7. Series & Parallel AC Circuits

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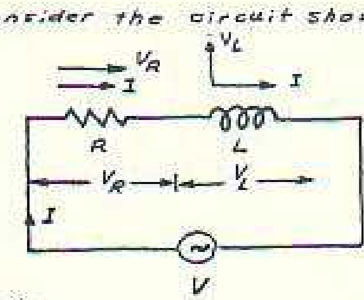
7.1 Series AC Circuits :

7.1.1 AC Through R and L : Consider the circuit shown below;

V = the rms value of the applied voltage.
I = the rms value of the resultant current.



$\bar{V} = \bar{V}_R + \bar{V}_L$

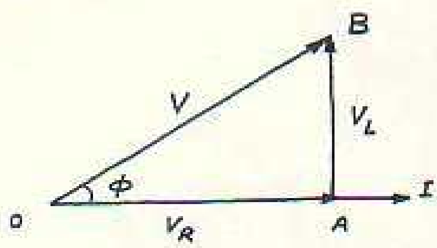
$\Rightarrow V_R = IR$ (in phase with I)
$V_L = I X_L$ (leading I by 90°)

The vector diagram for these voltage drops can be obtained as:

$$\Rightarrow V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$



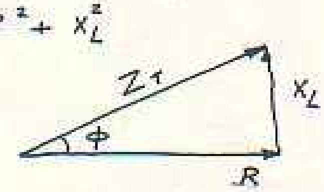
voltage triangle

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_T}$$

$\Rightarrow$ Z is known as the impedance of the circuit

$$Z_T = \sqrt{R^2 + X_L^2}$$

$$Z_T^2 = R^2 + X_L^2$$



Impedance traingle

* The phase difference angle ϕ can be determined as:

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{\omega L}{R}$$

$\cos\phi = \frac{R}{Z_T}$

$$\therefore \boxed{\phi = \tan^{-1}\frac{X_L}{R}}$$

It is clear that the current I lags behind the applied voltage V by an angle (ϕ), then if:

$v = V_m \sin\omega t \Rightarrow i = I_m \sin(\omega t - \phi)$

where $I_m = \frac{V_m}{Z}$

$\Rightarrow v_R = I_m R \sin(\omega t - \phi)$

$\Rightarrow v_L = I_m X_L \sin(\omega t - \phi + 90^\circ)$

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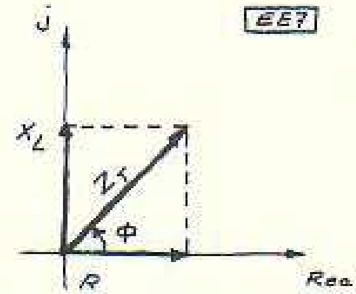
* In phasor notation

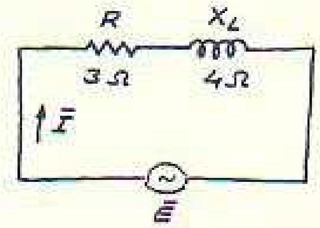
$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$$
$$= R\angle 0 + X_L \angle 90$$
$$\Rightarrow \therefore \bar{Z}_T = R + jX_L$$



Impedance diagram

Example

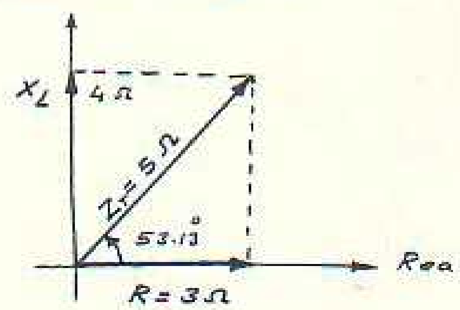
For the circuit shown, determine the total impedance and draw the impedance diagram



Solution : $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R\angle 0^\circ + X_L \angle 90^\circ = R + jX_L$

$$= 3 + j4$$

$$\Rightarrow \therefore \bar{Z}_T = 5 \angle 53.13^\circ$$



Note that, the angle ϕ (= 53.13° in this example) is always positive in the impedance diagram.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{4}{3}$$
$$= 53.13^\circ$$

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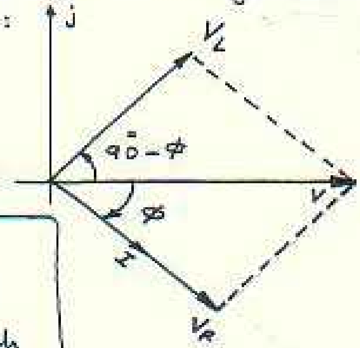
The phasor diagram

: The phasor diagram of the voltages of the RL series circuit can be as shown:

$$\bar{V} = \bar{V}_R + \bar{V}_L = \bar{I}R + \bar{I}\bar{X}_L$$
$$= IR\angle{-\phi} + IX_L\angle{90-\phi}$$



phasor diagram

كذلك تسمى بالقدرة الحقيقية (true power)

> The total power (average power) in (Watts) delivered to the circuit is given by the product of V and the component of the current I which is in phase with the applied voltage V

⇒ Then:

$$\boxed{P_T = VI\cos\phi}$$ Watts

⇐ where $\cos\phi$ is the power factor

* **The power factor**: The power factor can be defined as the cosine of the angle (lead or lag) between the current and voltage;

⇒ Power factor = $\cos\phi$

(2) or it may be defined as the ratio $\frac{R}{Z}$ (see the impedance triangle)

(3) or it may be defined as the ratio $\frac{\text{true power}}{\text{apparent power}}$

⇒ Power factor = $\frac{\text{true power}}{\text{apparent power}}$ ⇒ $\frac{\text{Watts}}{\text{Volt-ampere}}$

* **Active and Reactive Components of the Circuit Current**:

from the phasor diag. ⇒

* The active component is that which is in phase with the applied voltage V, i.e ($I\cos\phi$); it is also called the wattful component.

* The reactive component is that which is in quadrature with V, i.e ($I\sin\phi$); it is also called the wattless component or the idle component.

