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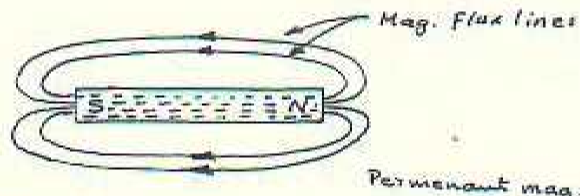
10. Magnetic Circuits

10.1 Definitions



* Magnetic Field Strength (H):

(Magnetizing force)

The magnetic field strength at any point within a magnetic field is numerically equal to the force experienced by an N-pole of 1 Weber placed at that point.

* Mag. flux lines;
  - radiate from the north pole to the south pole returning to north through the bar.
  - form continuous loops.
  - have equal spacing within the core and symmetric distribution outside the mag. material.
  - don't intersect.
  - pass with greater ease through mag. materials than through air.

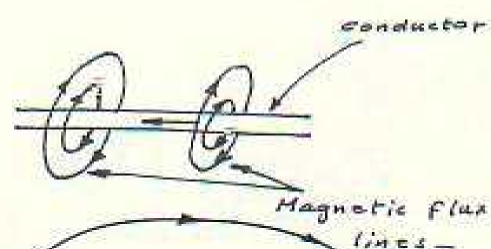
* Magnetic Flux Density (B):

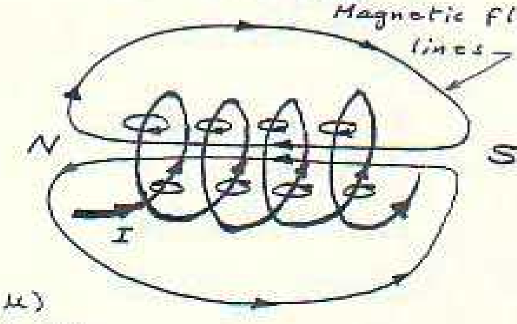
It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter ($\bar{B}$);

$$B = \frac{\Phi}{A} \quad Wb/m^2 \text{ or tesla } (T)$$

where Φ is the flux in Weber, and A is the area (m^2)

- B can be defined also; as the number of flux lines per unit area.





* Permeability (μ)

The absolute permeability (μ) of a magnetic material is defined as

$$\boxed{\mu = \frac{B}{H}}$$

When a magnetic material is placed in a magnetic field, it gets magnetized by induction

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* When (H) is established in air (vacuum), then the corresponding flux density developed in the air is:

$$B_o = \mu_o H$$

$\Rightarrow$ μ_o = constant value $= 4\pi \times 10^{-7}$ F/m.

where μ_o is the permeability of the air (vacuum). In general we have:

$$\mu = \mu_r \mu_o$$

where: μ_r is the relative permeability of magnetic materials (it is unitless quantity)

Reluctance (S)

: It is a quantity determined by the physical characteristics of a material that will provide an indication of reluctance of that material to the setting up of magnetic flux lines in the material. It is measured in (At/Wb). The reluctance S of a material is given by:

$$S = \frac{l}{\mu A}$$

$\Rightarrow$ where l is the length of the magnetic path.
A is the cross-section area.

10.2 Ohm's Law for Magnetic Circuit:

* In Electric Circuits, we have

$$I = \frac{V}{R} \Rightarrow \text{Effect} = \frac{\text{Cause}}{\text{Opposition}}$$

* Similarly, in Magnetic Circuits, we have

$$\Phi = \frac{MMF}{S}$$

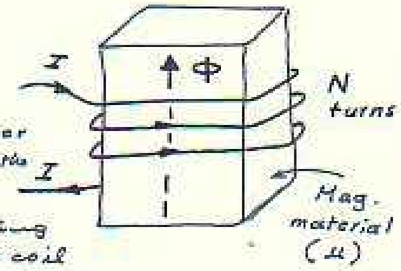
$\Rightarrow$ MMF = NI = magnetomotive force

S = Magnetic Reluctance of the material.

Φ = Magnetic Flux created in the mat.

So Φ can be written as →

$$\Phi = \frac{MMF}{S} = \frac{NI}{S}$$



where N is the number of turns of the coil.

I Current passing through the coil

* MMF is also given as:

$$MMF = Hl$$

⇒ where H is the magnetic field strength (At/m).
and l is the length of the magnetic path (m).

Hence the flux (Φ) can be written as

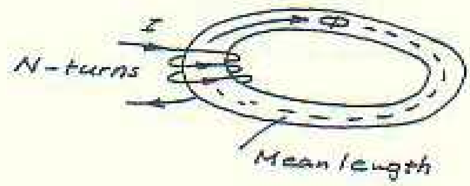
$$\Phi = \frac{mmf}{S} = \frac{NI}{S} = \frac{Hl}{S}$$

$\underline{\underline{H}}$ is sometimes called <u>the magnetizing force</u>

<u>Example</u>: For the magnetic circuit shown, if:

$NI = 40$ At
& $l = 0.2$ m

Calculate H.



