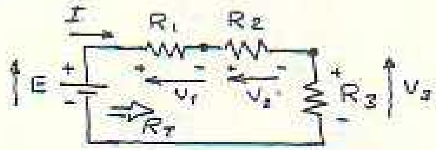
2. Circuit Transformations

2.1 Series Circuits

Series Circuits: Two elements are in series if they have only one point in common that is not connected to other current carrying elements of the network.

For the series cct. shown, using KVL we have:



$$E = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$
$$= IR_T$$

$$\Rightarrow \boxed{R_T = R_1 + R_2 + R_3}$$

$$\therefore \boxed{I = \frac{E}{R_T}}$$

In general, for a series cct. consisting N resistors, then the total resistance of such a cct. R_T is given as

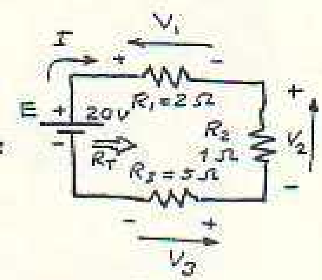
$$R_T = R_1 + R_2 + R_3 + \text{-----} R_N$$

Example: For the cct. shown;

a. Find the total resistance

b. Calculate the current I

c. Determine the voltages V_1, V_2 and V_3



Solution:

a. $R_T = R_1 + R_2 + R_3$
$= 2 + 1 + 5 = \underline{8\,\Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20}{8} = \underline{2.5\text{ A}}$

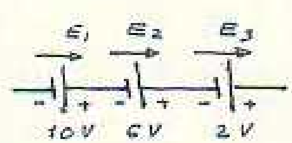
c. $V_1 = IR_1 = (2.5)(2) = 5\text{ V}$
$V_2 = IR_2 = (2.5)(1) = 2.5\text{ V}$
$V_3 = IR_3 = (2.5)(5) = 12.5\text{ V}$

ملاحظة
$V_1 + V_2 + V_3 = E$
$5 + 2.5 + 12.5 = 20\text{ V}$
$\therefore 20\text{ V} = 20\text{ V}$

2.1.1 Voltage Sources in Series

Voltage Sources in Series: The net voltage will the algebraic sum of all sources that are connected in series.

Example:



$\Rightarrow$

$E_T = 18V$

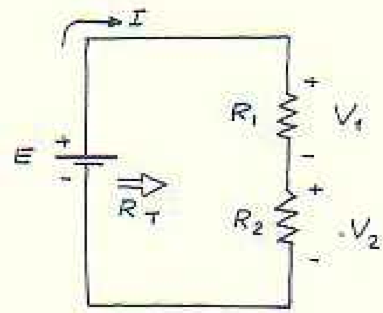
$$E_T = E_1 + E_2 + E_3$$
$$= 10 + 6 + 2 = 18\text{ V}$$

ملاحظة - لو عكس ربط أحد مصادر البطاريات
على التوالي فإن ذلك مخالف لقانون كيرشوف للتيار

EE2

2.1.2 Voltage Divider Rule

: Consider the series cct. shown.



we have:

$$R_T = R_1 + R_2$$

and $$I = \frac{E}{R_T}$$

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right).R_1 = \frac{E.R_1}{R_T}$$

Similarly;

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right).R_2 = \frac{E.R_2}{R_T}$$

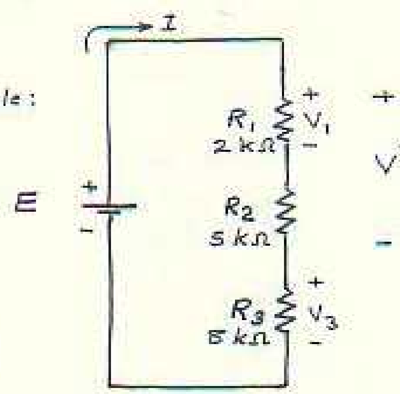
Hence, we can write:

$$\boxed{V_x = \frac{E.R_x}{R_T}}$$ ⟸ Voltage divider rule

This means that "the voltage divider rule" can be understood to state that:

> The voltage across a resistor in a series circuit is equal to the value of that resistor times the the total applied voltage across the series elements divided by the total resistance of the series elements.

Example: Determine the voltages V_1, V_3, and V' for the cct. shown.



Solution: Using the voltage divider rule:

$$V_1 = \frac{E.R_1}{R_T} = \frac{(45).(2\times10^3)}{(2+5+8)\times10^3} = \underline{6\,V}$$

$$V_3 = \frac{E.R_3}{R_T} = \frac{(45)(8\times10^3)}{(2+5+8)\times10^3} = \underline{24\,V}$$

EE2

$$V' = \frac{E \cdot R'}{R_T} \Leftarrow R' = (2+5)\times 10^3\ \Omega = 7\times 10^3\ \Omega$$

$$= \frac{(45)\cdot(7\times 10^3)}{(2+5+8)\times 10^3}$$

$$= \underline{21\ V}$$

نلاحظ انه :

$$E = V_3 + V'$$

$$45\ V = 24\ V + 21\ V$$

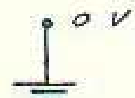
$$45\ V = 45\ V$$

نلاحظ ان فرق الجهد على طرفي اي عنصر من عناصر دائرة التوالي يتناسب مع مقاومة ذلك العنصر ، أي ان المقاومة الكبيرة يقابلها فرق جهد كبير والمقاومة الصغيرة يقابلها فرق جهد صغير ، وفي جميع الاحوال يجب ان يكون مجموع فروق الجهد مساوياً الى فولتية المصدر .

NOTATION

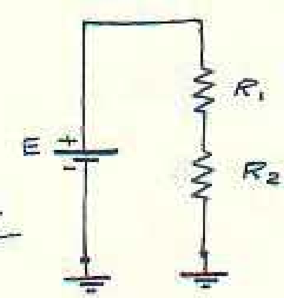
ground potential

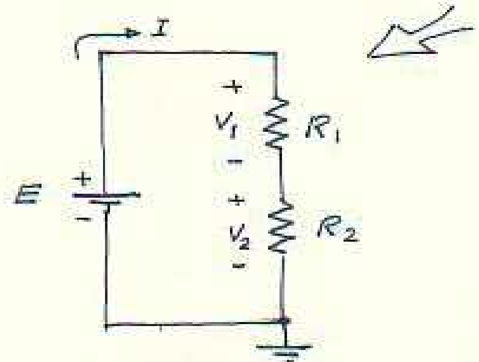
It is common, for safety purposes and as a reference to ground electrical and electronic systems. The symbol for the ground connection is :



with its defined potential level (zero volts). As a consequence, the cct. might need to be redrawn in the ordinary form so as to be analyzed.

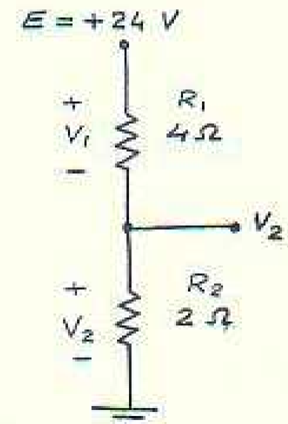
Example : For the cct. shown ; with ground potentials connected for the source and to elements; this cct. can be redrawn to make it easy analyzing it.





EE2

Example: Using the voltage divider rule, determine the voltages V_1 and V_2 for the cct shown:

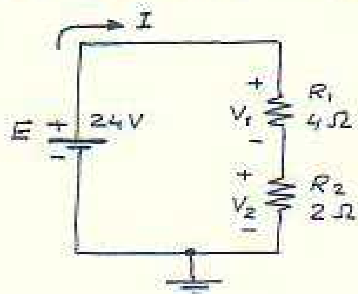


Solution: Redrawing the cct. with the standard battery symbol, then the cct. will be as shown below:

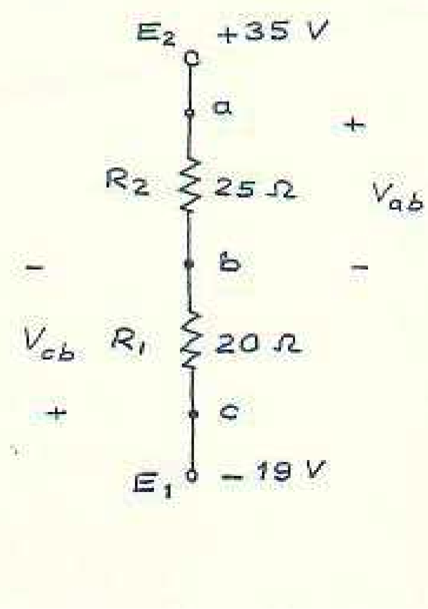
$$\therefore \; V_1 = \frac{R_1 E}{R_T} = \frac{(4)\cdot(24)}{4+2} = \underline{16\ V}$$

and

$$V_2 = \frac{R_2 E}{R_T} = \frac{(4)\cdot(24)}{4+2} = \underline{8\ V}$$



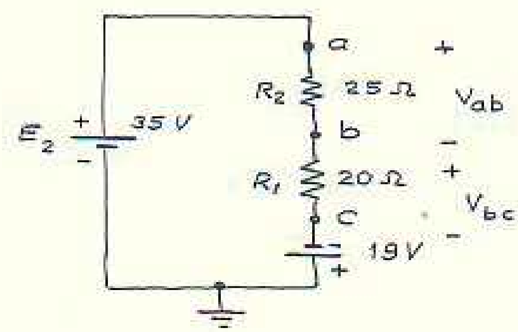
Example: For the cct. shown, determine V_{ab}, V_{cb} and V_b.