⇒ According to these definitions, we have also two components of power each relating its corresponding current component

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* Active, Reactive and Apparent Power (The Power Triangle):

* The Apparent Power:

It is the product of the rms values of the applied voltage and the circuit current

$$\Rightarrow \text{Apparent power} = S = VI = (IZ).I = I^2Z$$

وحدة قياس
volt-ampere (VA)

* The Active Power:

It is the power which is actually dissipated in the circuit resistance.

$$\Rightarrow \text{Active power} = P = I^2R = VI\cos\phi \qquad \text{watts (W)}$$

* The Reactive Power:

It is the power developed in the <u>inductive</u> reactance of the circuit.

$$\Rightarrow \text{Reactive Power} = Q = I^2X_L = I^2(Z\sin\phi)$$

$$\therefore Q = VI\sin\phi$$

وحدة قياس
Volt-amper reactive (VAR)

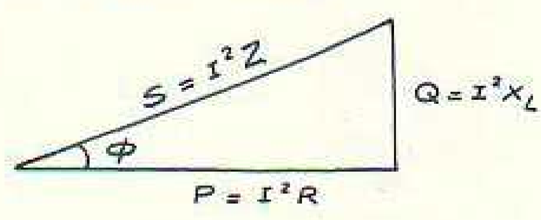
These three powers are shown in the power triangle:

From the power triangle:

$$S^2 = P^2 + Q^2$$

or

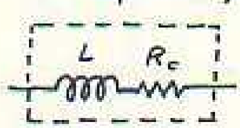
$$S = \sqrt{P^2 + Q^2}$$



The power triangle

* The quality factor of the Coil:

It is defined as the reciprocal of the power factor of the coil. Hence:



coil

$$\Rightarrow Q_{factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R_c} = \frac{\sqrt{R_c^2 + X_L^2}}{R_c}$$

if R_c is too small compared with X_L, then:

$$\boxed{Q_{factor} = \frac{X_L}{R_c}}$$

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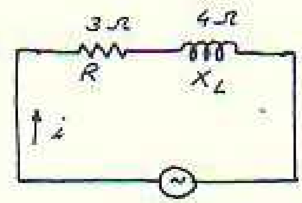
Example: For the circuit shown, draw the phasor diagram of the voltages across each element and the applied voltage, and determine :- The power factor.
- The active and reactive power.
- The apparent power.



$v = 141.4 \sin\omega t$

Solution:

* $\because v = 141.4 \sin\omega t \Rightarrow \bar{V} = 100\angle 0^\circ$

* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R + jX_L$
$= 3 + j4$
$= 5\angle 53.13^\circ$

* The current $\bar{I}$ $\Rightarrow \bar{I} = \dfrac{\bar{V}}{\bar{Z}_T}$

$\therefore \bar{I} = \dfrac{100\angle 0^\circ}{5\angle 53.13^\circ} = 20\angle -53.13^\circ$

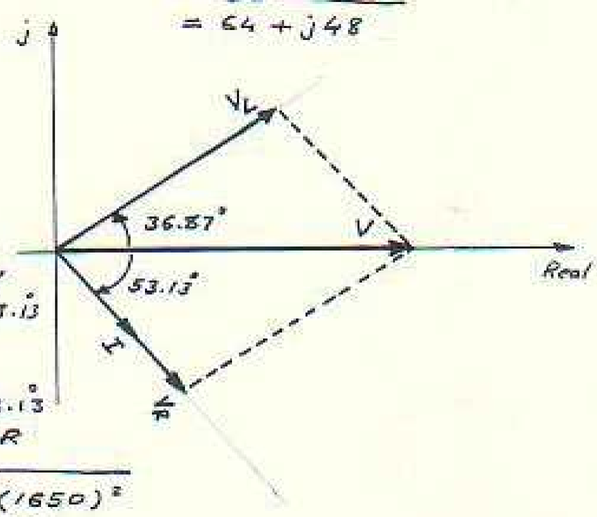
* The voltage drops $\bar{V}_R$ and $\bar{V}_L$:

$\bar{V} = \bar{V}_R + \bar{V}_L$ $\Rightarrow \bar{V}_R = \bar{I}R = (20\angle -53.13^\circ)(3)$
$= 60\angle -53.13^\circ$
$= 36 - j48$

$\therefore \bar{V} = 36 - j48 + 64 - j48$ $\Leftarrow V_L = \bar{I}\bar{X}_L = (20\angle -53.13)(4\angle 90^\circ)$
$= 100$ $= 80\angle 36.87^\circ$
$= 100\angle 0^\circ$ $= 64 + j48$

[illegible]

* The phasor diagram



* **Powers**

active power (real) (average) $P = I^2R = (20)^2(3) = 1200\,W$
or $P = VI\cos\phi = (100)(20)\cos 53.13^\circ$
$= 1200\,W = 1.2\,kW$

reactive power $Q = VI\sin\phi = (100)(20)\sin 53.13^\circ$
$= 1600\,VAR = 1.6\,kVAR$

$S = \sqrt{P^2 + Q^2} = \sqrt{(1200)^2 + (1650)^2}$
$= 1968\,VA = 1.968\,kVA$

* The power factor: $P.f = \cos\phi = \cos 53.13^\circ = \underline{0.6 \text{ lagging}}$

اعتماداً على التيار بالنسبة للفولتية

7.1.2 AC Through R and C

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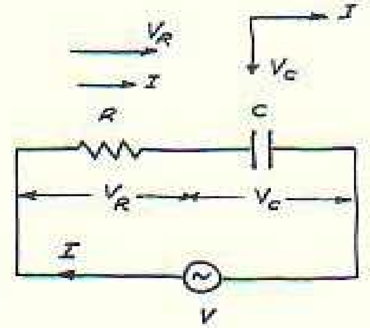
Consider the circuit shown, where



V = rms value of the applied voltage.
I = rms value of the resultant current.

