rightarrow I = 4\ A$$

$$\therefore V_1 = 2I = \underline{\underline{8V}} \quad \text{and} \quad V_2 = 3I = \underline{\underline{12V}}$$

# Tutorial Sheet № 1

## Basic Concepts & Basic Laws

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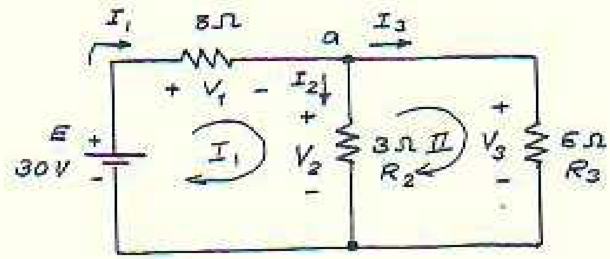
**Example**

Using Kirchoff's laws, find the currents and voltages in the circuit shown.




**Solution**:

ملاحظة : المطلوب في المثال ايجاد كل من :
$I_1$ , $I_2$ , $I_3$ , $V_1$ , $V_2$ & $V_3$
باستخدام قانوني كيرشوف

* Using Ohm's law:

$$V_1 = I_1 R_1 = 8 I_1$$
$$V_2 = I_2 R_2 = 3 I_2$$
$$V_3 = I_3 R_3 = 6 I_3$$

* Applying KCL at node a :

$$I_1 = I_2 + I_3 \Rightarrow \boxed{I_1 - I_2 - I_3 = 0} \quad Eq(1)$$

* Applying KVL to loop 1 ;

$$E - V_1 - V_2 = 0 \Rightarrow 30 - V_1 - V_2 = 0$$

$$\Rightarrow 30 - 8I_1 - 3I_2 = 0$$

$$\therefore \boxed{I_1 = \frac{30 - 3I_2}{8}} \quad Eq(2)$$

* Applying KVL to loop 2 ;

$$V_2 - V_3 = 0 \Rightarrow V_2 = V_3$$

وهذا واضح لأن
$R_2$ & $R_3$ متوازيتان

$$\therefore 6I_3 = 3I_2$$

$$\boxed{\therefore I_3 = \frac{I_2}{2}} \quad Eq(3)$$

⇒ From Eq(1), Eq(2) & Eq(3)

$$\frac{30 - 3I_2}{8} - I_2 - \frac{I_2}{2} = 0 \Rightarrow I_2 = \underline{\underline{2\ A}}$$

$$\text{and } I_1 = \frac{30 - 3I_2}{8} = \frac{30 - 3(2)}{8} = \underline{\underline{3\ A}}$$

$$I_3 = \frac{I_2}{2} = \frac{2}{2} = 1\ A$$

TS1

$\Rightarrow \because V_1 = 8 I_1 \Rightarrow \therefore V_1 = 8(3) = 24\ V$

Similarly $V_2 = 3 I_2 = 3(2) = 6\ V$

and $V_3 = 6 I_3 = 6(1) = 6\ V$

للتأكد من صحة الحل :

$I_1 = I_2 + I_3$

$3 = 2 + 1 \Rightarrow 3 = 3$

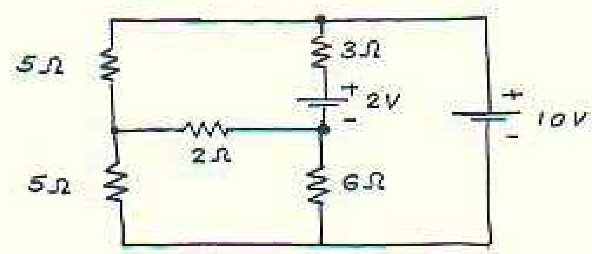
كذلك فإن تطبيق قانون كيرشوف للفولتية على المسار المغلق رقم 1 ينتج ما يأتي :

$E = V_1 + V_2$

$\therefore 30 = 24 + 6$

$\therefore 30 = 30$ ⟵ ∴ الحل صحيح

Example: Determine the number of branches, nodes and independent loops in the circuit shown.