EE2

Solution: The cct. is redrawn as shown;

$$\therefore I = \frac{E_1 + E_2}{R_T} = \frac{19+35}{20+25} = 1.2\ A$$



$$V_{ab} = I R_2 = (1.2)(25) = 30\ V$$

and $V_{cb} = -V_{bc} = -I R_1$

$$= -(1.2)(20) = -24\ V$$

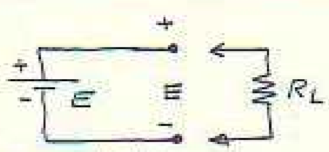
$$V_c = -E_1 = -19\ V$$

ملاحظة ١/ يمكن الحل باستخدام voltage divider rule مع ملاحظة ان $V_{cb} = -V_{bc}$.

ملاحظة ٣/ : ان V_a تعني جهد النقطة a بالنسبة الى الارضي.

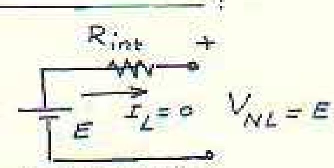
ملاحظة ٢/ : ان V_{ab} تعني فرق جهد النقطة a عن النقطة b ، فاذا كانت V_{ab} موجبة فان هذا يعني ان V_a اكبر من V_b والعكس صحيح.

Internal Resistance of Voltage Sources:



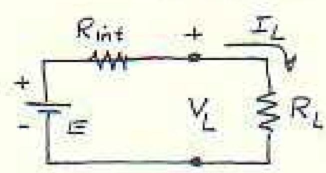
Ideal Voltage source

(1)



Actual Voltage source (at no load)

(2)



Actual Source connected to load R_L

(3)

- Applying KVL on figure (3): $E - I_L R_{in} - V_L = 0$
- Applying KVL on figure (2): $E = V_{NL}$

Substituting, then: $V_{NL} - I_L R_{in} - V_L = 0$

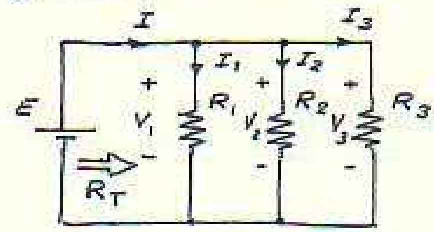
$$\boxed{\therefore V_L = V_{NL} - I_L R_{in}}$$

2.2 Parallel Circuits

EE2

: Two branches or elements or networks are in parallel if they have <u>two</u> points in common.

For the parallel cct. shown ; using KCL , we have :



$$I = I_1 + I_2 + I_3$$

$$= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

since $V_1 = V_2 = V_3 = E = V$

$$\therefore \; I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$E = V_1 = V_2 = V_3$$

$$I = \frac{E}{R_T} = \frac{V}{R_T}$$

$$\therefore \; \frac{V}{R_T} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore \; \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general , for N resistors connected in parallel, then :

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N}}$$

Notation

Conductance G

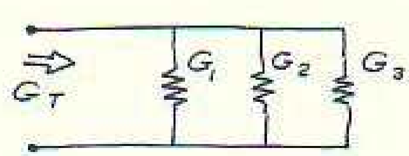
For parallel networks, it is common to use the idea of conductane in the cct. analysis. The conductance (G) is defined as:

$$G = \frac{1}{R} \qquad \text{Siemens} \qquad (S)$$

So, we can write the total conductance G_T for the parallel cct shown, as :

$$G_T = G_1 + G_2 + G_3$$

$$\Rightarrow \quad R_T = \frac{1}{G_T}$$



<u>NOTE</u> that, the total resistance R_T of parallel resistors is <u>always less than</u> the value of the <u>smallest resistor</u>.

Special Cases: The general relation for the total resistance of parallel resistors is:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

*** Case 1** For equal resistors in parallel, ie, when

$$R_1 = R_2 = R_3 \cdots R_N = R$$

then;

$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots \frac{1}{R}}_{N}$$

$$\therefore \frac{1}{R_T} = N\left(\frac{1}{R}\right) \Rightarrow \boxed{R_T = \frac{R}{N}}$$

*** Case 2**: For two resistors in parallel, then R_T is given as:

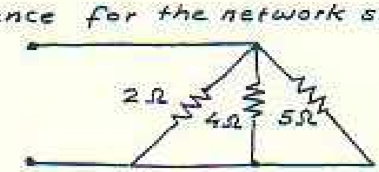
$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

This means that, the total resistance of the two parallel resistors is the product of the two divided by their sum.

*** Case 3**: For three resistors in parallel, then R_T is given as:

$$\boxed{R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

Example: Determine the total resistance for the network shown:



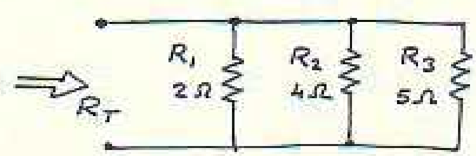
Solution Redraw the cct. to be as shown;

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$= 0.5 + 0.25 + 0.2$$

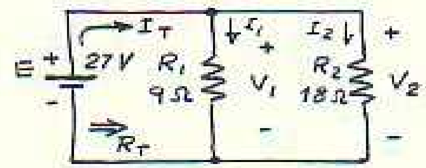
$$= 0.95 \Rightarrow \therefore R_T = \frac{1}{0.95} = \underline{1.053\ \Omega}$$



EE2

Example: For the parallel network shown;

a. Calculate R_T
b. Determine I_T
c. Calculate I_1 and I_2
d. Determine the power to each resistive load.
e. Determine the power delivered by the source and compare it with the total power dissipated by the resistive elements.



Solution:

a. $$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9)(18)}{9 + 18} = \frac{162}{27} = \underline{6\ \Omega}$$

b. $$I_T = \frac{E}{R_T} = \frac{27}{6} = \underline{4.5\ A}$$

c. $$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27}{9} = \underline{3\ A}$$

and

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27}{18} = \underline{1.5\ A}$$

نلاحظ ان : $I_T = I_1 + I_2$

d. $$P_1 = I_1 V_1 = E I_1 = (27)(3) = \underline{81\ W}$$

$$P_2 = I_2 V_2 = E I_2 = (27)(1.5) = \underline{40.5\ W}$$

e. $$P_s = E I_T = (27)(4.5) = 121.5\ W$$

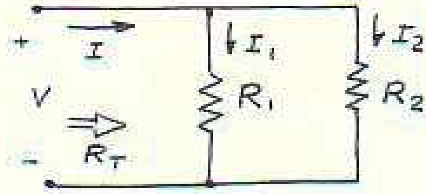
$$P_s = P_1 + P_2 = 81 + 40.5 = 121.5\ W$$

* نلاحظ ان مجموع القدرة المستهلكة في المقاومتين R_1 و R_2 يساوي القدرة التي يجهزها المصدر E

2.2.1 Current Divider Rule

: Consider the parallel cct. shown;

We have; $I = \frac{V}{R_T} \Rightarrow V = I R_T$



and $$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

but $$I_1 = \frac{V}{R_1} = \frac{I R_T}{R_1} = I \cdot \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1}$$

$$\therefore \boxed{I_1 = I \cdot \frac{R_2}{R_1 + R_2}}$$

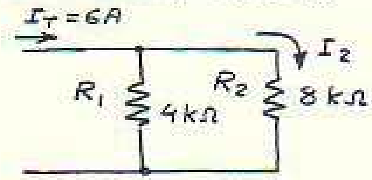
EE1

Similarly, for I_2, we have:

$$I_2 = \frac{V}{R_2} = \frac{I R_T}{R_2} = I . \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_2}$$

$$\therefore \boxed{I_2 = I . \frac{R_1}{R_1 + R_2}}$$

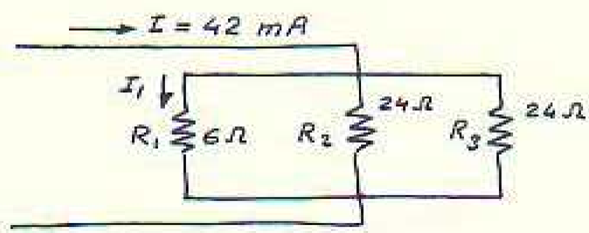
Example: Determine the current I_2 for the network shown;



solution: $I_2 = I_T . \frac{R_1}{R_1 + R_2}$

$$= 6 \frac{4 \times 10^3}{(4+8) \times 10^3} = 6 . \frac{4}{12} = \underline{2 \ A}$$

Example: Find the current I_1 for the network shown;



Solution:

$$I_1 = I . \frac{R_T}{R_1}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

or

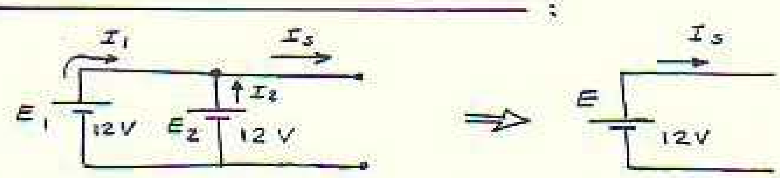
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{24} + \frac{1}{24}$$

$$\Rightarrow R_T = 4 \ \Omega$$

$$\therefore I_1 = 42 \times 10^{-3} \frac{4}{6}$$

$$= 28 \times 10^{-3} = \underline{28 \ mA}$$

2.2.2 Voltage Sources in Parallel : EE2



$E_1 = E_2 = E$ and $I_S = I_1 + I_2$

To increase the current rating of the source, two or more batteries in parallel of the same terminal voltage would be used.

