3. Techniques of Circuit Analysis

EE3

3.1 Determinants

: Consider the two simultaneous equations

$$a_1 x + b_1 y = C_1$$

and

$$a_2 x + b_2 y = C_2$$

where x and y are the unknown variables, and a_1, a_2, b_1, b_2, C_1 and C_2 are constants.

Using the determinants, the following formats are obtained for each of the variables; x and y:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where; $\Delta_1 = c_1 b_2 - c_2 b_1$

$\Delta = a_1 b_2 - a_2 b_1$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where; $\Delta_2 = a_1 c_2 - a_2 c$

Δ, Δ_1 and Δ_2 are called second order determinants, since it contains two rows and two columns.

Third order determinants are used to solve three simultaneous linear equations. Consider, the following three simultaneous equations:

$$a_1 x + b_1 y + C_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + C_3 z = d_3$$

The unkown variables, x, y, and z are determined as follows:

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$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

The third order determinant can be evaluated as:

4(-) 5(-) 6(-)

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \Longrightarrow \left| \begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array} \right.$$

1(+) 2(+) 3(+)

$$\Longrightarrow \Delta = \overset{1(+)}{a_1 b_2 c_3} + \overset{2(+)}{b_1 c_2 a_3} + \overset{3(+)}{c_1 a_2 b_3} - \overset{4(-)}{a_3 b_2 c_1} - \overset{5(-)}{b_3 c_2 a_1} \overset{6(-)}{- c_3 a_2 b}$$

$$\therefore \ \Delta = a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - a_3 b_2 c_1 - b_3 c_2 a_1 - c_3 a_2 b_1$$

<u>Example</u>: Find x and y:

$$\begin{aligned} -x + 2y &= 3 \\ 3x - 2y &= -2 \end{aligned} \longrightarrow x = \frac{\begin{vmatrix} 3 & 2 \\ -2 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{(3)(-2)-(-2)(2)}{(-1)(-2)-(3)(2)} = \frac{-2}{-4} = \frac{1}{2}$$

and $$y = \frac{\begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{(-1)(-2)-(3)(3)}{-4} = \frac{-7}{-4} = \frac{7}{4}$$

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Example: Find x, y and z, for the following simultaneous equation

$$x - 2z = -1$$
$$3y + z = 2$$
$$x + 2y + 3z = 0$$

Solution: Arrange the equations to be as:

$$1x + 0y - 2z = -1$$
$$0x + 3y + 1z = 2$$
$$1x + 2y + 3z = 0$$

then

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -1 & 0 & -2 \\ 2 & 3 & 1 \\ 0 & 2 & 3 \end{vmatrix} \begin{matrix} -1 & 0 \\ 2 & 3 \\ 0 & 2 \end{matrix}}{\begin{vmatrix} 1 & 0 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 3 \\ 1 & 2 \end{matrix}}$$

$$\therefore x = \frac{(-1)(3)(3) + (0)(1)(0) + (-2)(2)(2) - \left[(0)(3)(-2) + (2)(1)(-1) + (3)(2)(0)\right]}{(1)(3)(3) + (0)(1)(1) + (-2)(0)(2) - \left[(1)(3)(-2) + (2)(1)(1) + (3)(0)(0)\right]}$$

$$= \frac{-15}{13} = -\frac{15}{13}$$

and

$$y = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix}}{\Delta} = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix} \begin{matrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{matrix}}{13}$$

$$= \frac{(1)(2)(3) + (-1)(1)(1) + (-2)(0)(0) - \left[(1)(2)(-2) + (0)(1)(1) + (3)(0)(-1)\right]}{13}$$

$$\therefore y = \frac{5+4}{13} = \frac{9}{13}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 1 & 0 & -1 \\ 0 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 3 \\ 1 & 2 \end{matrix}}{13}$$

$$\therefore z = \frac{(1)(3)(0) + (0)(2)(1) + (-1)(0)(2) - \left[(1)(3)(-1) + (2)(2)(1) + (0)(0)(0)\right]}{13}$$