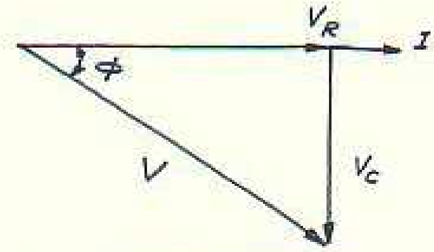
$$V_R = IR \quad \text{(in phase with } I\text{)}$$
$$V_L = IX_C \quad \text{(lagging } I \text{ by } 90^\circ\text{)}$$

* In Vector Notations:

$$\bar{V} = \sqrt{\bar{V}_R^2 + \bar{V}_c^2}$$
$$= \sqrt{I^2R + I^2X_c^2}$$
$$= I\sqrt{R^2 + X_c^2}$$

or $$I = \frac{V}{\sqrt{R^2 + X_c^2}} = \frac{V}{Z}$$

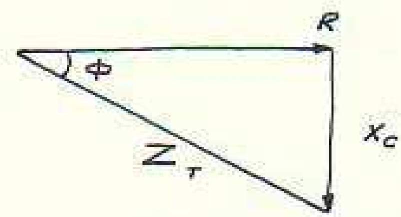
$$Z = \sqrt{R^2 + X_c^2}$$



voltage triangle

* $$\phi = \tan^{-1}\frac{X_c}{R}$$

$$\therefore \text{P.f} = \cos\phi = \frac{R}{Z}$$



impedance triangle

It is clear that I lead V by an angle ϕ. Hence if:

$$v = V_m \sin \omega t$$

then:

$$i = I_m \sin(\omega t + \phi)$$

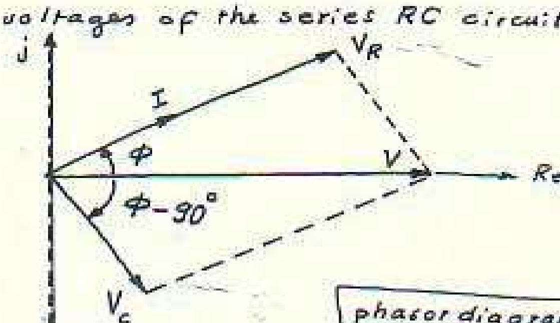
so that the current i lead the applied voltage v by an angle ϕ, and

$$v_R = I_m R \sin(\omega t + \phi)$$

$$v_c = I_m X_c \sin(\omega t + \phi - 90^\circ)$$

* The phasor diagram of the voltages of the series RC circuit can be as shown:

$$\bar{V} = V\angle 0$$
$$\bar{I} = I\angle\phi$$
$$\bar{V}_R = V_R\angle\phi$$
$$\bar{V}_c = V_c\angle\phi - 90^\circ$$



phasor diagram

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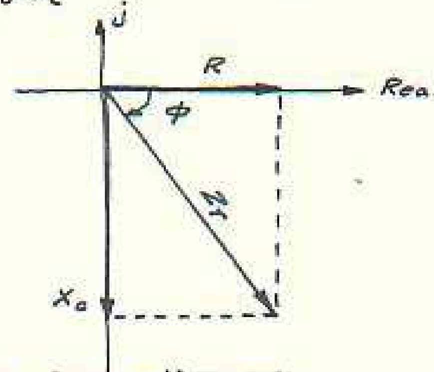
* The impedance diagram

in phasor notation

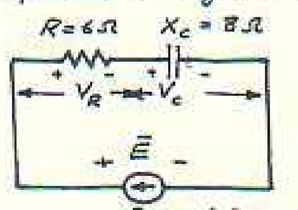
$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R\angle 0^\circ + X_c\angle -90^\circ = R - jX_c$$

$$\therefore \bar{Z}_T = \sqrt{R^2 + X_c^2} = Z_T \angle \phi$$



⇒ Note that ϕ is always negative for RC circuits.

Example: For the circuit shown, draw the phasor diagram.



Solution:

* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$
$= 6\angle 0^\circ + 8\angle -90^\circ$
$= 6 - j8 = 10\angle -53.13^\circ$

* $\bar{E}$?

$$\bar{E} = \bar{I}\bar{Z}_T = (5\angle 53.13^\circ)(10\angle -53.13^\circ) = \underline{50\angle 0}$$

$\bar{V}_R$?

$$\bar{V}_R = \bar{I}R = (5\angle 53.13^\circ)(6) = \underline{30\angle 53.13^\circ}$$

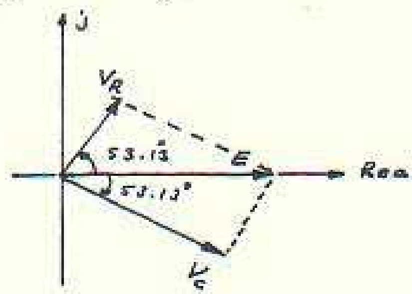
$\bar{V}_c$?

$$\bar{V}_c = \bar{I}\bar{X}_c = (5\angle 53.13^\circ)(8\angle -90^\circ) = 40\angle -36.87^\circ$$

* you can find that:

$$\bar{E} = \bar{V}_R + \bar{V}_c$$

using the above values.



* Similarly, as in the case of series RL circuit, the active (average or true) power, reactive power can be determined.

* The active power P is

$$\boxed{P = VI\cos\phi}$$

$P = I^2R$

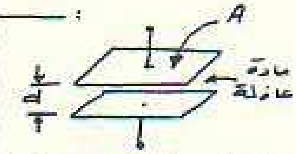
* The reactive power Q is:

$Q = I^2X_c$

$$\boxed{Q = VI\sin\phi}$$

* and the apparent power $S = \sqrt{P^2 + Q^2}$

* Dielectric Loss and the Power Factor of a Capacitor:



For parallel-placi capacitor

$$C = \epsilon \frac{A}{d}$$

C = capacitance (F)
ϵ = dielect constant
A = Area of plate m^2
d = Separation m

* A pure (ideal) capacitor is one in which there are no losses and whose current lead the voltage by 90° as shown:

* In practice, it is impossible to get such a capacitor although close approximation is achieved by proper design.

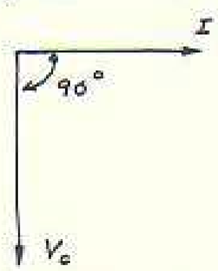
* In every capacitor, there is always some dielectric loss, and hence absorbs part of the power from the circuit. Due to this loss, the phase angle is somewhat less than 90°.

* This dielectric loss appears as heat

* ψ is the phase difference given by:

$$\psi = 90^\circ - \phi$$

where ϕ is the actual phase angle.

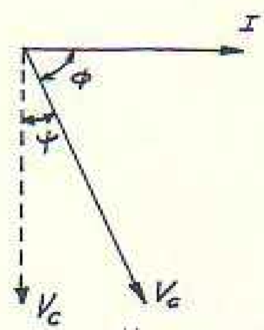


V_c and I for ideal (pure) capacitor

* Since ψ is generally small $\Rightarrow \sin\psi \approx \psi$

$$\therefore \boxed{\tan\psi \approx \psi \approx \cos\phi}$$

* It should be noted that dielectric loss increases with the frequency of the applied voltage.