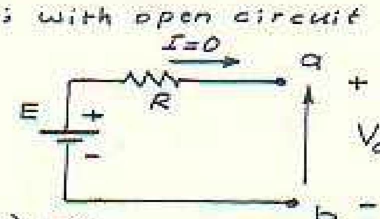
2.3 Open and Short Circuits

: We often need to apply the open and short circuits in the analysis fof electric networks.

*** Open Circuit** : An open circuit is simply two isolated terminals not connected by an element of any kind.

Consider the circuit shown ; with open circuit terminals a and b.

$V_{\text{open circuit}} = V_{oc} = V_{ab} = E$

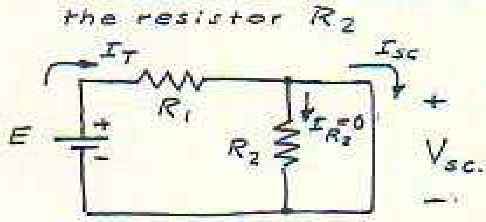


since I (in the open circuit) = 0

In general : An open circuit CAN HAVE a potential difference (voltage) across its terminals but the current is always ZERO.

*** Short Circuit** : A short circuit is a direct connection of zero ohms across an element or combination of elements.

Consider the circuit shown , with a short circuit across the resistor R_2

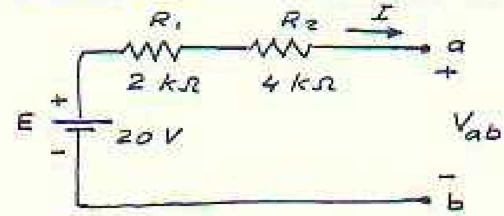


$I_{sc} = I_T = \frac{E}{R_1}$ since $I_{R_2} = 0$

$V_{\text{short cct.}} = V_{sc} = \underline{\underline{0}}$

In general: A short circuit CAN CARRY a current of any level but the potential difference (voltage) across its terminals is always ZERO.

Examples: (a). For the network shown, determine V_{ab}.

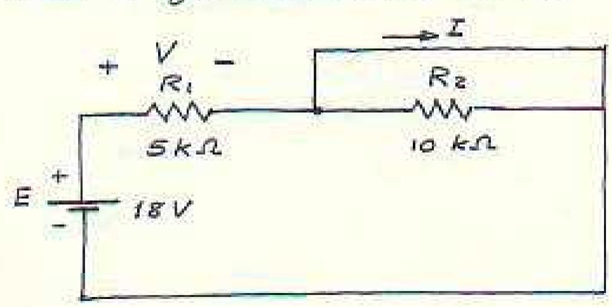


Solution: - We have an open cct across the terminal a and b, the

$$I = 0 \Rightarrow V_1 = 0 \quad \text{and} \quad V_2 = 0$$

- Applying KVL $\Rightarrow V_{ab} = E = 20\,V$

(b). Calculate I and V for the network shown;



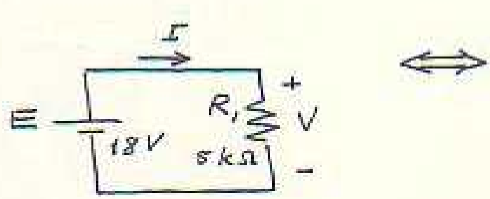
Solution: - We have a short cct. across R_2

$\Rightarrow$ No current through R_2

Note: the cct can be redrawn to be as shown

$$\Rightarrow I = \frac{E}{R_T} = \frac{E}{R_1 + 0} = \frac{18}{5\,k\Omega}$$

$$= \underline{3.6\ mA}$$



$$\therefore V = I R_1 = E = \underline{18\,V}$$

2.4 Source Transformation :

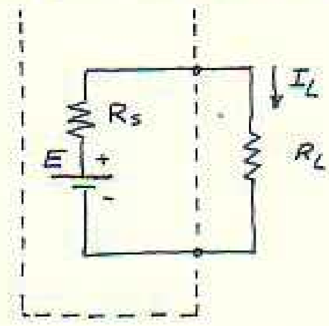
EE2

It is often necessary or convenient to have a voltage source rather than a current source or a current source rather than a voltage source.

In the cct. shown, we have a voltage source connected to a load resistance R_L

We have :

$$I_L = \frac{E}{R_T} = \frac{E}{R_S + R_L}$$



Voltage Source

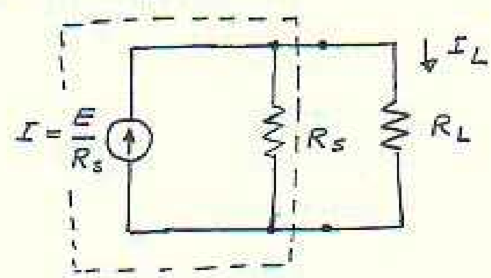
Multiplying the numerator by $(R_S/R_S = 1)$, we have :

$$I_L = \frac{(R_s/R_s)\,E}{R_s + R_L} = \frac{R_s(E/R_s)}{R_s + R_L}$$

$I = \frac{E}{R_S}$

$$\therefore \; I_L = \frac{R_s \cdot I}{R_s + R_L}$$

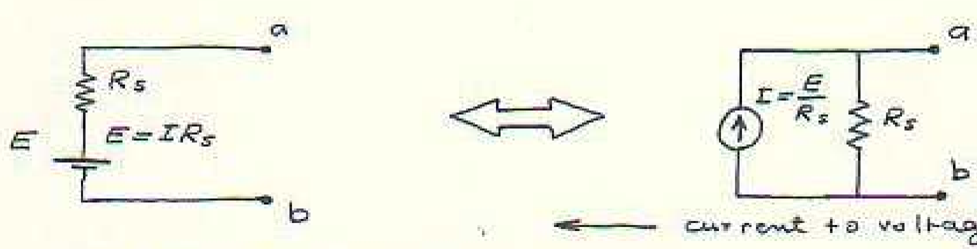
This is a current divider equation, which can be represented by the circuit below; which is the equivalent cct. of the voltage source.



Current source

In general

A voltage source with voltage E and series resistor R_S can be replaced by a current source with a current I and parallel resistor R_S as shown:

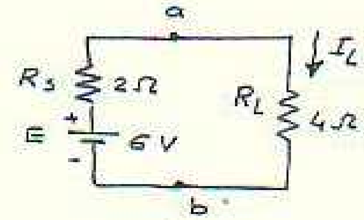


← current to voltage source

voltage source to current source →

EE2

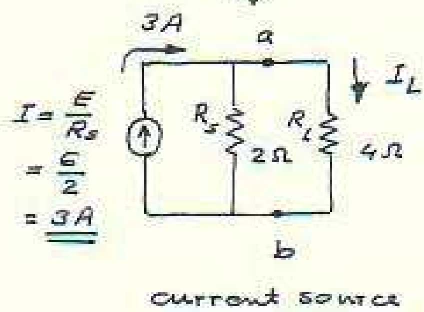
Example: Convert the voltage source, in the cct below, to a current source, then calculate the current through the load for each source.



Solution:

* For the voltage source cct;

$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2+4}$$
$$= \underline{\underline{1\ A}}$$

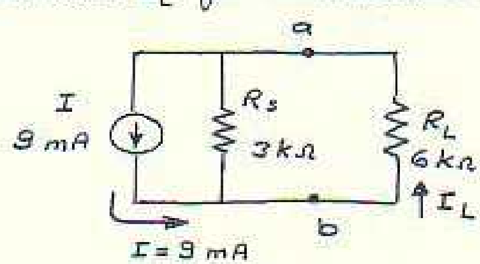


* For the current source cct;

$$I_L = \frac{I R_s}{R_s + R_L} = \frac{(3)(2)}{2+4}$$
$$= \underline{1\ A}$$

نلاحظ ان I_L متساوي في الحالتين وهذا صحيح.

Example: Convert the current source, in the cct. shown, to a voltage source and determine I_L for each source.



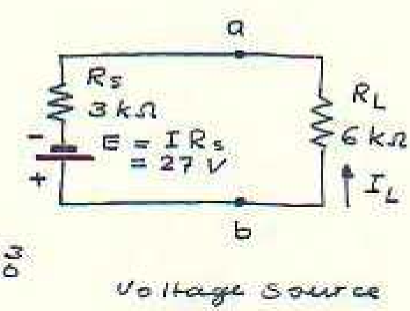
Solution:

* For the current source cct;

current divider rule ⇒
$$I_L = \frac{I \cdot R_s}{R_s + R_L} = \frac{(9\times10^{-3})(3\times10^{3})}{(3+\ )\times10^{3}}$$
$$= 3\times10^{-3} = \underline{3\ mA}$$

* For the voltage source cct;

$$I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L} = \frac{27}{(3+6)\times10^{3}}$$
$$= 3\times10^{-3} = \underline{3\ mA}$$



2.3 Series-Parallel Circuits:

EE2

Example: For circuit shown,

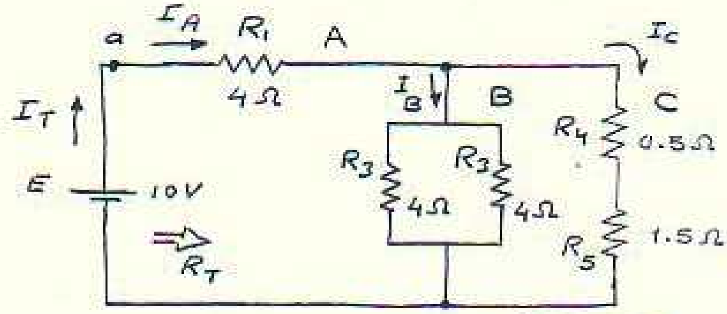


Fig (1)