$$\Rightarrow z = \frac{0-1}{13} = -\frac{1}{13}$$

3.2 Loop (Mesh) Current Method

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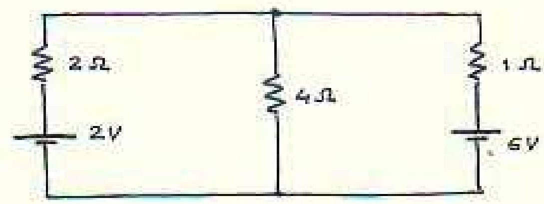
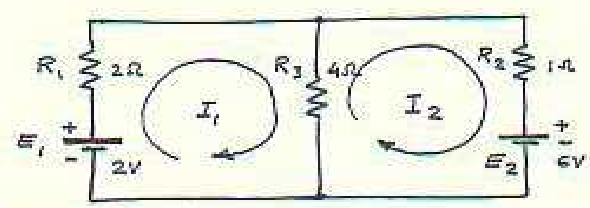
Consider the circuit shown below:



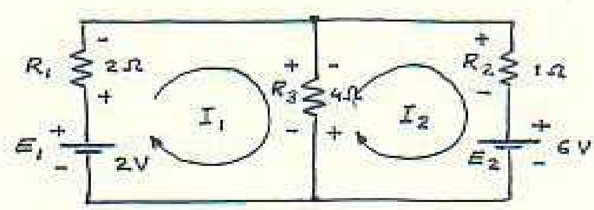
To analyze this circuit using the loop (mesh) method, the following steps must be followed.

STEP 1: Assign a distinct current in the clockwise direction to each independent loop of the network



Note: there are only two independent loops.

STEP 2: Indicate the polarities <u>within</u> each loop for <u>each resistor</u> as determined by the assumed direction of loop current for that loop.



STEP 3: Apply (KVL) around each closed loop in the clockwise direction.

<u>for loop 1</u> $\Rightarrow E_1 - V_1 - V_3 = 0 \Rightarrow E_1 - I_1R_1 - (I_1 - I_2)R_3 = 0$

$$2 - 2I_1 - 4(I_1 - I_2) = 0$$

<u>for loop 2</u> $\Rightarrow -E_2 - V_3 - V_2 = 0 \Rightarrow -E_2 - R_3(I_2 - I_1) - I_2R_2 = 0$

$$-6 - 4(I_2 - I_1) - 1I_2 = 0$$

Notes : * If a resistor has two or assumed currents through it, the total current must be taken into account.

* The polarity of the voltage source is unaffected by the loop currents passing through it.

STEP 4 : Solve the resulting simultaneous equations for the assumed loop currents.

⇒ The equations for loop 1 and loop 2 are rewritten to be as:

Loop 1 $$2 = 6 I_1 - 4 I_2$$

Loop 2 $$-6 = -4 I_1 + 5 I_2$$

Solving by determinants, then :

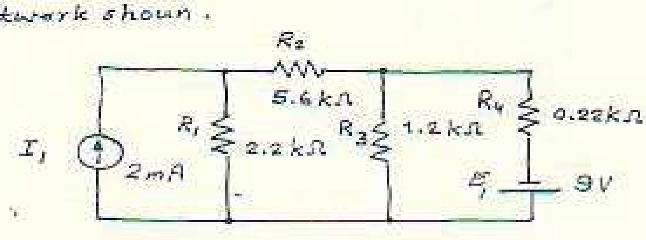
$$I_1 = \frac{\begin{vmatrix} 2 & -4 \\ -6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}} = \frac{(2)(5) - (6)(4)}{(6)(5) - (4)(4)} = \frac{-14}{+14} = \underline{\underline{-1\ A}}$$

and

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ -4 & -6 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}} = \frac{(-6)(6) - (4)(2)}{+14} = \frac{-28}{+14} = \underline{\underline{-2\ A}}$$

$$\therefore I_{4\Omega} = I_1 - I_2$$
$$= -1 - (-2)$$
$$= 1\ A \quad \text{in the direction of } I_1$$

Example : Using the mesh analysis, determine the current through the 9V battery for the network shown.