<u>Solution</u>:

$$mmf = Hl$$
$$\& \quad mmf = NI \implies \therefore NI = Hl$$

$$\therefore H = \frac{NI}{l} = \frac{40}{0.2} = 200 \text{ At/m}.$$

<u>Note</u>: If the relative permability (μ_r) of the core material is given to be (1350), then the magnetic flux density (B) can be determined as:

$$B = \mu H = \mu_0 \mu_r H$$
$$= (4\pi \times 10^{-7})(1350)(200)$$
$$= 0.34 \ \text{wb/m}^2$$

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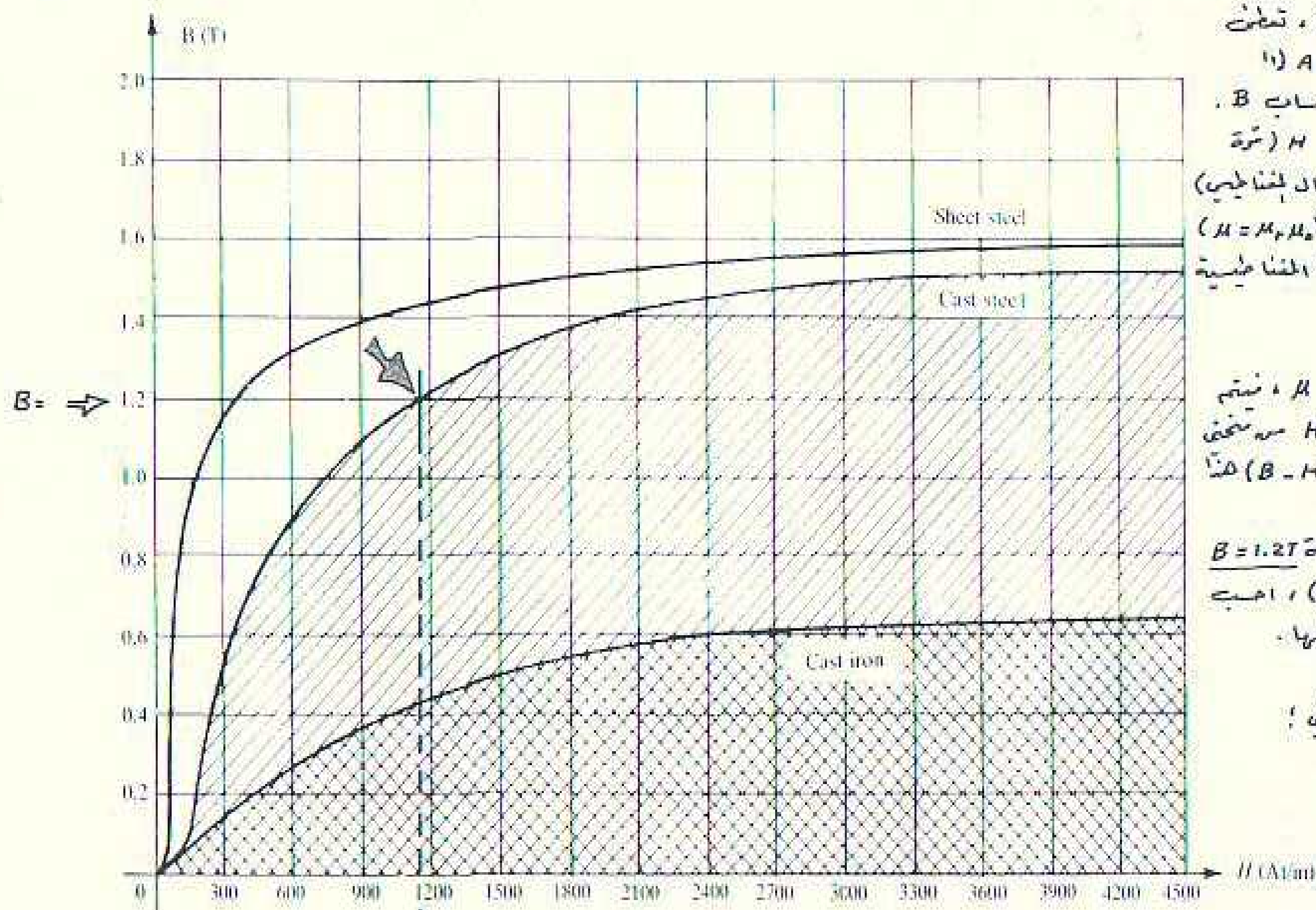

The (B_H) curve of some magnetic materials.

* ملاحظة هامة :

في الكثير من المسائل المتعلقة بالدوائر المغناطيسية ، تعطى قيمة Φ وبمعرفة A (المقطع العرضي) يتم حساب B .

ولأجل حساب قيمة H (قوة التمغنط أو شدة المجال المغناطيسي) لابد من معرفة قيمة ($\mu = \mu_r \mu_0$) للمادة المكونة للدائرة المغناطيسية حينئذ : $H = \frac{B}{\mu}$

اما إذا لم تعطَ قيمة μ ، فيتم الحصول على قيمة H من منحنى التمغنط او منحنى (B_H) هذا .

مثال : اذا كانت قيمة $B = 1.2T$ لمادة (cast steel) ، احسب قيمة H المقابلة لها .

الجواب : انظر المنحنى ؛
$H = 1125$ At/m .

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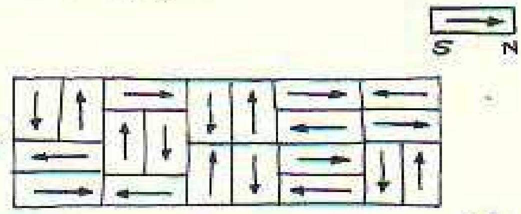
10.3 Hysteresis