Solution:

$$R_1 = R_A = 4\Omega$$

$$R_B = R_2 // R_3 = \frac{R_N}{N} = \frac{4}{2} = 2\Omega .$$

$$R_C = R_4 + R_5 = 0.5 + 1.5 = 2\Omega$$

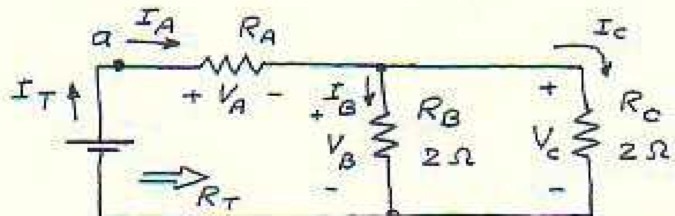


Fig (2)

$$R_{B//C} = \frac{R_N}{N} = \frac{2}{2} = 1\ \Omega$$

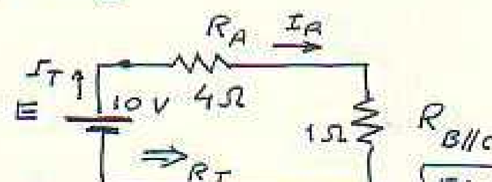


Fig (3)

From Fig. 3 $\Rightarrow \therefore R_T = R_A + R_{B//C} = 4 + 1 = 5\Omega$

$\therefore I_T = \dfrac{E}{R_T} = \dfrac{10}{5} = 2\ A$

From Fig (2) $I_B = I_c = \dfrac{I_A}{2} = \dfrac{2}{2} = 1\ A$

From Fig (1) $\Rightarrow I_{R_4} = I_{R_5} = I_c = 1\ A$

From fig (2) $\Rightarrow$ $V_A = I_A R_A = (2)(4) = 8\ V$

$V_B = I_B R_B = (1)(2) = 2\ V$

$V_c = I_c R_c = (1)(2) = 2\ V$

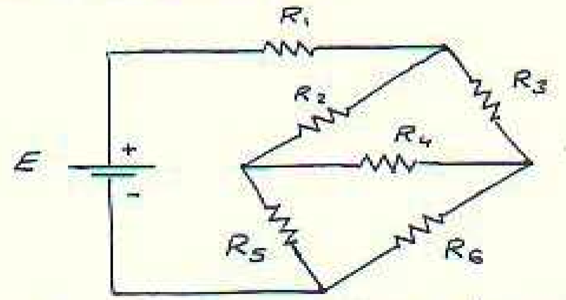
For check KVL: $\sum_o V = 0 \Rightarrow E - V_A - V_B = 0 \Rightarrow 10 = 8 + 2$

$\therefore 10\ V = 10\ V$

EE2

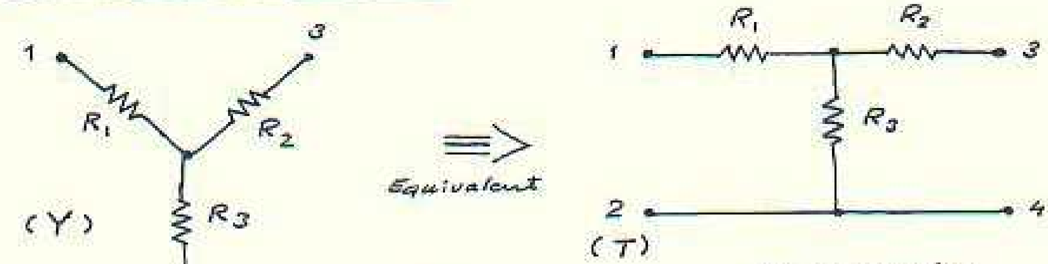
Wye - Delta Transformation

: There are some cases often arises in circuit analysis, when the resistors are neither in parallel nor in series. For example, consider the circuit shown:



In this circuit, R_1, R_2, R_3 ... R_6 are neither in series nor in parallel

* Wye (Y or star) connection

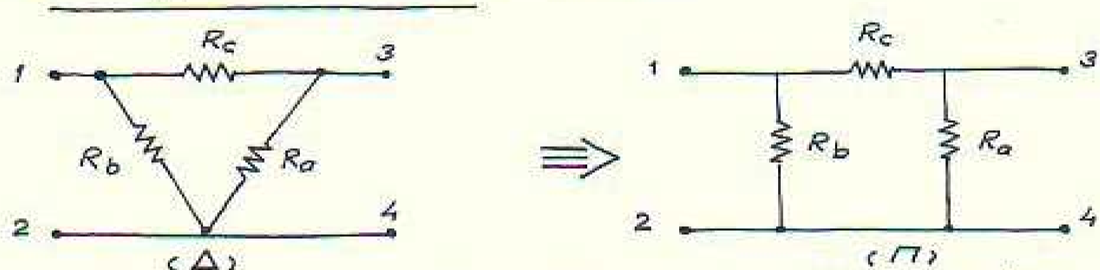


Y or star circuit connection

T- connection

Y and T connections

* Delta (Δ or π) circuit connection



Delta circuit connection & its equivalen Π- connection

Δ $\qquad$ Y

* Delta to Wye Transformation :

EE2

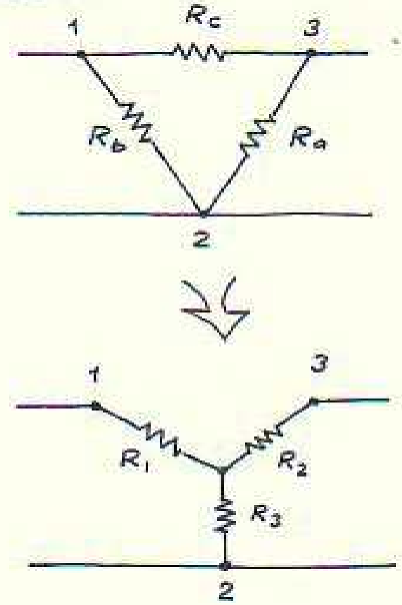
- We have Δ and want to get its equivalent star circuit

- Consider the Δ circuit shown; to be transformed into its equivalent star shown below:

$$R_{12} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{13} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{23} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$



Delta

$$R_{12} = R_1 + R_3$$

$$R_{13} = R_1 + R_2$$

$$R_{23} = R_2 + R_3$$

$$\therefore \quad R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad \cdots (1)$$

and

$$R_1 + R_2 = \frac{R_c(R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (2)$$

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (3)$$

Subtraction Eq.(3) from Eq.(1) and adding the resulting equation to Eq.(1) results in:

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

Similarly;

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

and;

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

IN GENERAL

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three resistors

* Wye to Delta Transformation

- We have Y connected circuit and want to get its equivalent Δ.
- Consider the Y circuit shown, its equivalent Δ is shown below;

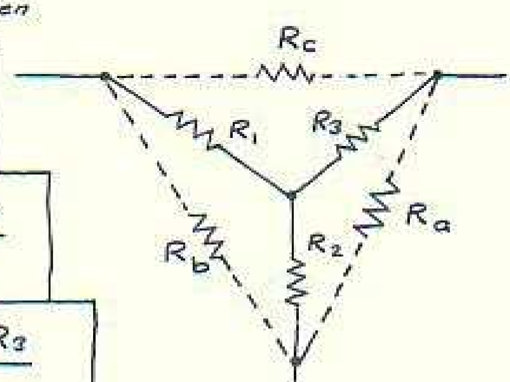
Using the previous sets of equations, then we have:

$$\boxed{R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}}$$

$$\boxed{R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}}$$

and

$$\boxed{R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}}$$



IN GENERAL

> Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

Notes:

* The Y and Δ networks are said to be <u>balanced</u> when:

$$R_1 = R_2 = R_3 = R_Y$$

and

$$R_a = R_b = R_c = R_\Delta$$

* Under <u>balance</u> condition, the conversion equations become:

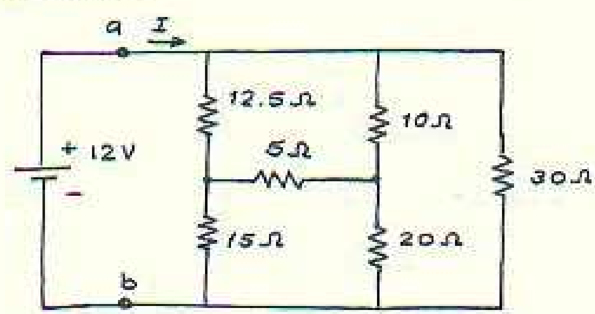
$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

or

$$\boxed{R_\Delta = 3R_Y}$$

Example

Obtain the quivalent resistance R_{ab} for the circuit shown and use it to find the current I



Solution

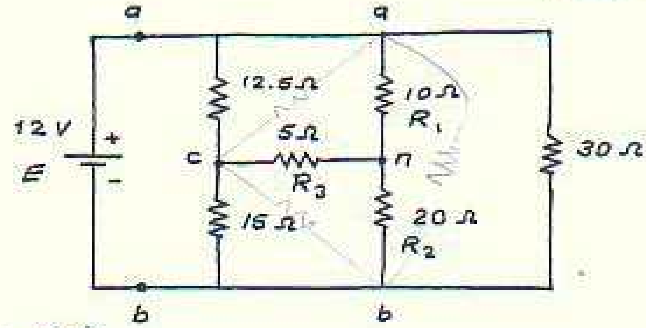
* We can't use the relations of series connected or parallel connected resistors to obtain R_{ab}.