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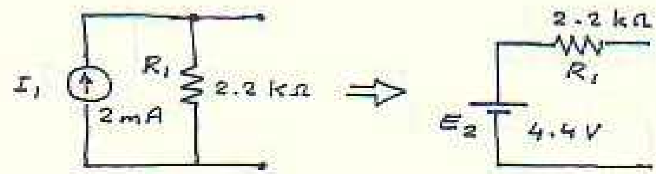
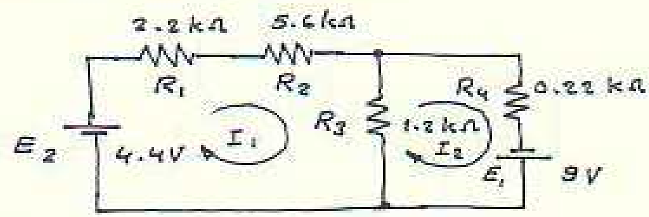
Solution:

* First, the current source has to be converted to a voltage source as shown:



$$E_2 = I_1 R = 2\times10^{-3} \times 2.2\times10^{3} = 4.4\ V$$

* The original circuit will be as shown;



∴ For loop 1, we have:

$$E_2 - I_1(R_1 + R_2 + R_3) + I_2 R_3 = 0$$

$$4.4 - I_1(2.2\times10^3 + 5.6\times10^3 + 1.2\times10^3) - 1.2\times10^3 I_2 = 0$$

$$\Rightarrow 9\times10^3 I_1 - 1.2\times10^3 I_2 = 4.4$$

for loop 2, we have:

$$E_1 - I_2(R_3 + R_4) + I_1 R_3 = 0$$

$$\Rightarrow -1.2\times10^3 I_1 + 1.42\times10^3 I_2 = 9$$

Solving for I_2

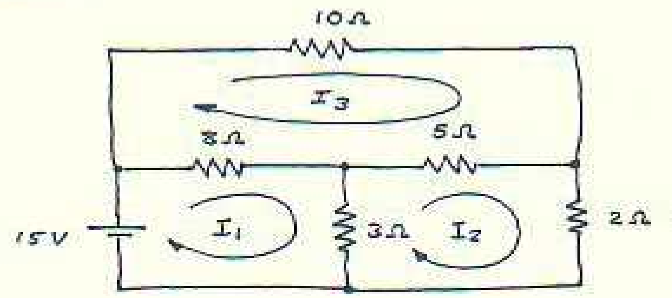
$$\therefore I_2 = \frac{\begin{vmatrix} 9\times10^3 & 4.4 \\ -1.2\times10^3 & 9 \end{vmatrix}}{\begin{vmatrix} 9\times10^3 & -1.2\times10^3 \\ -1.2\times10^3 & 1.42\times10^3 \end{vmatrix}}$$

$$= \frac{86.28}{11.34\times10^3} = 7.608\times10^{-3}$$

$$\therefore I_2 = \underline{7.608\ mA}$$

$\boxed{EE3}$

Example: Find the current through the $10\,\Omega$ resistor of the network shown.



Solution: The loop equations are:

Loop 1:

$$(8+3)I_1 - 3I_2 - 8I_3 = 15$$

Loop 2:

$$(3+5+2)I_2 - 3I_1 - 5I_3 = 0$$

Loop 3:

$$(5+8+10)I_3 - 8I_2 - 5I_2 = 0$$

Rearrange, then:

$$\begin{aligned} 11I_1 - 3I_2 - 8I_3 &= 15 \\ -3I_1 + 10I_2 - 5I_3 &= 0 \\ -8I_1 - 5I_2 + 23I_3 &= 0 \end{aligned}$$

$$\therefore\ I_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \underline{1.22\ A}$$

$$\therefore\ I_3 = I_{10\,\Omega} = 1.22\ A$$

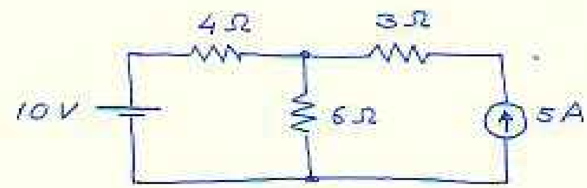
Special Cases — Mesh Analysis with Current Sources

CASE 1:

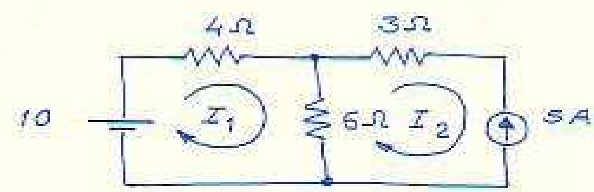
When a current source exists only in one mesh. This will simplify the analysis.

Example:

Calculate the circuit currents in the circuit shown, using mesh analysis.