V_c and I for Actual capacitor.

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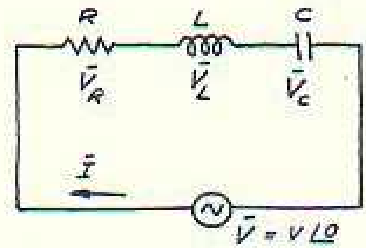
7.2.3 AC Through RLC series circuit :

$\bar{Z} = R + jX_L - jX_c$

$= R + j(X_L - X_c) = Z\angle\phi$



$\bar{I} = \dfrac{V\angle 0}{Z\angle\phi} = I\angle{-\phi}$

$\bar{V}_R = \bar{I}R = (I\angle\phi)(R\angle 0^\circ)$

$= IR\angle{-\phi}$

$X_L = \omega L$

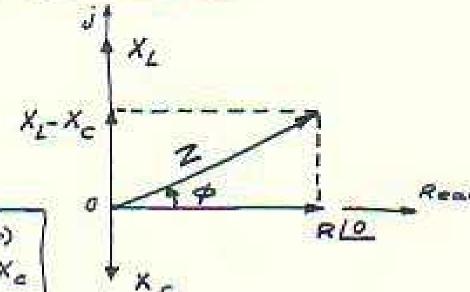
$X_C = \dfrac{1}{\omega C}$

$\bar{V}_L = \bar{I}\bar{X}_L = (I\angle{-\phi})(X_L\angle 90^\circ) = IX_L\angle 90^\circ - \phi$

$V_c = \bar{I}X_c = (I\angle{-\phi})(X_c\angle{-90^\circ}) = IX_c\angle{-(90^\circ + \phi)}$

* The impedance diagram :

$\phi = \tan^{-1}\dfrac{X_L - X_c}{R}$



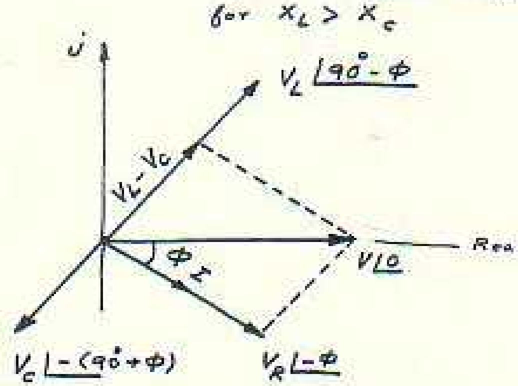
impedance phasor diagram for $X_L > X_c$

> The angle ϕ may be (+ve) or (−ve) depending on the values of X_L and X_c

* The voltage phasor diagram

$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_c$

$\therefore\ \bar{V} = \bar{I}R + \bar{I}\bar{X}_L + \bar{I}X_c$



* The active, reactive and the apparent powers can be determined as mentioned previously.

* When $X_L > X_c \Rightarrow X_L - X_L = (+ve) \Rightarrow \phi$ = positive in the impedance diagram.

* When $X_L < X_c \Rightarrow X_L - X_c = (-ve)$ $\hookrightarrow \phi$ = (−ve) in the voltage phasor diagram.
so $\phi \Rightarrow$ (−ve) in the impedance diagram, ϕ = (+ve) in the phasor diagram.

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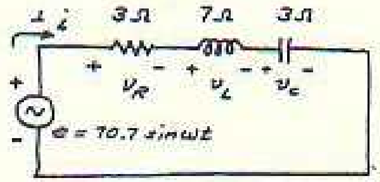
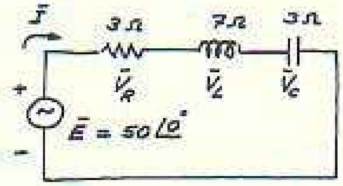
Example: For the circuit shown, determine:

- $\bar{Z}_T$, and draw the impedance diagram.
- $\bar{I}$, $\bar{V}_R$, $\bar{V}_L$, $\bar{V}_C$ in the phasor domain, and draw the phasor diagram.
- i, v_R, v_L, v_C in the time domain.
- The power factor of the circuit.
- The active, reactive and the apparent powers.



Solution: In phasor notation the circuit is redrawn as:



* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3$

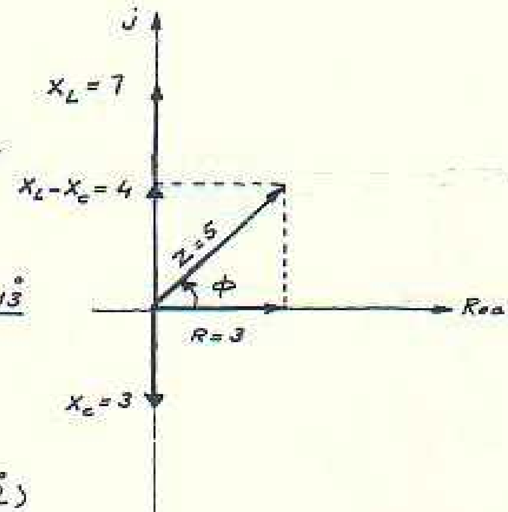
$= R\angle 0 + X_L\angle 90° + X_C\angle -90°$

$= 3 + j7 - j3$

$= 3 + j4$

$\therefore \bar{Z}_T = 5\angle 53.13°$

* The impedance diagram is ⇒



$$\bar{I} = \frac{\bar{E}}{\bar{Z}} = \frac{50\angle 0}{5\angle 53.13°} = 10\angle -53.13°$$

$\bar{V}_R = \bar{I}R = (10\angle -53.13°)(3\angle 0°)$

$= 30\angle -53.13$

$V_L = \bar{I}\bar{X}_L = (10\angle -53.13)(7\angle 90°)$

$= 70\angle 36.87°$

$V_C = \bar{I}\bar{X}_C = (10\angle -53.13°)(3\angle -90)$

$= 30\angle -143.13°$

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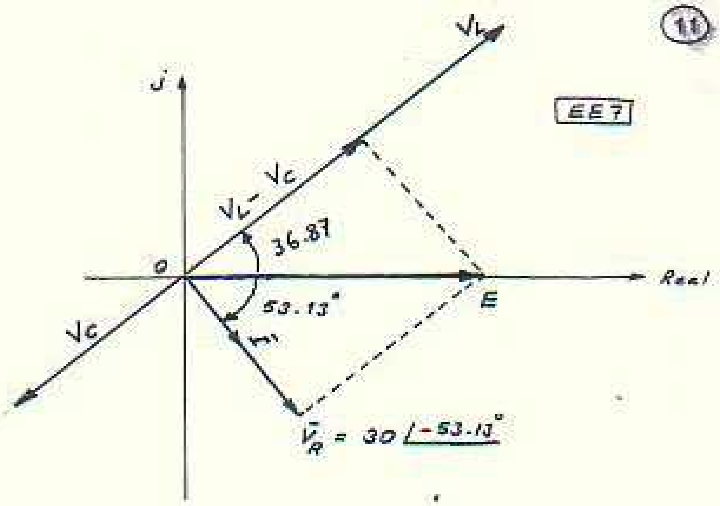
* The phasor diagram