1852 1890

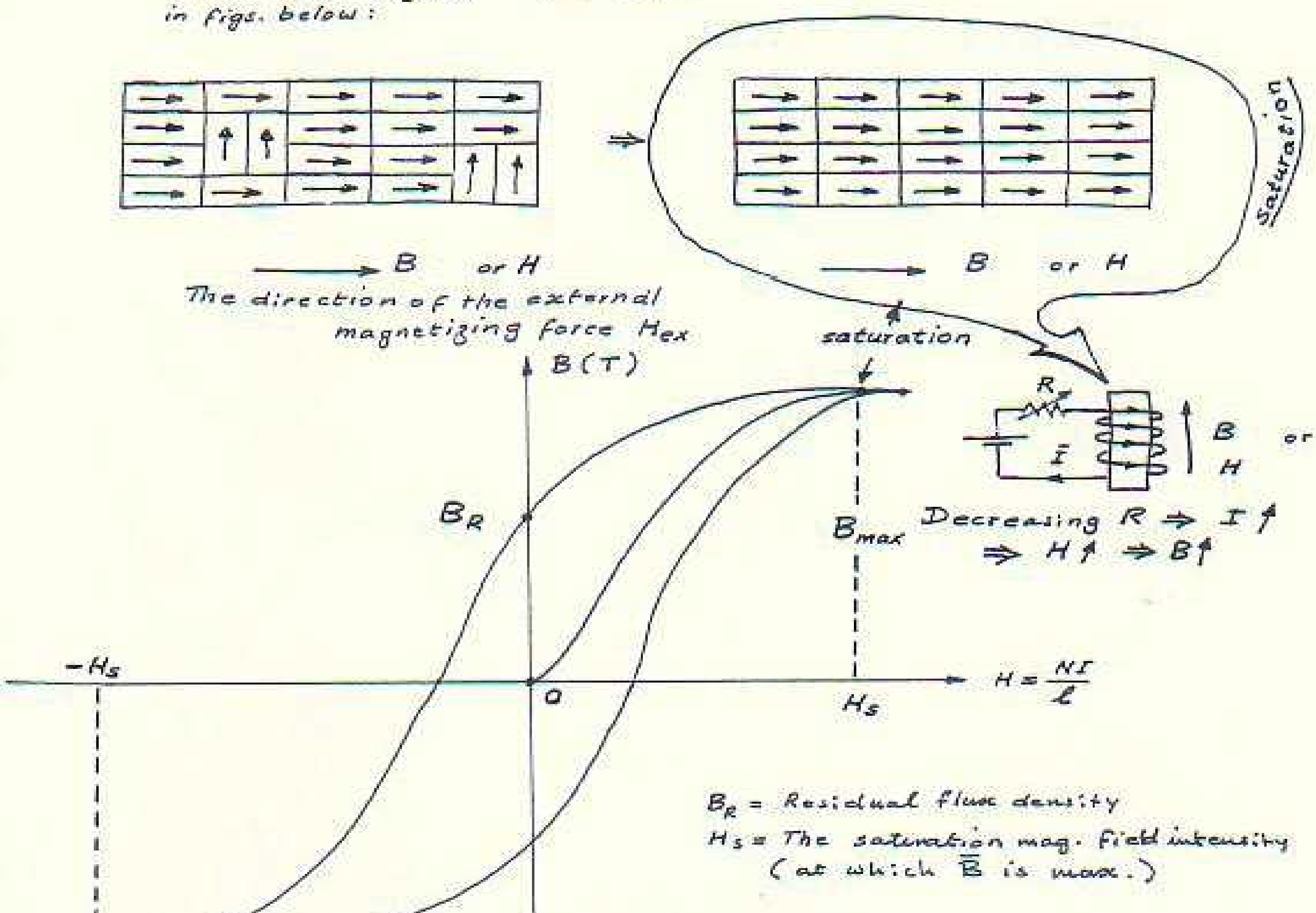
_____________ : (Weber & Edwing's Molecular Theory)

The basic assumption of this theory is that 'molecules of all substances are inherently magnets in themselves, each having N and S poles. In unmagnetized state, the net magnetic field due to these tiny molecular magnetic dipoles is zero, as shown in the figure:

* If this magnetic material is placed in a magnetic field or under the influence of a magnetic force, then these molecular magnets start turning themselves to be orientented more or less along straight lines parallel to the direction of the magnetizing force, as shown in figs. below:



Unmagnetized material ($\bar{B}_T = 0$)



Hysteresis Loop

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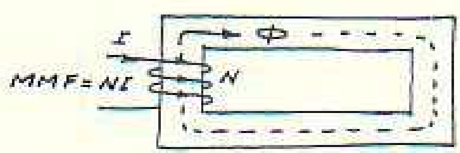
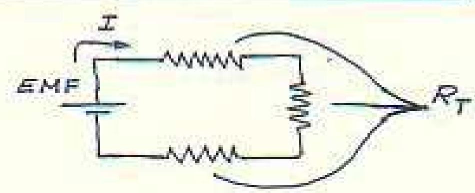
This linear arrangement of the molecular magnets results in N-polarity at one end and S-polarity at the other. As the small magnets turn more nearly in the direction of the magnetising force, it requires more and more of this force to produce a given turning moment thus accounting for the magnetic saturation.

10.3 Ampére's Circuital Law

: Comparing Electric circuit with the Magnetic Circuit, there may be some analogy between them. The table below summarizes such an analogy.

نقاط التشابه — Similarities

Magnetic Circuit	Electric Circuit
	
1. Flux = $\frac{mmf}{Reluctance}$ $\Phi = \frac{NI}{S}$	Current = $\frac{emf}{Resistance}$ $I = \frac{emf}{R_T}$
2. mmf (Ampere.turns)	emf (volts)
3. Flux, Φ (Webers = Wb)	current, I (Amperes, A)
4. Flux density, $\vec{B}$ (Wb/m^2)	current density, $\vec{J}$ (A/m^2)
5. Reluctance, $S = \frac{\ell}{\mu A} = \frac{\ell}{\mu_r \mu_o A}$	Resistance, $R = \rho \frac{\ell}{A}$ =
6. Permeance (= 1/Reluctance)	Conductance (= 1/Resistance)
7. Total mmf = $\Phi S_1 + \Phi S_2 + \cdots \Phi_n S_n$	Total emf = $IR_1 + IR_2 + \cdots IR_n$
$\Sigma mmf = 0$ قانون أمبير	قانون كرشوف $\Sigma emf = 0$
Ampere's Circuital Law	Kirchhoff's Voltage Law

نقاط الاختلاف — Differences

1. Flux does not actually "flow" in the sense in which an electric current flows.
2. Reluctance is not constant; it is small for small values of B and large for large values of B.