* We try to use Δ-Y transformations or Y-Δ to get R_{ab}.

* If we transform the Y consisting of :

$R_1 = 10\,\Omega$
$R_2 = 20\,\Omega$
and $R_3 = 5\,\Omega$

∴ the equivalent △ circuit contains :



$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$= \frac{10\times20 + 20\times5 + 5\times10}{10} = \frac{350}{10}$$

$$= 35\,\Omega$$

Similarly ;

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{20} = \frac{350}{20} = 17.5\,\Omega$$

and :

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{5} = \frac{350}{5} = 70\,\Omega$$

$12.5 \,//\, 17.5 = \frac{12.5\times17.5}{12.5\times17.5} = 7.3\,\Omega$

$15 \,//\, 35 = \frac{15\times35}{15+35} = 10\,\Omega$

$70 \,//\, 30 = \frac{70\times30}{70+30} = 21\,\Omega$

$$\therefore R_{ab} = (7.3 + 10.5) \,//\, 30$$

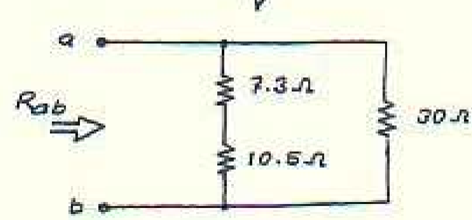
$$= \frac{17.8\times21}{17.8+21}$$

$$\Rightarrow R_{ab} = \underline{9.632\,\Omega}$$

$$\therefore I = \frac{E}{R_{eq}} = \frac{E}{R_{ab}}$$

$$= \frac{12}{9.632}$$

$$\Rightarrow I = \underline{1.246\ A}$$



Tutorial Sheet № 2

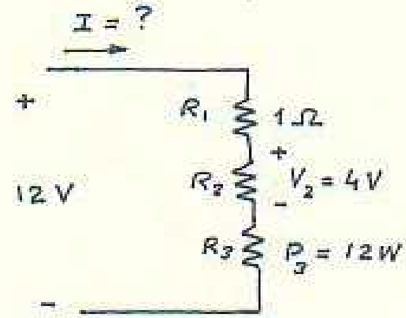
TS2

ملاحظة : جميع اسئلة الكتاب والامثلة المحلولة فيه مطلوبة

Example 1: Three resistors are connected in series across a 12 V battery. The first resistor has a value of 1Ω, the second has a voltage drop of 4 V, and the third has a power dissipation of 12 W.

Calculate the value of the circuit current.

Solution:



$R_2 = ?$
$R_3 = ?$

We have $\quad P_3 = I^2 R_3 = 12\ W$ —— ①

and $\quad V_2 = I R_2 = 4\ V$ —— ②

From ②

$$\therefore\ I = \frac{4}{R_2}$$

From ① & ②

$$\left(\frac{4}{R_2}\right)^2 R_3 = 12$$

$$\therefore\ R_3 = \frac{3}{4} R_2^2$$

From the circuit shown, we have:

$$12 = I(R_1 + R_2 + R_3) = I(1 + R_2 + R_3)$$

Substituting for I and R_2, we have:

$$12 = \frac{4}{R_2}\left(1 + R_2 + \frac{3}{4} R_2^2\right)$$

$$\therefore\ 3R_2^2 - 8R_2 + 4 = 0$$

$$\therefore\ R_2 = \frac{8 \pm \sqrt{64-48}}{6} = 2\Omega \text{ or } \frac{2}{3}\Omega$$

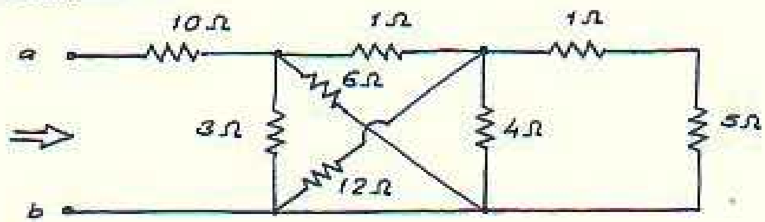
$$\therefore\ R_3 = \frac{3}{4} R_2^2 \Rightarrow R_3 = 3\Omega \text{ or } \frac{1}{3}\Omega$$

$$\therefore\ I = \frac{12}{R_1 + R_2 + R_3} = \frac{12}{1+2+3} = \underline{2\ A}$$

or

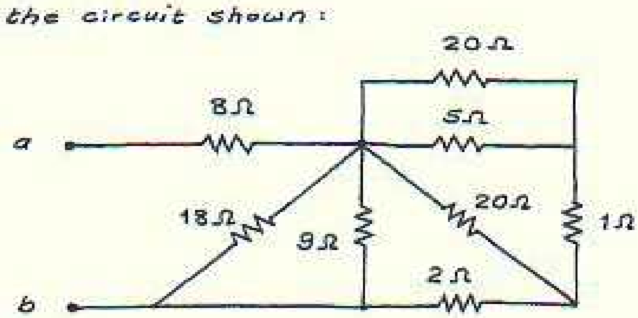
$$I = \frac{12}{1 + \left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)} = \underline{6 A}$$

Practice Problem : Calculate the equivalent resistance R_{ab} in the circuit shown.



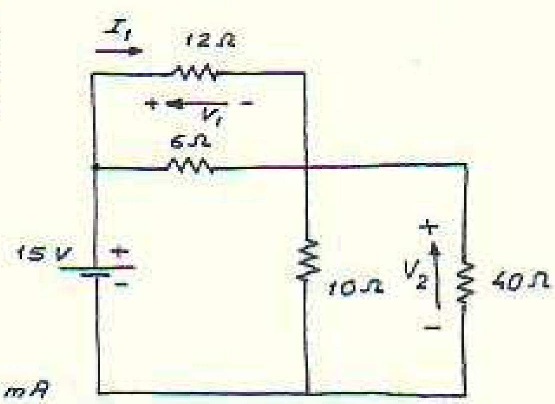
Answer : $R_{ab} = 11.2\ \Omega$

Practice Problem : Find R_{ab} for the circuit shown:



Answer : $R_{ab} = 11\ \Omega$

Practice Problem : Find V_1 and V_2 in the circuit shown. Also calculate I_1 and I_2 and the power dissipated in the 12 Ω and 40 Ω resistors.



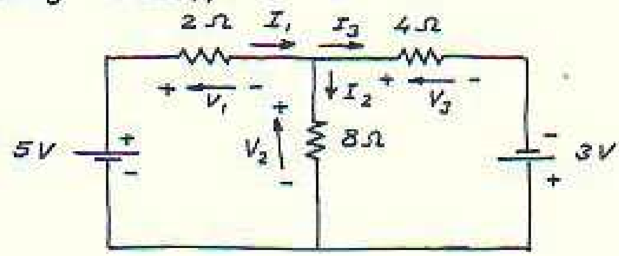
Answer :

$V_1 = 5V$, $V_2 = 10V$

$I_1 = 416.7\ mA$, $I_2 = 250\ mA$

$P_1 = 2.083\ W$, $P_2 = 2.5\ W$

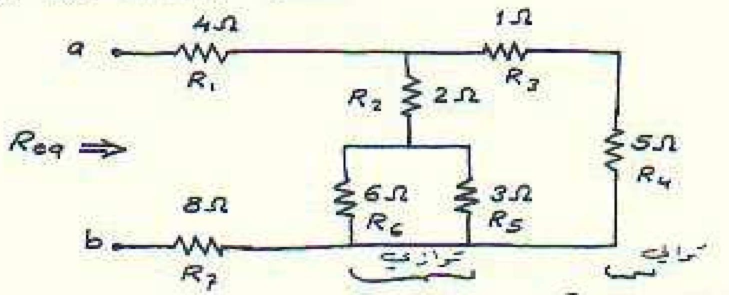
Practice Problem: Find the currents and voltages in the circuit shown, using Kirchoff's laws.



Answer: $V_1 = 3V$, $V_2 = 2V$, $V_3 = 5V$

$I_1 = 1.5\,A$, $I_2 = 0.25\,A$, $I_3 = 1.25\,A$

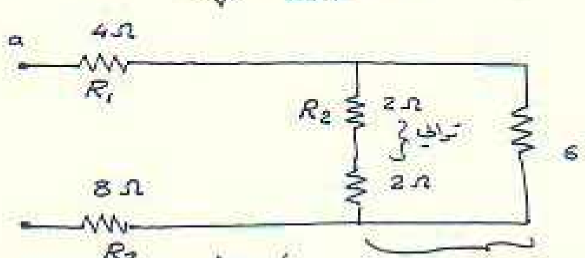
Example: Find R_{eq} for the circuit shown.



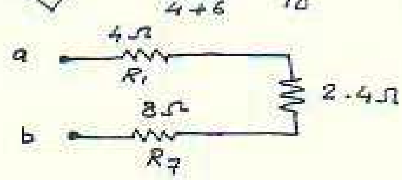
Solution:

$R_5 // R_6 = \frac{6 \times 3}{6+3} = 2\,\Omega$

$R_4 + R_3 = 5 + 1 = 6$



$\frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4\,\Omega$

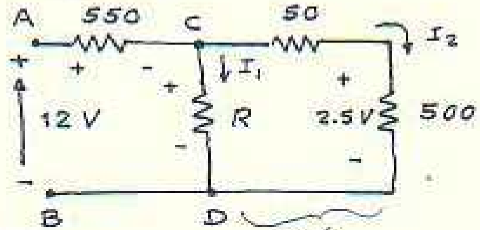


$\therefore R_{eq} = R_{ab} = 4 + 2.4 + 8$

$= \underline{14.4\,\Omega}$

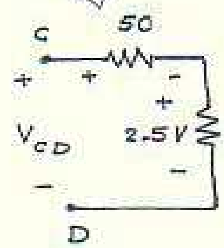
$\boxed{TS2}$

Example: What is the value of the unknown resistor R in the circuit shown, if the voltage drop across the $500\,\Omega$ resistor is 2.5 V ? All resistors are in Ohms.