* The time domain

$$i = \sqrt{2}\,(10)\sin(\omega t - 53.13°) = 14.14 \sin(\omega t - 53.13°)$$
$$v_R = \sqrt{2}\,(30)\sin(\omega t - 53.13°) = 42.42 \sin(\omega t - 53.13)$$
$$v_L = \sqrt{2}\,(70)\sin(\omega t + 36.87°) = 98.98 \sin(\omega t + 36.87°)$$
$$v_c = \sqrt{2}\,(30)\sin(\omega t - 143.13°) = 42.42 \sin(\omega t - 143.13°)$$

* The total power

$$P_T = VI\cos\phi = (50)(10)\cos 53.13° = \underline{300\ W}$$

$$\boxed{\text{or}}\quad P_T = I^2 R = (10)^2(3) = \underline{300\ W}$$

from the voltage phasor diagram ⟺ * The power factor :

$$p.f = \cos\phi = \cos 53.13° = 0.6\ \underline{\text{lagging}}$$

$$\text{or } p.f = \cos\phi = \frac{R}{Z_T} = \frac{3}{5} = 0.6 \text{ lagging}$$

$P \Rightarrow$ * Active power = true power = $VI\cos\phi = (50)(10)\cos 53.13° = 300\ W$

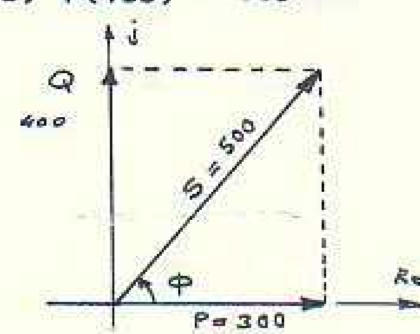
$Q \Rightarrow$ Reactive power = $Q = VI\sin\phi = (50)(10)\sin 53.13° = 400\ VAR$

$S \Rightarrow$ Apparent power = $S = \sqrt{P^2 + Q^2} = \sqrt{(300)^2 + (400)^2} = 500\ VA$

∴ The power triangle

$$\boxed{\bar{S} = P + jQ}$$

$\bar{S} \Rightarrow$ Complex apparent power.



7.2 Parallel AC Circuits

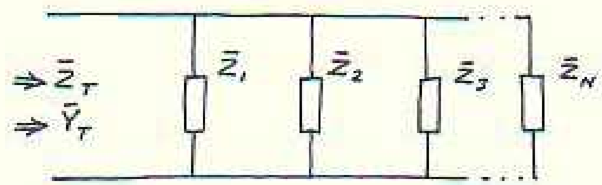
* Admittance and Susceptance

- In the dc circuit analysis, we had used the term conductance to represent the reciprocal of the resistance R; ie:

$$G = \frac{1}{R} \qquad \text{where } G \text{ is the conductance}$$

The total conductance of the paralled circuit is then found by adding the conductance of eachb branch.

- In AC circuit analysis, we define the admittance ($\bar{Y}$) as equal to $1/\bar{Z}$. For the parallel circuit shown:



* The total admittance Y_T:

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \dots \bar{Y}_N$$

then since $\bar{Y} = \frac{1}{\bar{Z}}$; so the total impedance $\bar{Z}_T$:

$$\frac{1}{\bar{Z}_T} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} + \dots \frac{1}{\bar{Z}_N}$$

- As mentioned earlier, for 2 branches parallel ac circuit, then:

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

- Also for 3 parallel branches;

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2 \bar{Z}_3}{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_3}$$

* INGENERAL, we have: $\bar{Z}_T = R \mp jX$, the

$$\bar{Y}_T = \frac{1}{R} \mp \frac{1}{jX} = \boxed{G \pm jB}$$

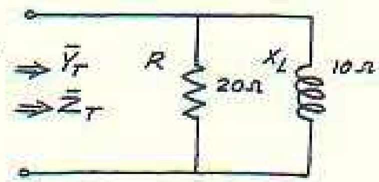
where: $G \Rightarrow$ Conductance $= \frac{1}{R}$ (Siemens, S) شس جوهري

$B \Rightarrow$ Susceptance $= \frac{1}{X}$ (Siemens, S)

Example EE7

: For the circuit shown;

a. Determine the admittance of each branch.
b. Find the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



Solution

ⓐ:

- $\bar{Y}_1 = \bar{G} = G\angle 0 = \frac{1}{R}\angle 0 = \frac{1}{20}\angle 0 = 0.05\angle 0$
  $= 0.05 + j0$
  $= 0.05$
- $\bar{Y}_2 = \bar{B}_L = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10}\angle -90^\circ = 0.1\angle -90^\circ$
  $= 0 - j0.1$
  $= -j0.1$

ⓑ: $\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = 0.05 - j0.1 = G - jB_L$

ⓒ: $\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.05 - j0.1} = \frac{1}{0.112\angle -63.43^\circ}$
$= 8.93\angle 63.43^\circ$

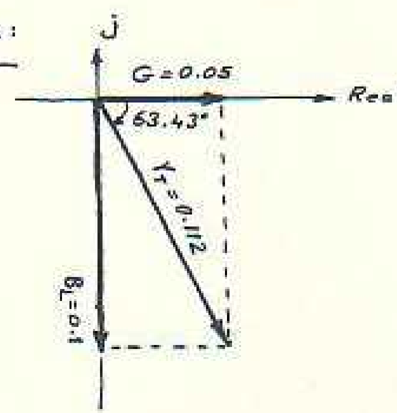
OR

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(20\angle 0)(10\angle 90^\circ)}{20 + j10}$$

$$= \frac{200\angle 90^\circ}{22\angle 26.57^\circ}$$

$= 8.93\angle 63.43^\circ$ ⟵ which is the same as calculated above.