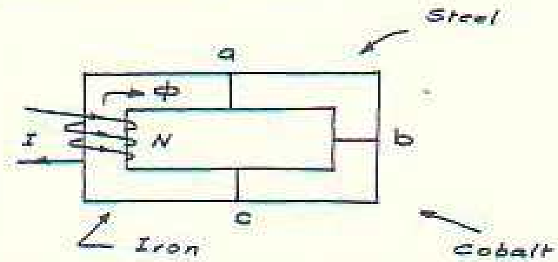
* It is clear, from the table, that Ampére's Circuital Law is:

$$\Sigma\, mmf = 0$$

When this law is applied to the magnetic circuit shown:

$$mmf_{total} = mmf_{ab} + mmf_{bc} + mmf_{ca}$$

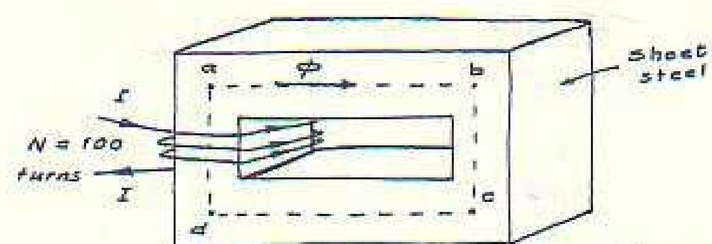
$$\therefore\quad NI = H_{ab}\,\ell_{ab} + H_{bc}\,\ell_{bc} + H_{ca}\,\ell_{ca}$$



ملاحظة مهمة → ⊕ All of the terms of the above equation are known except the magnetizing force for each portion of the magnetic circuit, which can be found by using the (B-H) curve if the flux density B is known.

10.4 Series Magnetic Circuits:

Example: Determine the current I required to establish a flux of 2.5×10^{-4} Wb in the core of the transformer of the Fig. shown.



Area (throughout) $= 0.2\times10^{-3}\ m^2$

$a\rightarrow b = c\rightarrow d = 0.05$ m

$b\rightarrow c = d\rightarrow a = 0.03$ m

Solution:

$$\Sigma\, mmf = 0 \quad\Rightarrow\quad NI = \Sigma H\ell$$

$$\Rightarrow NI = H_{ab}\,\ell_{ab} + H_{bc}\,\ell_{bc} + H_{cd}\,\ell_{cd} + H_{da}\,\ell_{da}$$

* Since the mag. cct is series ⟹ Φ is the same in all parts
* Since the cross-sectional area is the same ⟹ B is the same for all parts
* Since the material is the same for all parts ⟹ $\bar{H}$ is the same for all parts

$$\therefore NI = 2Hl_{ab} + 2Hl_{bc}$$

$$= 2H(l_{ab} + l_{bc})$$

↘ to find H ⇒

لإيجاد $\bar{H}$

١. إما من العلاقة $\bar{B} = \mu\bar{H}$ بشرط أن تكون قيمة μ معلومة

٢. أو من منحنى $(B-H)$ وفي كلتا الحالتين يجب الحصول أولاً على قيمة $\bar{B}$ من العلاقة: $B = \frac{\Phi}{A}$

$$\therefore B = \frac{\Phi}{A} = \frac{2.5\times10^{-4}}{0.2\times10^{-3}}$$

$$= 1.25\ T$$

تم استخراج قيمة $\bar{H}$ من منحنى B-H في حل ⇒ $\therefore H = 420\ At/m$

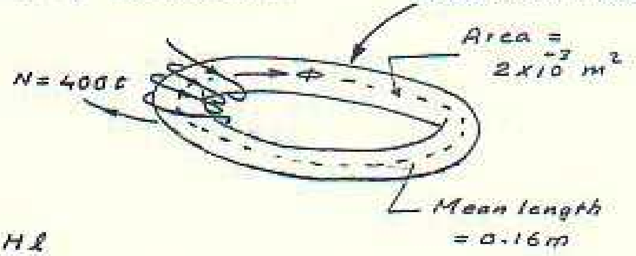
$$\therefore NI = 2(420)(0.05 + 0.03)$$

$$\text{or } (100)I = 67.2 \quad \Rightarrow \quad \underline{\underline{I = 0.672\ A}}$$

Example: For the series magnetic circuit shown;

(a). Find the value of I to establish a magnetic flux of $\Phi = 4\times10^{-4}$ Wb.

(b). Determine the permeability of the material under these conditions



Solution:

(a). $\Sigma mmf = 0$

⇐ المادة متكونة من جزء واحد مستمر.

$$\Rightarrow NI = \Sigma Hl = Hl$$

Since $\Phi = 4\times10^{-4}$ Wb

& $A = 2\times10^{-3}$ m

$$\therefore B = \frac{\Phi}{A} = \frac{4\times10^{-4}}{2\times10^{-3}} = \underline{0.2\ T}$$

إذا لم تعط قيمة μ للمادة فلابد من استخدام منحنى B-H للمادة مع استخراج قيمة H لقيمة معلومة من B.