Solution

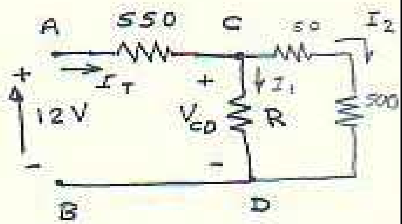
Using the voltage divider rule;



* The voltage drop across the $50\,\Omega$ resistor is :

$$V_{50\Omega} = 2.5 \times \frac{50}{500}$$
$$= 0.25 \text{ V}$$

$$\therefore \; V_{CD} = 2.5 + 0.25$$
$$= 2.75 \text{ V}$$



* $V_{550\Omega} = 12 - V_{CD}$

$$= 12 - 2.75 = 9.25 \text{ V}$$

$$\therefore \; I_T = \frac{V_{550\Omega}}{550} = \frac{9.25}{550} = 0.0168 \text{ A}$$

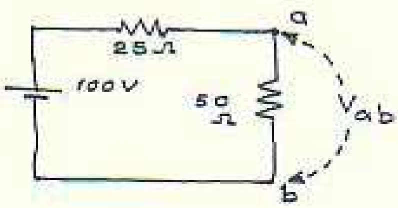
$$I_2 = \frac{V_{500}}{500} = \frac{2.5}{500} = 0.005 \text{ A}$$

$$\therefore \; I_1 = I - I_2 = 0.0168 - 0.005 = 0.0118 \text{ A}$$

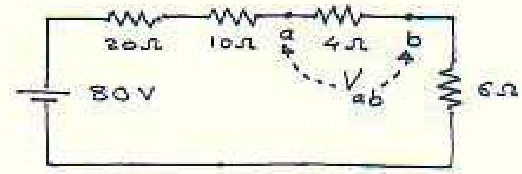
$$\therefore \; R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = 233\,\Omega$$

TS2

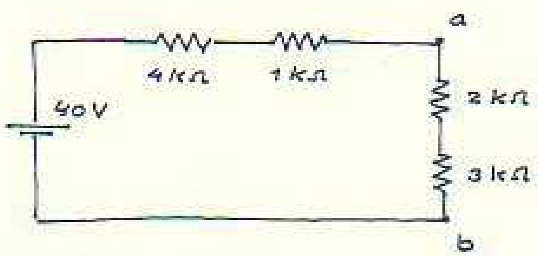
Example: For the networks shown, find V_{ab} (with polarity).



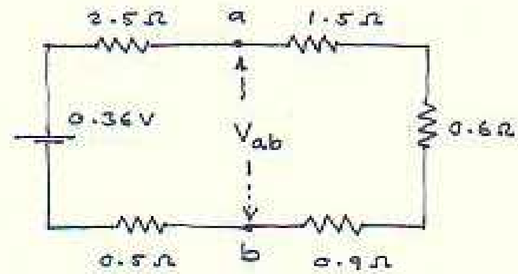
(a)



(b)

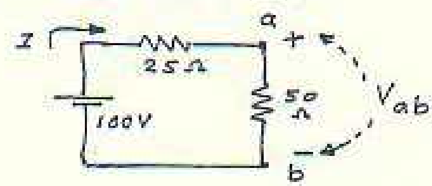


(c)



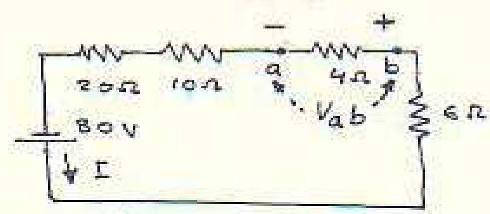
Solution: (a). Using the voltage divider rule; then

$$\therefore \underset{+\,-}{V_{ab}} = 100 \cdot \frac{50}{25+50} = 66.67 \text{ V}$$



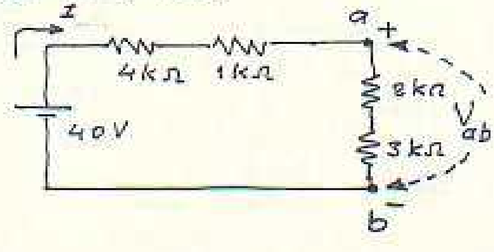
(b). Using the voltage divider rule, then:

$$\therefore \underset{-\,+}{V_{ab}} = 80 \frac{4}{20+10+4+6} = 8 \text{ V}$$



(c). Using the voltage divider rule, then:

$$\underset{+\,-}{V_{ab}} = \frac{(2+3)\times 10^{3}}{(4+1+2+3)\times 10^{3}} = 20 \text{ V}$$

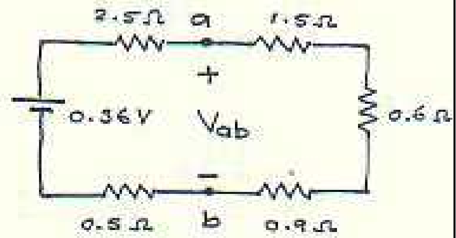


TS2

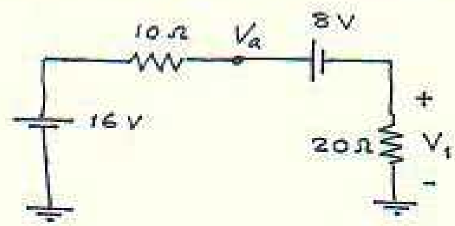
(d). Using the voltage divider rule, then;

$$\underset{+\ -}{V_{ab}} = 0.36\,\frac{(1.5+0.6+0.9)}{2.5+1.5+0.6+0.9+0.5}$$

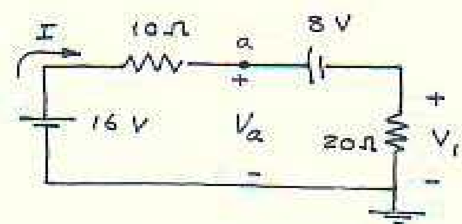
$$= 0.18\ V$$



Example : Determine the voltage V_a and V_1, for the network shown.



Solution : Redraw the circuit to be as shown;



using KVL $\quad V_a = 8 + V_1$

$$= 8 + I(20)$$

$$I = ?$$

$$I = \frac{16-8}{20+10}$$

$$= \frac{8}{30}$$

$$\therefore V_a = 8 + \frac{8}{30}(20) = 8 + \frac{16}{3} = \frac{40}{3}\ V$$

$$= \underline{13.33\ V}$$

$\boxed{OR}$

using KVL

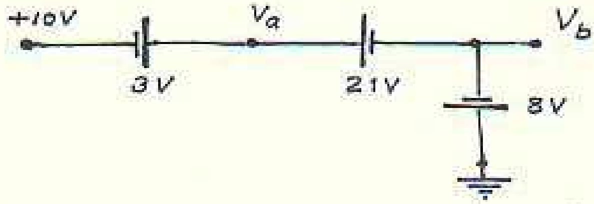
$$16 = I(10) + V_a \Rightarrow 16 = \frac{8}{30}(10) + V_a$$

$$\therefore V_a = 16 - \frac{8}{3} = \frac{40}{3} = \underline{13.33\ V}$$

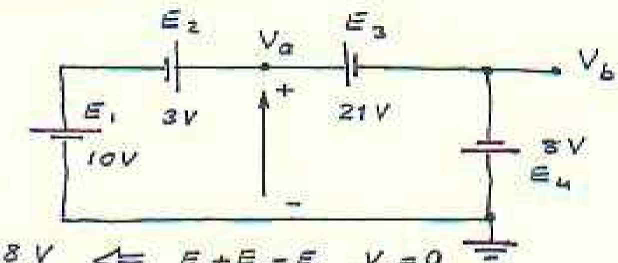
$$V_1 = I(20) = \frac{8}{3}(20) = \frac{16}{3} = \underline{5.33\ V}$$

TS2

Example: Determine the voltage V_{ab} for the network shown;



Solution: Redraw the cct. to be as shown;



$V_b = -8\ V$

$V_b = -E_4$ directly the drop of point $\underline{b}$ with respect to the ground.

$\boxed{OR}$ using KVL

$V_b = 10 + 3 - 21 = -8\ V \Leftarrow E_1 + E_2 - E_3 \quad V_b = 0$

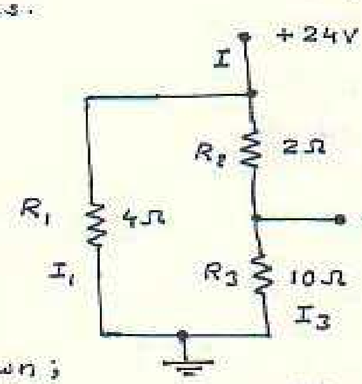
KVL $\quad V_a = 10 + 3 = 13\ V \Leftarrow E_1 + E_2 - V_a = 0$

$\therefore V_{ab} = V_a - V_b$

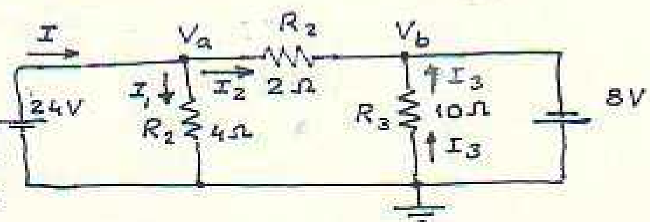
$= 13 - (-8) = 21\ V$

OR
$V_a - E_3 + E_4 = 0$
$\therefore V_a = E_3 - E_4$
$= 21 - 8 = 13\ V$

Example: For the network shown, find the currents I, I_1 and I_3 and indicate their directions.