Solution:



I_2 is known

$$\underline{I_2 = -5A}$$

⇒ We need ONE equation

(KVL) mesh ① ⇒ $10 = I_1(4+6) - I_2(6) \Rightarrow 10 = 10I_1 + 30$

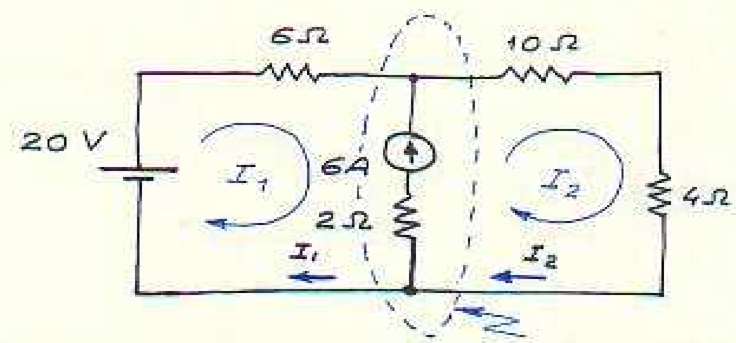
$$\Rightarrow I_1 = \frac{10-30}{10} = \underline{-2\ A}$$

CASE 2:

When a current source exists between TWO meshes. In this case a supermesh is created.

Example:

Calculate all branch currents in the circuit shown, using mesh current method.



Solution:

In this circuit, we have a supermesh

* To solve for the branch currents, KVL and KCL must be applied.

Applying KCL ;

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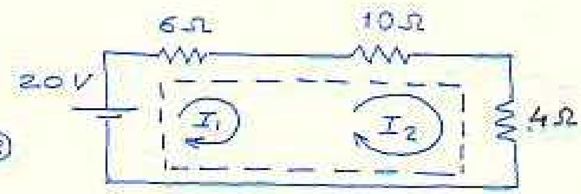
$$I_2 = I_1 + 6 \quad \cdots (1)$$

To apply KVL to the supermesh ; see the equivalent circuit below

$$\Rightarrow 20 = 6I_1 + (10+4)I_2$$

$$6I_1 + 14I_2 = 20 \quad \cdots (2)$$



applying KVL on supermesh

From Equ(1) and Equ(2)

$$I_1 = -3.2\ A$$

and

$$I_2 = 2.8\ A$$

Summary

: The supermesh has the following properties :

1. The current source in the supermesh is not completely ignored.
2. It provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KCL & KVL.

3.3 Nodal Voltage Method

EE3

: To analyze a given circuit using this method, the following steps have to be followed.

STEP1
: Convert all voltage sources to current sources.

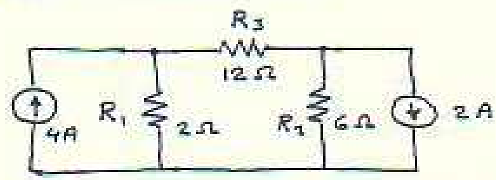
STEP2
: Determin the number of nodes within the network.

STEP3
: Pick a reference node and label the remaining nodes as V_1, V_2 and so on.

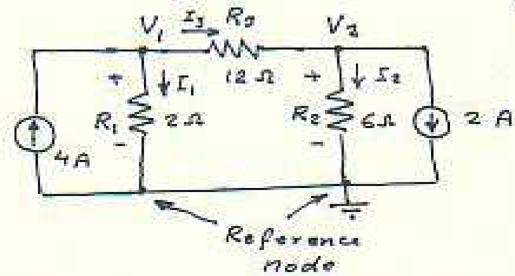
STEP4
: Apply (KCL) at each node except the reference.

STEP5
: Solve the resulting equations for the nodal voltages.

Example
: Using the nodal voltage method, determine the currents in R_1, R_2 and R_3, in the circuit shown.