⇒ From the (B–H) curve, at $\bar{B} = 0.2$ T

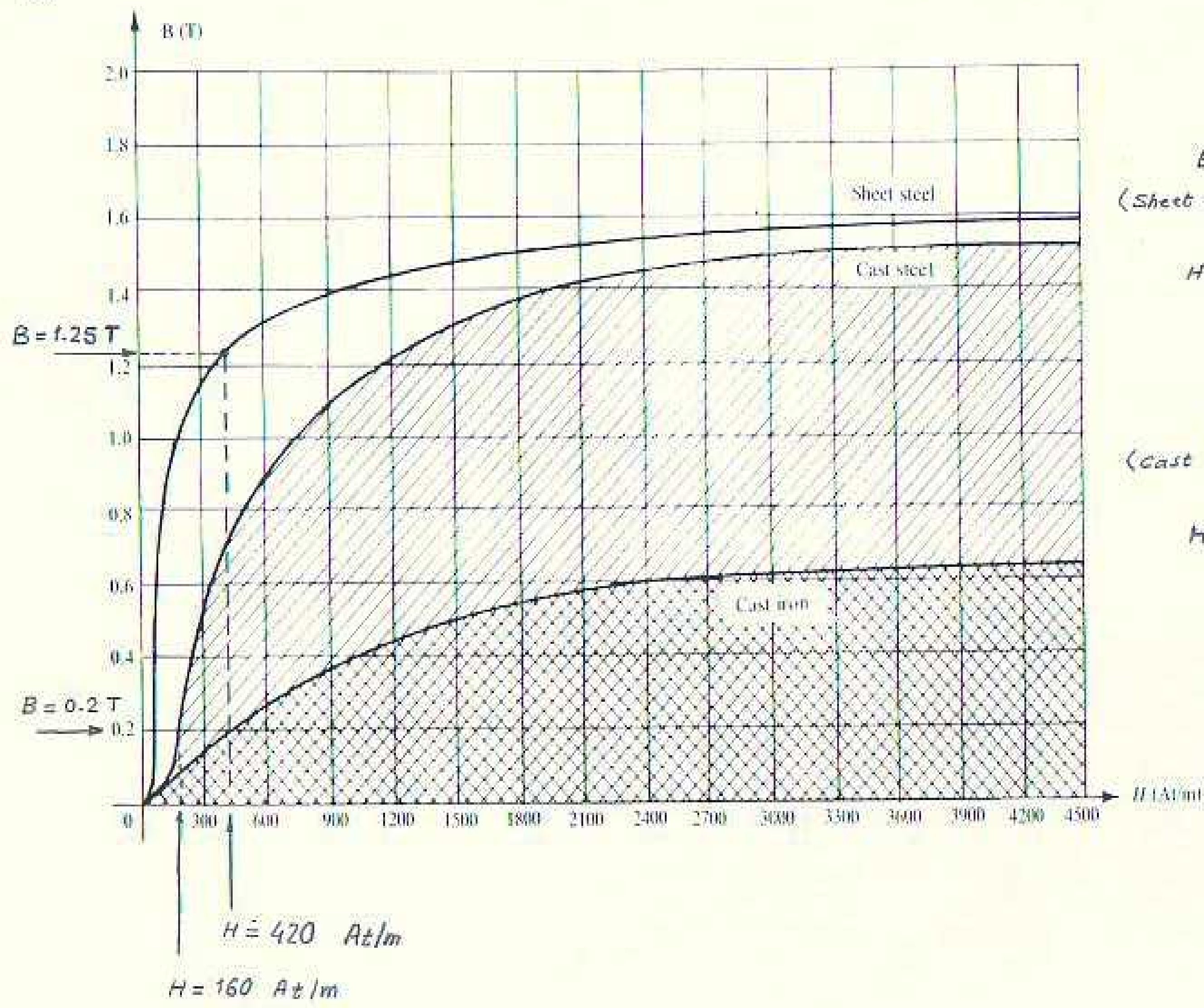
⇒ $\bar{H} = 160$ At/m

$$\therefore NI = Hl \Rightarrow I = \frac{Hl}{N} = \frac{(160)(0.16)}{400}$$

$$= \underline{64\ mA}$$

(b).

$$\mu = \frac{B}{H} = \frac{0.2}{160} = 1.25\times10^{-3}\ H/m$$



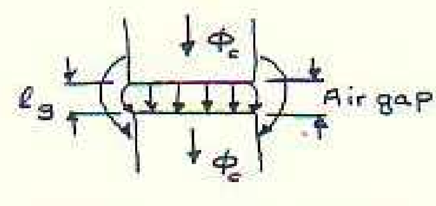
⊛ لقيمة :
$B = 1.25\ T$
لمادة (Sheet steel)
فان :
$H \doteq 420\ At/m$

⊛ لقيمة :
$B = 0.2\ T$
لمادة (Cast steel)
فان :
$H \doteq 110\ At/m$

Air Gaps

: In many practical applications, air gaps form essential parts in magnetic circuits such as air gaps in motor, generators, meters .. etc.

* In our calculations, the effect of fringing in the air gap is assumed to be small and can be neglected, so that;



$$\Phi_c = \Phi_g$$

ie, the flux in the core is equal to the flux in the gap.

* The magnet flux density of the air gap is given by:

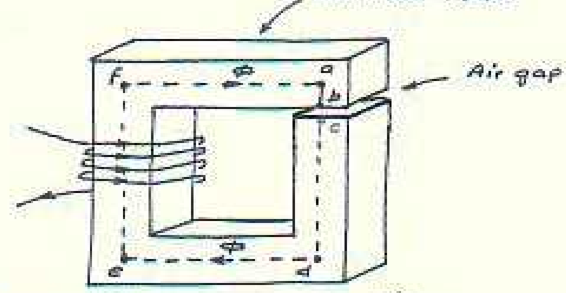
$$B_g = \frac{\Phi_g}{A} = \frac{\Phi_c}{A}$$

* The magnetizing force of the air-gap is then determined by:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}} = 7.97 \times 10^5 \, B_g$$

Example

: Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit shown.



Area (throughout) $= 1.5 \times 10^{-4}$ m²
$l_{ab} = 3.4 \times 10^{-3}$ m
$l_{bc} = 1.6 \times 10^{-3}$ m
$l_{cd} = 45 \times 10^{-3}$ m
$l_{de} = l_{ef} = l_{fa} = 58 \times 10^{-3}$ m

Solution

⇨

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Solution:

$$B = \frac{\Phi}{A} = \frac{0.75\times10^{-4}}{1.5\times10^{-4}} = 0.5\ T$$

* Φ هو نفسه في جميع اجزاء الدائرة لكونها دائرة توالي
* اما B فانه يعتمد على قيمة Φ و A وبما ان Φ و A ثابتين فان B هي نفسها في جميع اجزاء الدائرة.
* أما H فانها تعتمد على B و على μ.