Solution: Redraw the cct to be as shown;



$I_3 = \frac{V_b}{R_3} = \frac{-8}{10} = -0.8\ A$

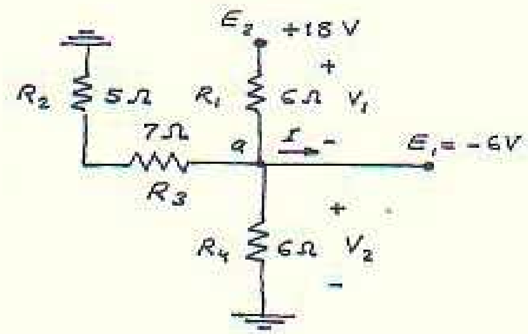
$I_1 = \frac{V_a}{R_2} = \frac{24}{4} = 6\ A$

$I_2 = \frac{V_a - V_b}{R_2} = \frac{24+8}{2} = 16\ A$

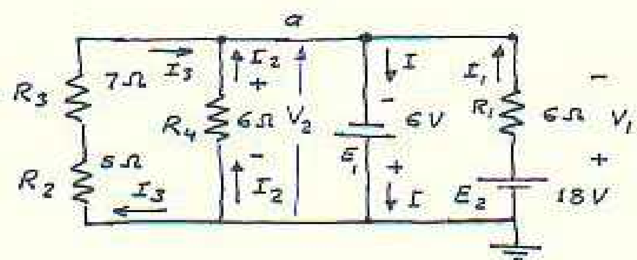
$I = I_1 + I_2 = 6 + 16 = 24\ A$

TS2

Example: For the network shown, determine the voltages V_1, V_2 and the current I.



Solution: Redraw the network to be as shown;



$$V_2 = -E_1 = -6\,V$$

Applying KVL, we have;

$$-E_1 + V_1 - E_2 = 0$$

$$\therefore\ V_1 = E_1 + E_2 = 18 + 6$$

$$= 24\,V$$

Applying KCL, then:

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{V_1}{R_1} = \frac{24}{6} = 4\,A$$

$$I_2 = \frac{V_2}{R_4} = \frac{6}{6} = 1\,A$$

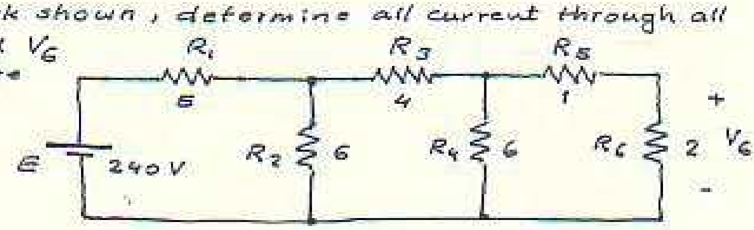
$$I_3 = \frac{E_1}{R_2 + R_3} = \frac{6}{7+5} = 0.5\,A$$

$$\therefore\ I = 4 + 1 + 0.5$$

$$= \underline{5.5\,A}$$

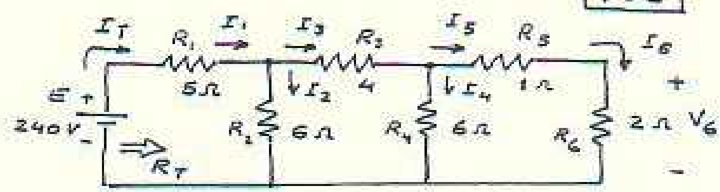
Example: For the network shown, determine all current through all branches, and V_6

All resistors are in Ohms.



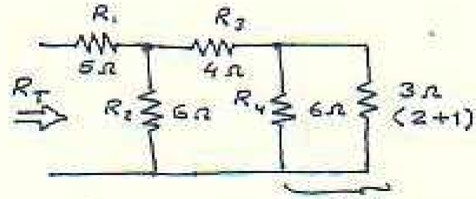
Solution:

TS2

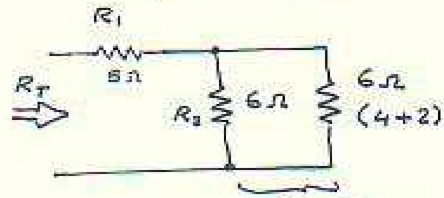


$$I_T = \frac{E}{R_T} \quad \Rightarrow \quad R_T = ?$$

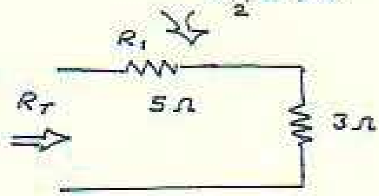
+ Find R_T



$$\frac{3(6)}{3+6} = 2\Omega$$



$$\frac{6}{2} = 3\Omega$$

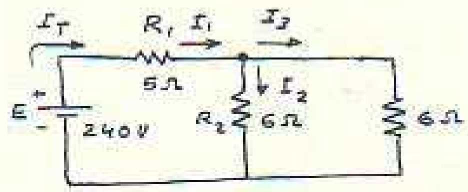


$$\therefore \; R_T = 5 + 3 = \underline{8\Omega}$$

$$\therefore \; I_T = \frac{240}{8} = \underline{30\ A}$$

$$I_1 = I_T = 30\ A$$

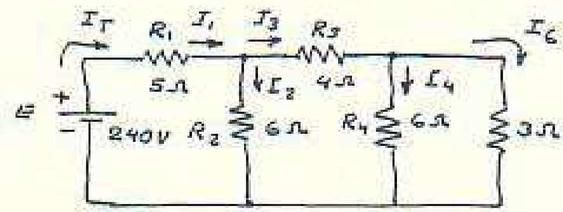
$$I_3 = \frac{I_T}{2} = \frac{30}{2} = \underline{15\ A}$$



Using the current division rule;

$$I_6 = I_3 \frac{6}{3+6} = 10\ A$$

$$\therefore \; V_6 = I_6 R_6 = (10)(2) = 20\ V$$



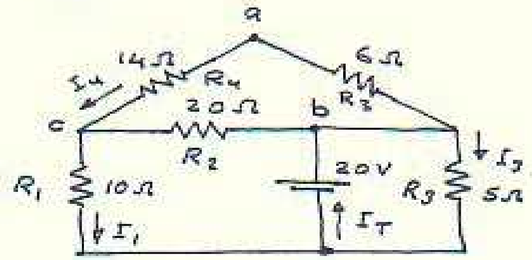
$$I_1 = I_2 + I_3 \Rightarrow I_2 = I_1 - I_3 = 30 - 15 = 15\ A$$

$$I_3 = I_4 + I_6 \Rightarrow I_4 = I_3 - I_6 = 15 - 10 = 5\ A$$

$$I_5 = I_6 = 10\ A$$

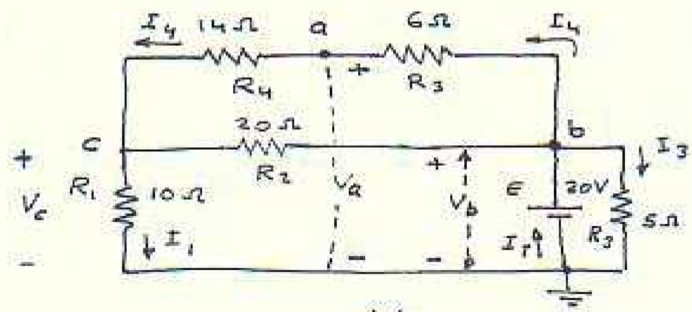
TS2

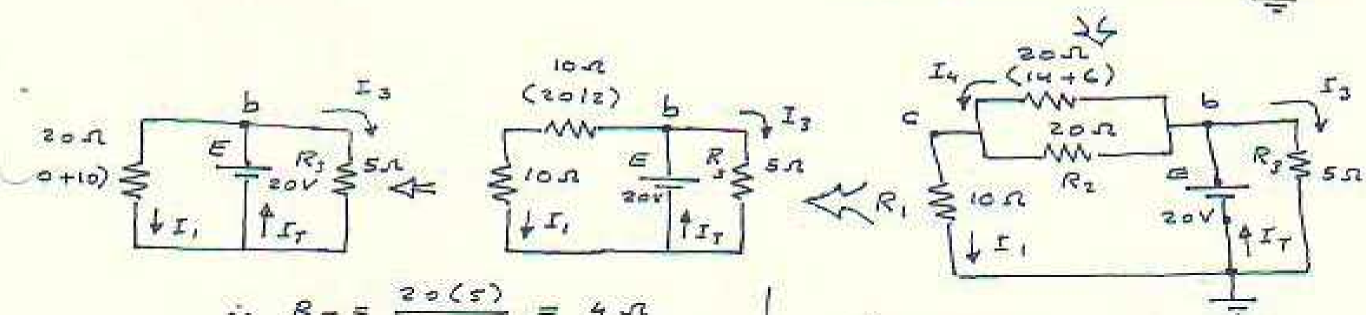
Example: For the network shown;
(a) Determine the currents I_T, I_1, I_3 and I_4.
(b) Calculate V_a and V_{bc}.