ⓓ: The admittance diagram:



Example

: For the circuit show;

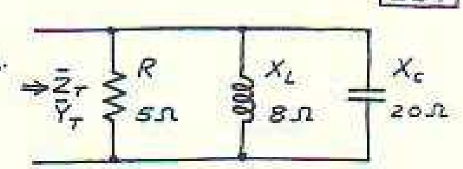
a. Determine the admittance of each branch.

b. Find the input admittance.

c. Calculate the input impedance.

d. Draw the admittance diagram.

EE7



Solution

: ⓐ.

$$\bar{Y}_1 = \bar{G} = \frac{1}{R\angle 0} = \frac{1}{5}\angle 0 = 0.2\angle 0^\circ = 0.2 + j0 = 0.2$$

$$\bar{Y}_2 = \bar{B}_L = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ = \frac{1}{8}\angle -90^\circ = 0.125\angle -90^\circ = -j0.125$$

$$\bar{Y}_3 = \bar{B}_C = \frac{1}{X_C\angle -90^\circ} = \frac{1}{X_C}\angle 90^\circ = \frac{1}{20}\angle 90^\circ = 0.05\angle 90^\circ = +j0.05$$

ⓑ.

$$\begin{aligned}\bar{Y}_T &= \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 \\ &= 0.2 - j0.125 + j0.05 \\ &= 0.2 - j0.075 \\ &= 0.2136\angle -20.56^\circ\end{aligned}$$

ⓒ.

$$\begin{aligned}\bar{Z}_T &= \frac{1}{\bar{Y}_T} = \frac{1}{0.2136\angle -20.56^\circ} \\ &= 4.68\angle 20.56^\circ\end{aligned}$$

$\boxed{\text{or}}$

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1 Z_3}$$

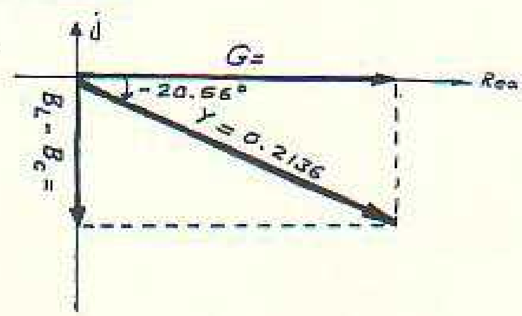
$$= \frac{(5\angle 0^\circ)(8\angle 90^\circ)(20\angle -90^\circ)}{(5\angle 0^\circ)(8\angle 90^\circ) + (8\angle 90^\circ)(20\angle -90^\circ) + (5\angle 0^\circ)(20\angle -90^\circ)}$$

$$= \frac{800\angle 0^\circ}{40\angle 90^\circ + 160\angle 0^\circ + 100\angle -90^\circ}$$

$$= \frac{800\angle 0^\circ}{j40 + 160 - j100} = \frac{800\angle 0^\circ}{160 - j60}$$

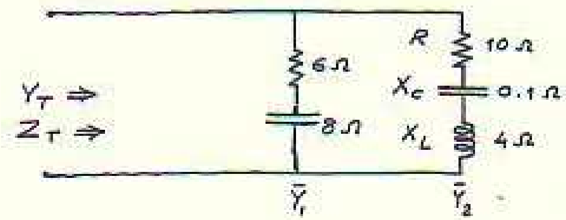
$$= \frac{800\angle 0^\circ}{170.88\angle -20.56^\circ} = \underline{\underline{4.68\angle 20.56^\circ}}$$

ⓓ. *The Admittance Diagram*



EE7

Example: Find the admittance of the circuit shown



Solution:

$$\bar{Z}_1 = 6 - j8$$

$$\Rightarrow \bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{6-j8} = \frac{6+j8}{6^2+8^2} = \frac{6}{100} + j\frac{8}{100}$$

$$= 0.06 + j0.08$$

$$\bar{Z}_2 = 10 + j4 - j0.1$$

$$= 10 + j3.9$$

$$\Rightarrow \bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10+j3.9} = \frac{10-j3.9}{10^2+3.9^2} = \frac{10}{115.21} - j\frac{3.9}{115.21}$$

$$= 0.087 - j0.034$$

$$\therefore \bar{Y}_T = \bar{Y}_1 + \bar{Y}_2$$

$$= 0.06 + j0.08 + 0.087 - j0.034$$

$$= 0.147 + j0.046$$

$$= 0.154 \angle 17.3762^\circ$$

$\Rightarrow$ OR you can try again to get $\bar{Y}_T$ as follows:

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(6-j8)(10+j3.9)}{(6-j8)+(10+j3.9)}$$

and proceed to get $\bar{Z}_T \Rightarrow Y_T$ must be the same value

$$\bar{Y}_T = \frac{1}{\bar{Z}_T} = 0.154 \angle 17.3762^\circ$$

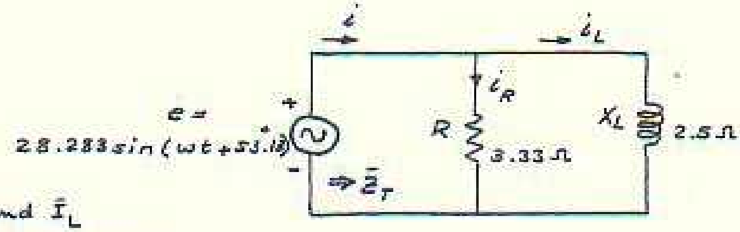
Illustrative Examples on R-L, R-C, and R-L-C Prallel AC Circuits

EE7

* R-L parallel ac circuits

Example:

For the circuit shown;



a. $\bar{Z}_T$

b. Draw the admittance diagram

c. The currents $\bar{I}$, $\bar{I}_R$, and $\bar{I}_L$

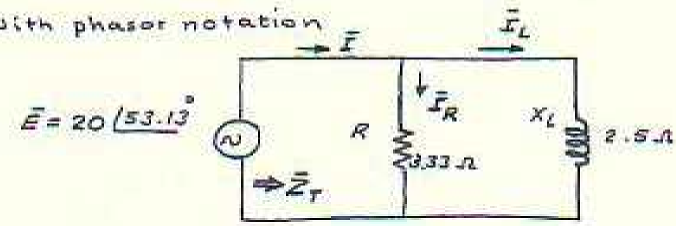
d. Draw the current phasor diagram.

e. Calculate the active, reactive, and complex apparent powers.

f. Determine the power factor.