∴ From the B-H curve;

$$H = 280\ At/m \quad \text{for cast steel}$$

* For the air-gap

$$H_g = \frac{B}{\mu_o} = \frac{0.5}{4\pi\times10^{-7}}$$

$$\therefore H_g = 3.985\times10^{5}\ At/m$$

$$\therefore NI = \Sigma H\ell$$

$$= H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd} + H_{de}\ell_{de} + H_{ef}\ell_{ef} + H_{fa}\ell_{fa}$$

$$= 3H_{de}\ell_{de} + H_{ab}\ell_{ab} + \overbrace{H_{bc}\ell_{bc}}^{H_g\ell_g} + H_{cd}\ell_{cd}$$

$$\therefore 200I = 3(280)(50\times10^{-3}) + (280)(3.4\times10^{-3}) + (3.985\times10^{5})(1.6\times10^{-3}) + (280)(45\times10^{-3})$$

$$\therefore 200I = 693.152$$

$$\Rightarrow I = \frac{693.152}{200} = 3.466\ A$$

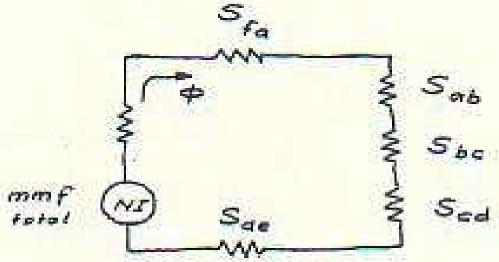
Note

The above results can be tabulated, for convenience, as:

	Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	ℓ (m)	Hℓ (At)
	ab	0.75×10^{-4}	1.5×10^{-4}	0.5	280	3.4×10^{-3}	0.952
air gap ⇒	bc	0.75×10^{-4}	1.5×10^{-4}	0.5	3.985×10^{5}	1.6×10^{-3}	637.6
	cd	0.75×10^{-4}	1.5×10^{-4}	0.5	280	45×10^{-3}	12.60
	de = ef = fa	0.75×10^{-4}	1.5×10^{-4}	0.5	280	50×10^{-3}	14.00

ملاحظة:

من الممكن الاستفادة من التناظر بين الدوائر الكهربائية والدوائر المغناطيسية لحل المثال اعلاه (وغيره من الامثلة الاخرى) وكما مبين ⟸



$$mmf_{total} = NI = \Sigma H\ell$$

كما مر معنا سابقاً.

$$OR \Rightarrow mmf_{total} = NI = \Sigma\Phi S = \Phi\Sigma S = \Phi(S_{fa} + S_{ab} + S_{bc} + S_{cd} + S_{de} + S_{ef})$$

حيث يتم حساب الـ (S) لكل جزء من اجزاء الدائرة $\left(S = \frac{\ell}{\mu A}\right)$ والحصول على النتيجة نفسها.

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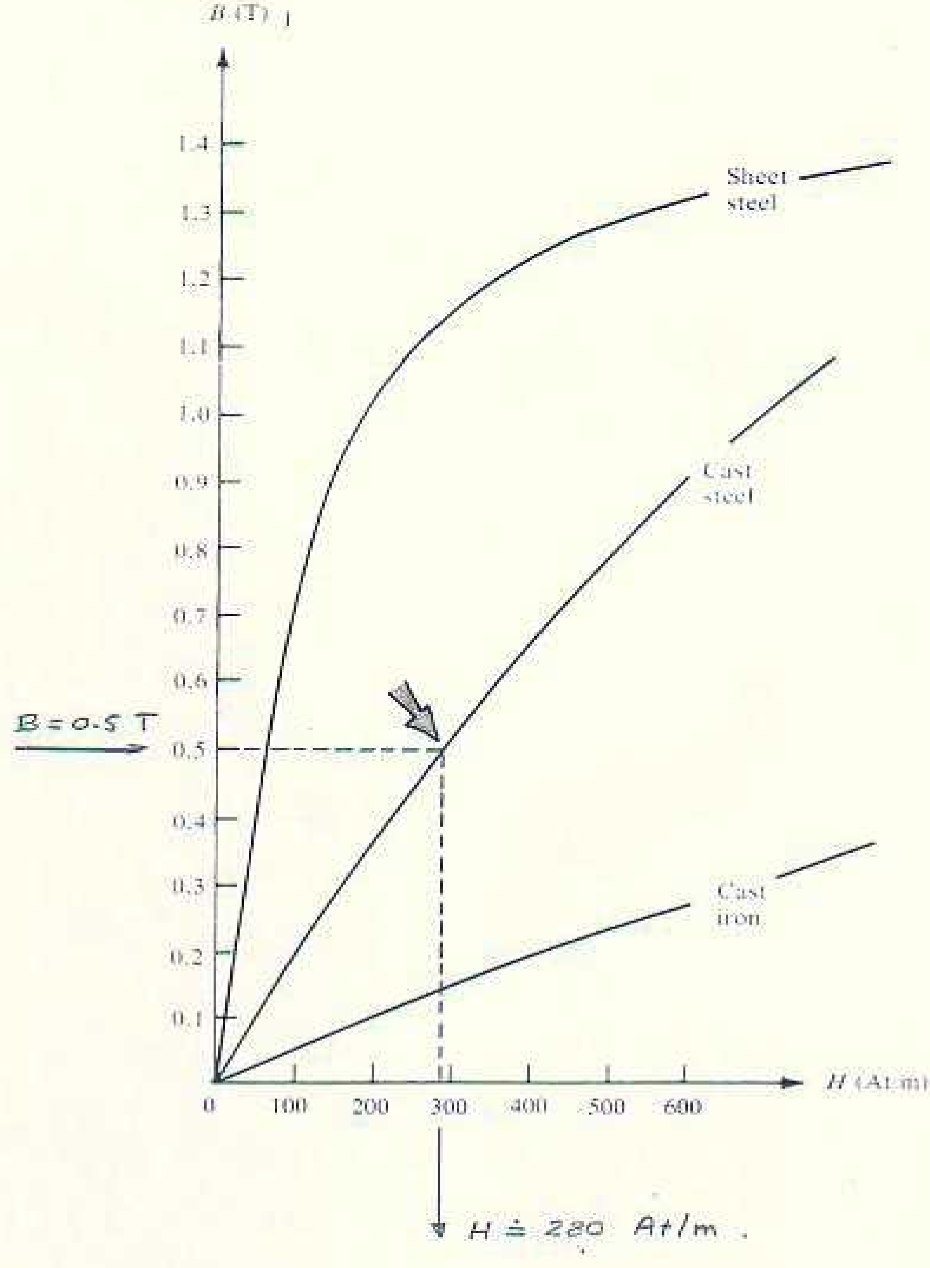
Hysteresis