Solution: Redraw the circuit to be as shown, we notice that the circuit consists of only series – parallel combination.

$I_T = \frac{E}{R_T} \Rightarrow R_T = ?$





$\therefore R_T = \frac{20(5)}{20+5} = 4\,\Omega$

$I_T = \frac{20}{4} = 5\,A$

$I_1 = \frac{E}{20} = \frac{20}{20} = 1\,A$

$I_3 = \frac{E}{5} = \frac{20}{2} = 4\,A$

$I_4 = \frac{I_1}{2} = \frac{1}{2} = 0.5\,A$

$V_b = 20\,V$

$V_a = ?$ Applying KUL

$V_a = I_4 R_4 + I_1 R_1$
$= 0.5(14) + 1(10) = 17\,V$

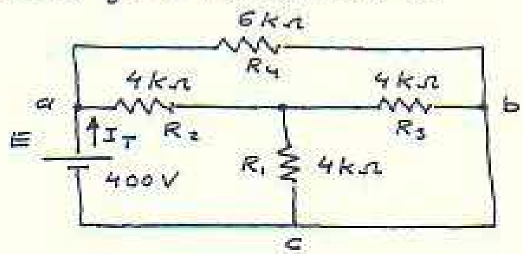
$\boxed{OR}$ KVL $V_a = V_b - I_4 R_6$
$= 20 - (0.5)(6) = \underline{17V}$

$V_c = I_1 R_1 = (1)(10) = 10\,V$

$\therefore V_{bc} = V_b - V_c = 20 - 10 = 10\,V$

TS2

Example: For the circuit shown, find the current I.



Solution: – There is no series or parallel circuit elements to be simplified.

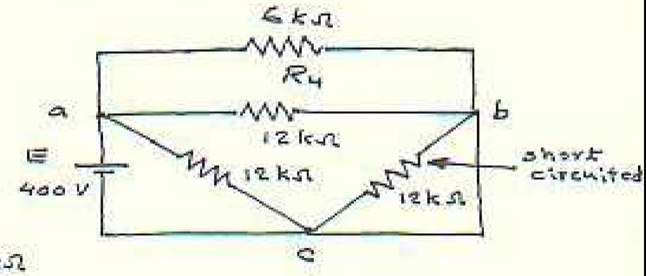
– Convert the star labled (abc) to a delta

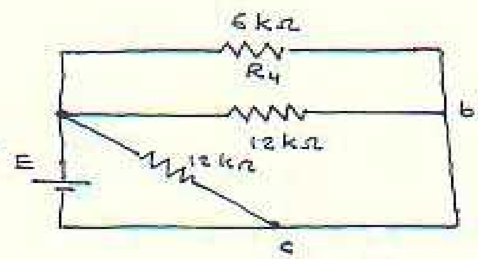
$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

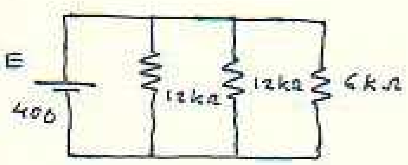
since $R_1 = R_2 = R_3 = 4\,k\Omega$

$\therefore R_A = 3R_1 = 3(4\,k\Omega) = 12\,k\Omega$

and $R_A = R_B = R_C = 12\,k\Omega$







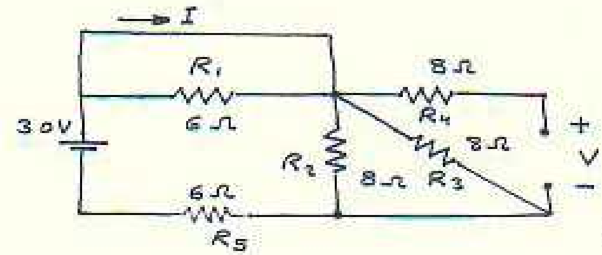
$$\therefore \quad R_T = \underbrace{\underbrace{12\,k\Omega \,//\, 12\,k\Omega}_{6\,k\Omega} \,//\, 6\,k\Omega}_{3\,k\Omega}$$

$\therefore R_T = 3\,k\Omega$

$$\therefore I_T = \frac{E}{R_T} = \frac{400}{3 \times 10^3} = 133.33\ mA$$

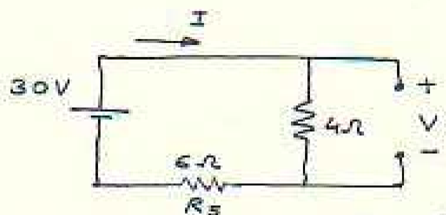
T52

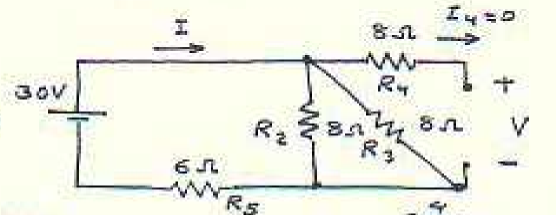
Example: For the circuit shown, calculate I and V.



Solution:

- Because of the short circuit ($R_1 = 6\Omega$) is cancelled.
- Because of the opencircuit, the current in R_4 is zero
- The circuit will be as shown.





$$R_T = \overbrace{(8 // 8)}^{4} + 6 = 10\ \Omega$$

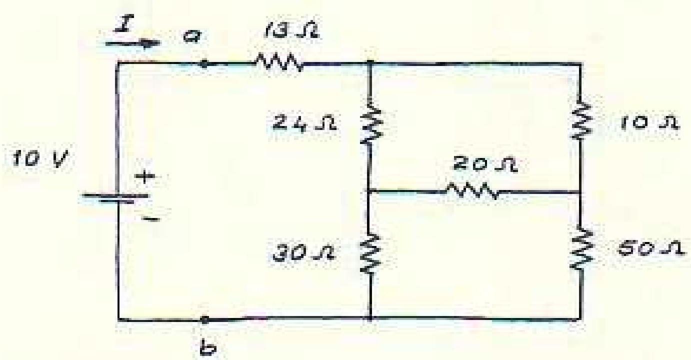
* $I = \dfrac{E}{R_T} = \dfrac{30}{10}$

$\therefore\ I = 3\ A$

* and $V = I R_2 = 3(4)$

$= 12\ V$

Practice Problem: For the bridge network shown, find R_{ab} and I.



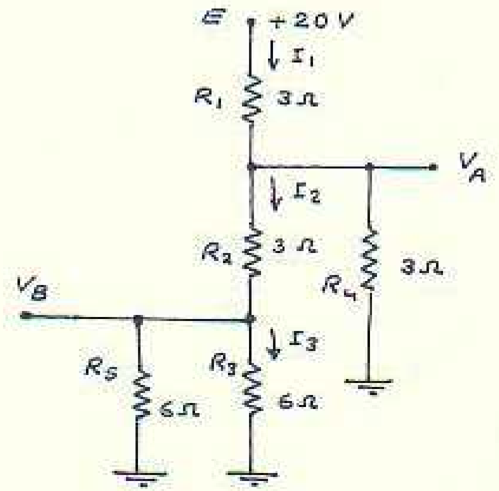
Answer:

$R_{ab} = \underline{40\ \Omega}$

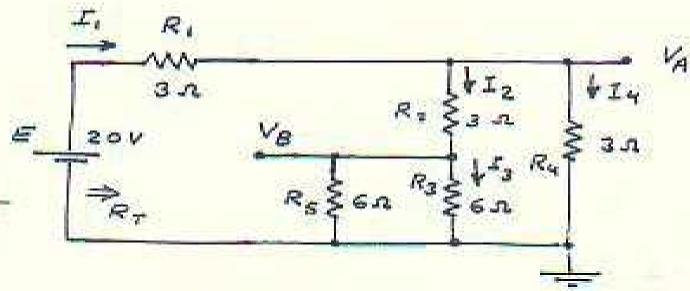
$I = \underline{0.25\ A}$

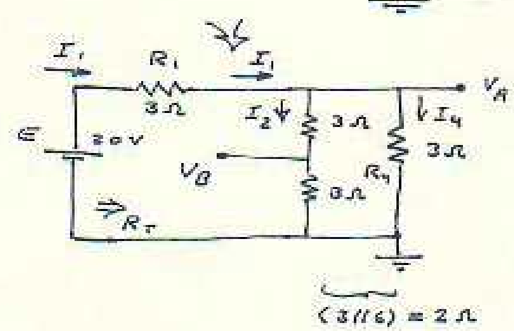
T52

<u>Example</u> : For the network shown ;
(a). Calculate the values of I_1 , I_2 and I_3 .
(b) Determine V_A and V_B .



<u>Solution</u> : Redraw the circuit to be as shown below;





(a). $I_1 = \frac{E}{R_T}$

$R_T = 3 + 2 = 5\,\Omega$

$\therefore I_1 = \frac{20}{5} = 4\,A$

ing Current
ision rule $I_2 = I_1 \frac{3}{3+6} = 4 \frac{3}{3+6}$

$\therefore I_2 = 1.333\ A$

$I_3 = \frac{I_2}{2} = \frac{1.333}{2}$

$= 0.6665\ A$

(b). $V_A = I_4 R_4$

$\Leftarrow I_4 = I_1 - I_2$
$= 4 - 1.333$
$= 2.667\ A$

$\therefore V_A = (2.667)(3) = 8\,V$

$V_B = I_3 R_3 = (0.6665)(6)$
$= 4V$