Solution

Draw the circuit with phasor notation



ⓐ. $\bar{Z}_T$:

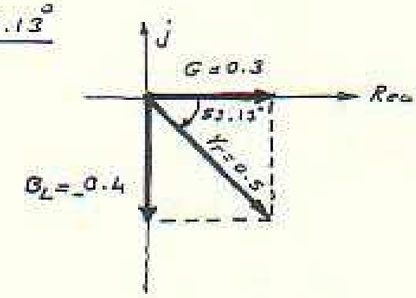
$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = G + B_L = \frac{1}{3.33}\angle 0° + \frac{1}{2.5}\angle -90°$$

$$= 0.3 - j0.4$$

$$= 0.5\angle -53.13°$$

$$\therefore \bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.5\angle -53.13°} = 2\angle 53.13°$$

ⓑ. The admittance diagram $\Rightarrow$



EE7

(c). $\bar{I} = \dfrac{\bar{E}}{\bar{Z}_T} = \dfrac{20\angle 53.13^\circ}{2\angle 53.13^\circ} = 10\angle 0^\circ$

$\bar{I}_R = \dfrac{\bar{E}}{R} = \dfrac{20\angle 53.13^\circ}{3.33\angle 0^\circ} = 6\angle 53.13^\circ$

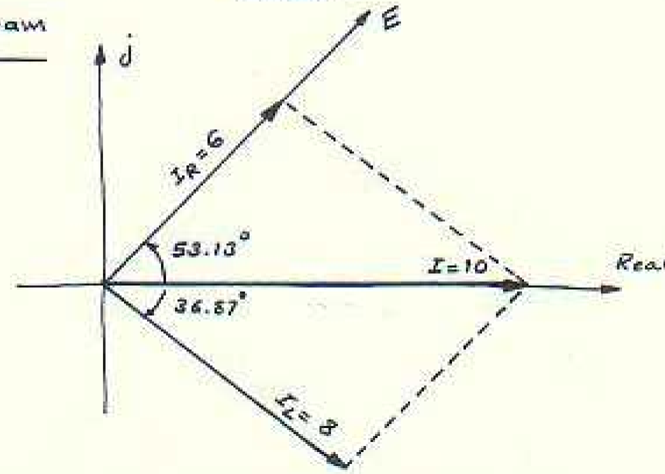
$\bar{I}_L = \dfrac{\bar{E}}{\bar{X}_L} = \dfrac{20\angle 53.13^\circ}{2.5\angle 90^\circ} = 8\angle -36.87^\circ$

* For check

using KCL $\Rightarrow \bar{I} = \bar{I}_R + \bar{I}_L$

$\Rightarrow \bar{I} = 6\angle 53.13^\circ + 8\angle -36.87$
$= (3.6 + j4.8) + (6.4 - j4.8)$
$= 10 + j0$
$= \underline{\underline{10\angle 0^\circ}}$ which is the same as calculated above

(d). The current phasor diagram



(e). Active power $= P = EI\cos\phi = (20)(10)\cos 53.13^\circ$
$= 120\ W$

Reactive power $= Q = EI\sin\phi = (20)(10)\sin 53.13^\circ$
$= 160\ VAR$

$\therefore$ Complex apparent power $= \bar{S} = P + jQ = 120 + j160$

$\therefore \quad \bar{S} = P^2 + Q^2 = 200\ VA$

from the phasor dig.

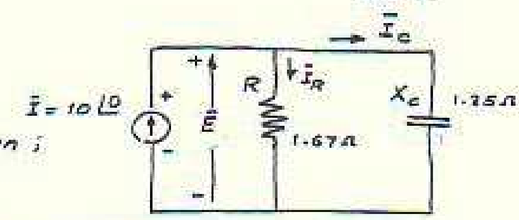
(f). The power factor $P.f = \cos\phi = \cos 53.13^\circ = 0.6$ lagging

or $\cos\phi = \dfrac{P}{EI} = \dfrac{E^2/R}{EI} = \dfrac{EG}{I} = \dfrac{G}{Y_T}$

ملاحظة : يمكن حساب $\bar{Z}_T$ حسب العلاقة $\Leftarrow \dfrac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$ والحصول على النتيجة نفسها .

EE7

* R-C parallel ac circuit



Example: For the circuit shown;

a. Determine $\bar{Z}_T$.

b. Draw the admittance diagram.

c. Calculate $\bar{E}$, I_R, and $\bar{I}_c$, and draw the current phasor diagram.

d. Active, reactive, and apparent powers.

e. Determine the power factor for the circuit.

Solution: ⓐ $\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = \dfrac{1}{R\angle 0} + \dfrac{1}{X_c\angle -90^\circ} = \dfrac{1}{1.67}\angle 0^\circ + \dfrac{1}{1.25}\angle 90$

$= 0.6 + j0.8 = \underline{1\angle 53.13^\circ}$

Also: $\bar{Z}_T = \dfrac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$

$\Rightarrow \bar{Z}_T = \dfrac{1}{Y_T} = \dfrac{1}{1\angle 53.13^\circ} = 1\angle -53.13^\circ$

ⓑ.

$\bar{E} = \dfrac{\bar{I}}{Y_T} = \dfrac{10\angle 0^\circ}{1\angle 53.13^\circ}$ or $\bar{E} = \bar{I}\bar{Z}_T$

$= 10\angle -53.13^\circ$

$\bar{I}_R = \bar{E}\bar{G}$ or $\bar{I}_R = \dfrac{\bar{E}}{R}$

$= (10\angle -53.13^\circ)(0.6)$

$= 6\angle -53.13^\circ$

$I_c = \bar{E}\bar{B}_c$ or $\bar{I}_c = \dfrac{\bar{E}}{\bar{X}_c}$

$= (10\angle -53.13^\circ)(0.8\angle 90)$

$= 8\angle 36.87^\circ$

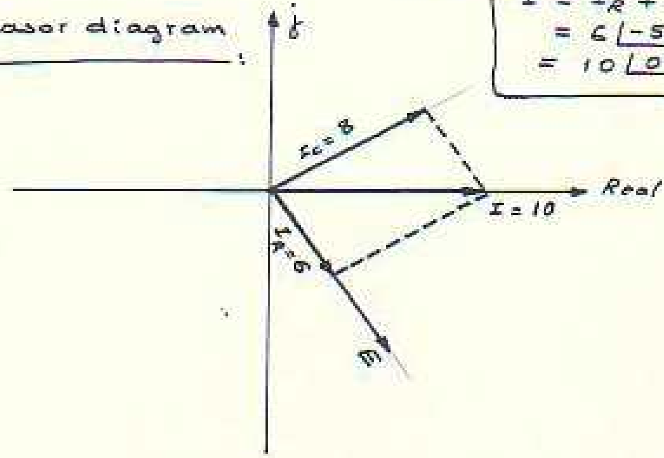
$\Rightarrow$ As a check?