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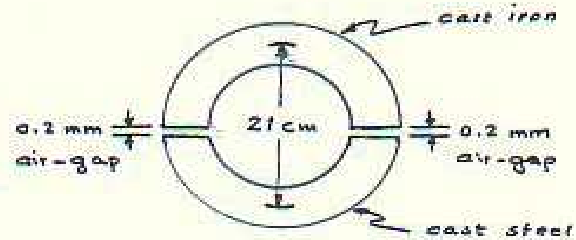
Example: A ring has a mean diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air gap of 0.2 mm. Find the Ampere-turns (total mmf) required to produce a flux of 8×10^{-4} Wb. Neglect fringing and leakage effects. Given that the relative permeabilities for cast steel & cast iron are 800 and 166 respectively.

Solution:

$\Phi = 8\times10^{-4}$ Wb

$A = 10\ cm^2 = 10\times10^{-4}\ m^2$

$$\therefore B = \frac{\Phi}{A} = \frac{8\times10^{-4}}{10\times10^{-4}} = 0.8\ Wb/m^2$$



$$\Sigma mmf = NI = \Sigma\Phi S = \Phi\Sigma S = \Phi\left(S_{\substack{cast\\iron}} + S_{\substack{air\\gap}} + S_{\substack{cast\\steel}} + S_{\substack{air\\gap}}\right)$$

* For the air gap

$$S_{\substack{air\\gap}} = \frac{l}{\mu_o A} = \frac{0.2\times10^{-3}}{4\pi\times10^{-7}(10\times10^{-4})}$$
$$=$$

* For the cast steel:

$$S_{\substack{cast\\steel}} = \frac{l}{\mu A} = \frac{l}{\mu_r\mu_o A}$$

$$\therefore S_{\substack{cast\\steel}} = \frac{0.33}{800\times4\pi\times10^{-7}\times10\times10^{-4}} = 3.28\times10^{5}$$

$l_{\substack{cast\\iron}} = \pi D/2 = 21\pi/2 = 0.33\ m = l_{cast\ steel}$

* For the cast iron

$$S_{\substack{cast\\iron}} = \frac{l}{\mu_r\mu_o A} = \frac{0.33}{166\times4\pi\times10^{-7}\times10\times10^{-4}} = 1.582\times10^{6}$$

$$\therefore \Sigma mmf = NI = 8\times10^{-4}(0.032\times10^{7} + 3.28\times10^{5} + 1.582\times10^{6})$$
$$= 1784\ At$$

ملاحظات: ١. من الممكن حل هذا المثال بطريقة ($\Sigma mmf = NI = \Sigma Hl$) كما رسمنا في الأمثلة السابقة والحصول على النتيجة نفسها.

٢. لا نحتاج في حل هذا المثال إلى استخدام منحني (B − H) وذلك لأن قيمتي μ_r لكل من المادتين المغناطيسيتين في الدائرة معلومة.

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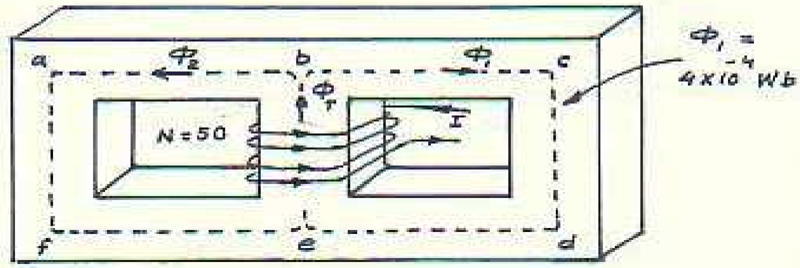
10.5 Series - Parallel Magnetic Circuits

: As in electric circuits, magnetic circuits may involve constructions lead to series - parallel connection.

Example : Determine the current I required to produce a flux of 4×10^{-4} Wb in the right limb of the magnetic circuit shown.

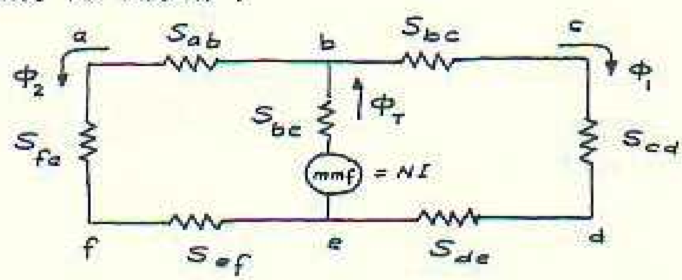


$$l_{ab} = l_{bc} = l_{de} = l_{ef} = 0.08 \text{ m}$$

$$l_{af} = l_{be} = l_{cd} = 0.05 \text{ m}$$

$$A \text{ (throughout)} = 6 \times 10^{-4} \text{ m}^2$$

Solution : An equivalent magnetic circuit analogous to the electric circuit is shown :