Solution:

* There are 7 element ⇒ no. of branches = 7
  $\Rightarrow b = 7$

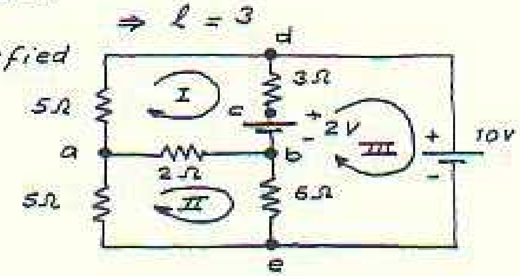
a, b, c, d, e ⟸ * There are 5 nodes as shown in the figure:
  $\Rightarrow n = 5$

I, II, III ⟸ * There are 3 independent loops:
  $\Rightarrow l = 3$

$\therefore b = l + n - 1$ is satisfied

since $7 = 3 + 5 - 1$

$\Rightarrow 7 = 7$



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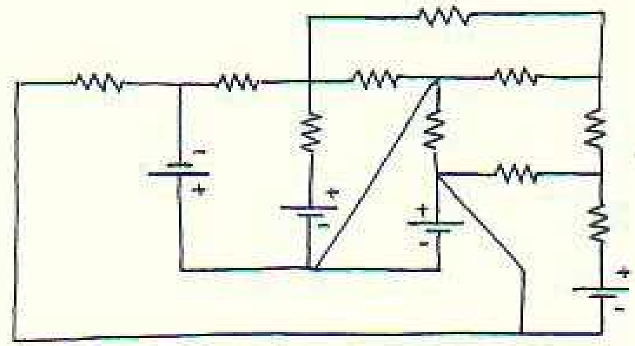
**Practice Problem**: Identify all nodes, branches and independent loops in the circuit shown in the figure.




**Answer**:

no. of nodes = 8 $\quad n = 8$

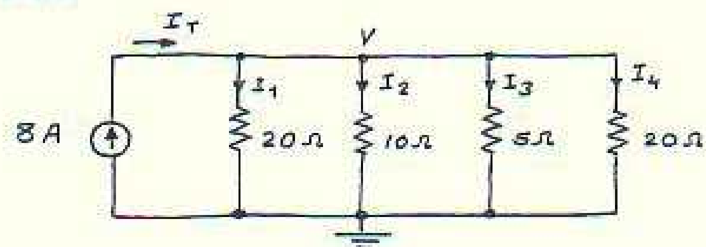
no. of branches = 14 $\quad b = 14$

no. of independent loops = 7 $\quad \ell = 7$

Check. Does this satisfy the fundamental theorem of network topology?

$$b = \ell + n - 1 = 7 + 8 - 1 = 14 \quad \underline{\text{YES}}$$

**Example**: Determine all currents and voltages in the circuit of the figure shown.




**Solution**: KCL ⇒

$$I_T = I_1 + I_2 + I_3 + I_4 \Rightarrow 8 = I_1 + I_2 + I_3 + I_4$$

Ohm's law

$$V = 20 I_1 \Rightarrow I_1 = V/20$$
$$= 10 I_2 \qquad I_2 = V/10$$
$$= 5 I_3 \qquad I_3 = V/5$$
$$= 20 I_4 \qquad I_4 = V/20$$

TS1

Substituting in the current equation;

$$\Rightarrow \quad 8 = \frac{V}{20} + \frac{V}{10} + \frac{V}{5} + \frac{V}{20}$$

$$\therefore \quad 160 = V + 2V + 4V + V$$

$$160 = 8V$$

$$\therefore \quad \underline{V = 20 \text{ Volts}}$$

$$\therefore \quad I_1 = \frac{V}{20} = \frac{20}{20} = 1 \text{ A}$$

$$I_2 = \frac{V}{10} = \frac{20}{10} = 2 \text{ A}$$

$$I_3 = \frac{V}{5} = \frac{20}{5} = 4 \text{ A}$$

$$I_4 = \frac{V}{20} = \frac{20}{20} = 1 \text{ A}$$

<u>Check</u>

$$I_T = I_1 + I_2 + I_3 + I_4$$

$$8 = 1 + 2 + 4 + 1$$

$$8 = 8 \quad \checkmark$$