$\bar{I} = \bar{I}_R + \bar{I}_c$

$= 6\angle -53.13 + 8\angle 36.87^\circ$

$= 10\angle 0^\circ$

ⓒ. The phasor diagram:



(d). Active power $= P = EI\cos\phi$

$= (10)(10)\cos 53.13^\circ$

$= 60\ W$

$*$ or $P = E^2G$

$= (10)^2(0.6)$

$= 60\ W$

Reactive power $= Q = EI\sin\phi$

$= (10)(10)\sin 53.13^\circ$

$= 80\ VAR$

$\therefore$ Apparent power $S = \sqrt{P^2 + Q^2} = \sqrt{(60)^2 + (80)^2}$

$= 100\ VA$

(e). The power factor

$p.f = \cos 53.13$

$= 0.6$ <u>leading</u> $\Leftarrow$ from the phasor diagram $\bar{I}$ leads $\bar{E}$

$*$ R-L-C parallel ac circuit:

Example: For the circuit shown;

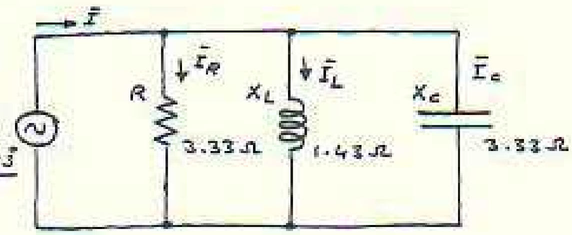
a. Determine $\bar{Z}_T$.

b. Calculate $\bar{I}$, $\bar{I}_R$, $\bar{I}_L$ & $\bar{I}_c$

c. Draw the phasor diag. $\bar{E} = 100\angle 53.13^\circ$

d. Calculate the active (real) power

e. Determine the power factor



Solution: (a). $\bar{Z}_T = ?$ $\Rightarrow$

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

$$= \frac{1}{R}\angle 0^\circ + \frac{1}{X_L}\angle -90^\circ + \frac{1}{X_c}\angle 90^\circ$$

$$= \frac{1}{3.33}\angle 0^\circ + \frac{1}{1.43}\angle -90^\circ + \frac{1}{3.33}\angle 90^\circ$$

$$= 0.3 - j0.7 + j0.3$$

$$= 0.3 - j0.4$$

$$= 0.5\angle -53.13^\circ$$

$$\therefore \bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.5\angle -53.13^\circ} = 2\angle 53.13^\circ$$

(b). $$\bar{I} = \frac{\bar{E}}{\bar{Z}_T} = \frac{100\angle 53.13^\circ}{2\angle 53.13^\circ} = 50\angle 0^\circ$$

$\bar{I}_R = \dfrac{\bar{E}}{R} = \checkmark$ يطلع نفسه $\Leftarrow$

$$I_R = \bar{E}\bar{G} = (100\angle 53.13^\circ)(0.3\angle 0^\circ)$$

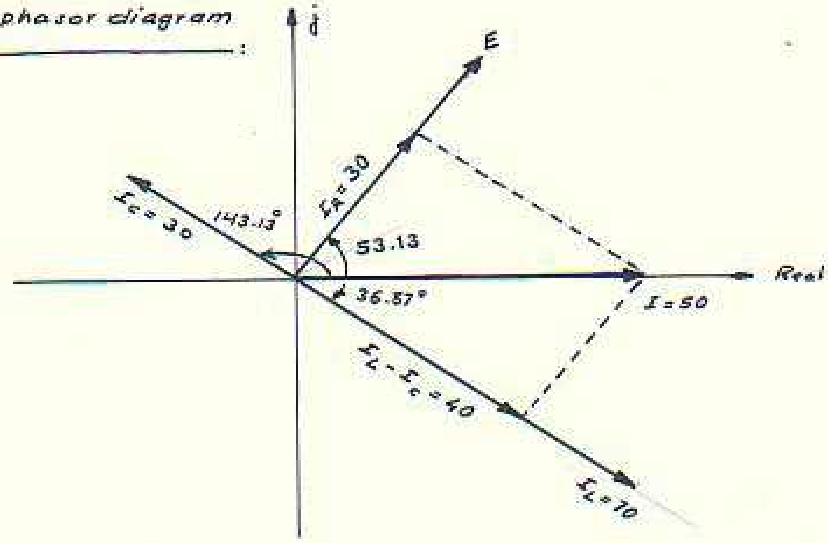
$$= 30\angle 53.13^\circ$$

$$\bar{I}_L = \bar{E}\bar{B}_L = (100\angle 53.13^\circ)(0.7\angle -90) = 70\angle -36.87^\circ \qquad \Rightarrow \bar{I}_L = \frac{\bar{E}}{\bar{X}_L} \quad \boxed{EE?}$$

$$I_c = \bar{E}\bar{B}_c = (100\angle 53.13^\circ)(0.3\angle 90) = 30\angle 143.13^\circ$$

$\boxed{\text{Prove that : } \bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_c}$

ⓒ. The current phasor diagram :



ⓓ. Active power (Real power) $= P = EI\cos\phi$

$$= (100)(50)\cos 53.13^\circ$$
$$= 3000 \text{ W}$$
$$= 3.0 \text{ kW}$$

$$\Rightarrow \text{or} \quad P = E^2 G = (100^2)(0.3) = 3.0 \text{ kW}$$

ⓔ. The power factor $\Rightarrow$ $p.f = \cos\phi$

$$= \cos 53.13$$
$$= 0.6 \text{ lagging}$$

⇐ from the phasor diagram.

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}$$

⇐ ملاحظة : من الممكن كذلك حساب Z_T والحصول على النتيجة نفسها .