* Due to symmetry;

$$\Phi_2 = \Phi_1 = 4 \times 10^{-4} \text{ Wb}$$

and $$\Phi_T = \Phi_1 + \Phi_2 = 2(4 \times 10^{-4}) = 8 \times 10^{-4} \text{ Wb}$$

* For each outside leg;

section bcdc & section efab ⇒ $$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4}}{6 \times 10^{-4}} = 0.667 \text{ T}$$

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* For the centre leg ;

section be

$$B = \frac{\Phi}{A} = \frac{8 \times 10^{-4}}{6 \times 10^{-4}} = 1.333 \text{ T}$$

* Since the magnetic circuit is made up of one material , the

H (from the B-H curve)

$$\therefore H_{bcde} \doteq 100 \text{ At/m}$$

and

$$H_{be} \doteq 650 \text{ At/m}$$

Results summarized in the table below ;

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	ℓ (m)
bcde or efab	4×10^{-4}	6×10^{-4}	0.667	100	0.21
be	8×10^{-4}	6×10^{-4}	1.333	650	0.05

* Using Ampère circuital law

$$NI = H_{be}\ell_{be} + H_{bcde}\ell_{bcde}$$

$$50\,I = (650)(0.05) + (100)(0.21)$$
$$= 53.5$$

$$\therefore I = \frac{53.5}{50} = \underline{\underline{1.070 \text{ A}}}$$

ملاحظة : من الممكن استخدام الصيغة

$$NI = \Sigma \Phi S$$
$$= \Phi_1 \cdot S_{bcde} + \Phi_T S_{be}$$

والحصول على النتيجة نفسها . وتستخدم هذه الطريقة في حالة كون Φ_T معلومة .

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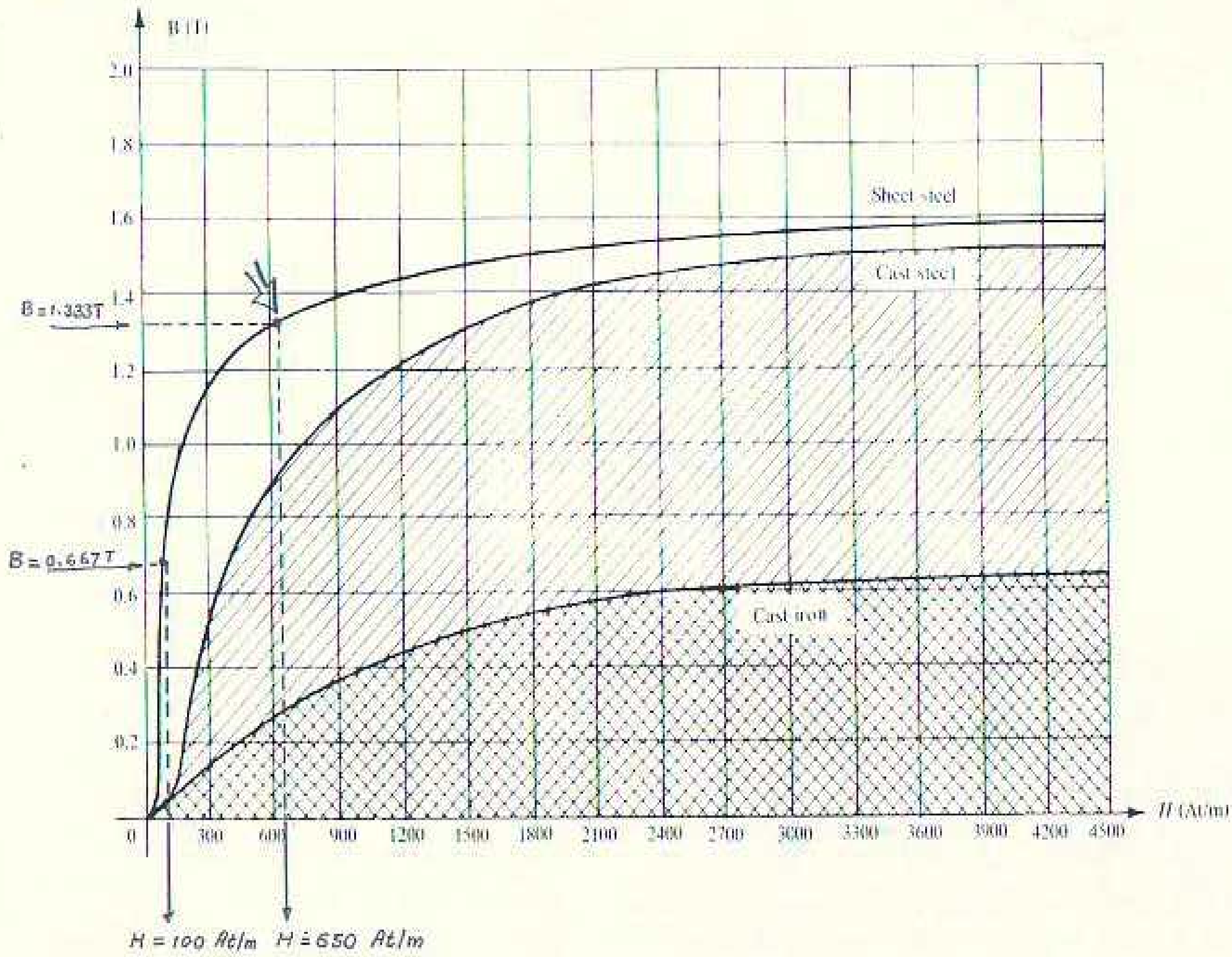
EE10


B (T)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
300
600
900
1200
1500
1800
2100
2400
2700
3000
3300
3600
3900
4200
4500
H (At/m)
Sheet steel
Cast steel
Cast iron
B = 1.333T
B = 0.667T
H = 100 At/m
H ≅ 650 At/m