Solution
: There are 3 nodes $\Rightarrow$ V_1, V_2 and the reference node (ground)



For node 1 :

$$4 - I_1 - I_3 = 0 \quad \Rightarrow \quad 4 - \frac{V_1}{R_1} - \frac{V_1 - V_2}{R_3} = 0$$

For node 2 :

$$I_3 - I_2 - 2 = 0 \quad \Rightarrow \quad \frac{V_1 - V_2}{R_3} - \frac{V_2}{R_2} - 2 = 0$$

EE3

Rearrange, then:

$$V_1\left(\frac{1}{2}+\frac{1}{12}\right)-V_2\left(\frac{1}{12}\right)=+4$$

$$V_2\left(\frac{1}{12}+\frac{1}{6}\right)-V_1\left(\frac{1}{12}\right)=-2$$

which produce:

$$\frac{7}{12}V_1-\frac{1}{12}V_2=4 \Rightarrow 7V_1-V_2=48$$

and;

$$-\frac{1}{12}V_1+\frac{3}{12}V_2=-2 \Rightarrow -1V_1+3V_2=-24$$

$$\therefore\ V_1=\frac{\begin{vmatrix}48 & -1\\ -24 & 3\end{vmatrix}}{\begin{vmatrix}7 & -1\\ -1 & 3\end{vmatrix}}=\frac{120}{20}=6V$$

$$V_2=\frac{\begin{vmatrix}7 & 48\\ -1 & -24\end{vmatrix}}{\begin{vmatrix}7 & -1\\ -1 & 3\end{vmatrix}}=\frac{-120}{20}=-6V$$

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Since V_1 is greater than V_2, the assumed direction of I_3 is correct.

$$\therefore\ I_3=\frac{V_1-V_2}{R_3}=\frac{6-(-6)}{12}=\frac{12}{12}=1\ A$$

$$I_1=\frac{V_1}{2}=\frac{6}{2}=3\ A$$

and

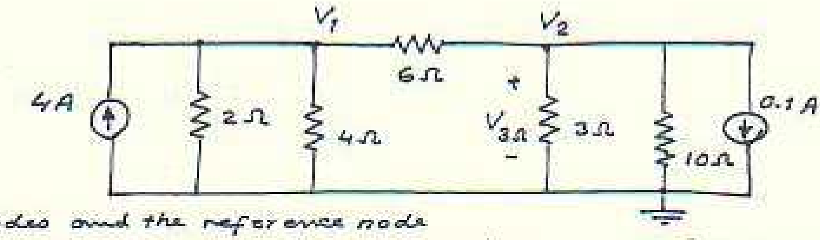
$$I_2=\frac{V_2}{R_2}=\frac{-6}{6}=\underline{-1\ A}$$

The (-ve) sign means that the current in the cct has the opposite direction.

EE3

Example: Find the voltage across the 3Ω resistor of the network shown;

Solution: Converting sources and choosing nodes as shown;



* We have two nodes and the reference node
⇒ Two nodal voltages are required for V_1 and V_2

For node 1

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_1 - \left(\frac{1}{6}\right)V_2 = 4$$

$$\Rightarrow \frac{11}{12}V_1 - \frac{1}{6}V_2 = 4 \Rightarrow \boxed{11V_1 - 2V_2 = 48}$$

For node 2

$$\left(\frac{1}{10}+\frac{1}{3}+\frac{1}{6}\right)V_2 - \left(\frac{1}{6}\right)V_1 = -0.1$$

$$\Rightarrow -\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1 \Rightarrow \boxed{-5V_1 + 18V_2 = -3}$$

$$\therefore V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33+240}{198-10} = \frac{207}{188}$$

$$\therefore V_{3\Omega} = 1.101 \text{ V}$$

Special Cases : Nodal Analysis with Voltage Sources

CASE 1 :

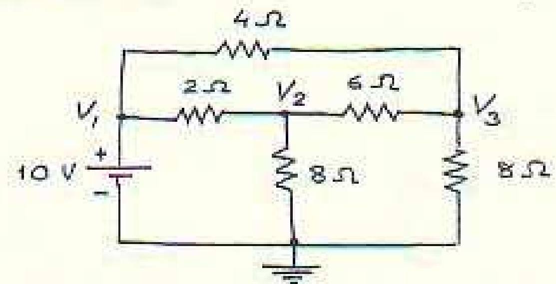
When a voltage source is connected between the referencenode and a non-reference node.

As shown in the Figure below

In this circuit, we have 3 node voltages:

V_1 , V_2 , and V_3

but $V_1 = 10\ V$



Thus, our analysis is somewhat amplified by this knowledge of the voltage at this node. We, now, need 2 nodal equations to analyse the circuit.

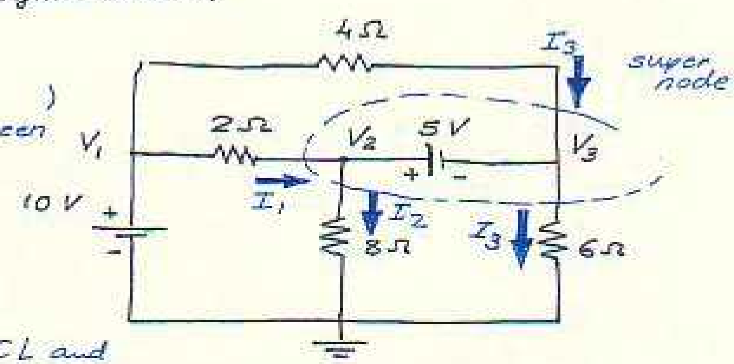
CASE 2 (SUPERNODE) :

When the voltage source is connected between two non-reference nodes. The two non-reference nodes form a supernode.

· As shown in the figure below:

The voltage source ( ) has been connected between the nodes V_2 and V_3.

[V_2 and V_3] is called a supernode.



⊛ To solve the circuit, KCL and KVL must be applied at the supernode:

Applying KVL at the supernode results in:

Applying KCL at the supernode results in:

$I_1 + I_4 = I_2 + I_3$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2 - 0}{8} + \frac{V_3 - 0}{6}$$

$V_1 = 10$ Volts

$$\Rightarrow \underbrace{\frac{10 - V_2}{2} + \frac{10 - V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6}}_{\Downarrow}$$

$$36 = 3V_2 + 2V_3$$

We have

$$5 = V_2 - V_3$$

Solving for V_2 and V_3 $\Rightarrow$ $\underline{V_2 = 9.2\ V}$ and

$\underline{V_3 = 4.2\ V}$

Summary: The supernode has the following properties:

1. The voltage source inside the supernode provides a constraint equation needed to solve for node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KVL & KCL.

4. Circuit Theorems

EE4

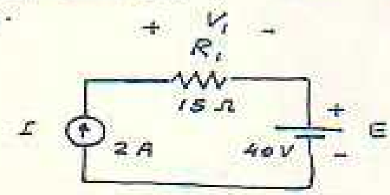
3.1 Superposition Theorem

: The theorem states that: "the current through (or the voltage across) an element in a linear bilateral network is equal to the algebraic sum of the currents (or voltages) produced independently by each source.

* To apply this theorem to find the current (or voltage) in a certain part of a network, remove the sources of the network and find the current (or voltage) in the existance of only one source each time. The resultant current (or voltage) will be the algebraic sum of currents (or voltages) due to all sources when acting independently once a time.

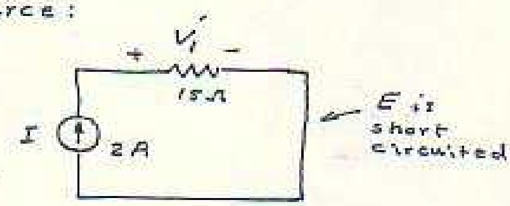
* Removing the sources means: SHORT CIRCUITING the voltage source and OPEN CIRCUITING the current source.

Example: Using the superposition theorem, determine V_1 for the network shown.

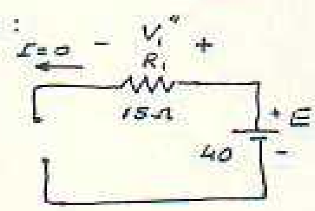


Solution: * Due to the current source:

$$V_1' = I R_1$$
$$= (2)(15)$$
$$= 30 \text{ V}$$



* Due to the voltage source:

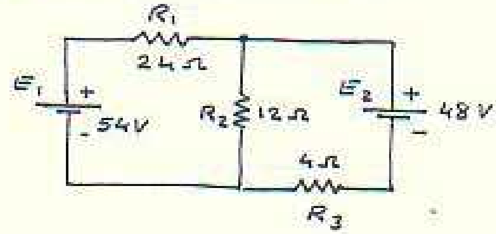


$$V_1'' = I_1 R_1$$
$$= (0)(15)$$
$$= 0 \text{ V}$$

$$\therefore V_1 = V_1' + V_1''$$
$$= 30 - 0 = 30 \text{ V}$$

EE4

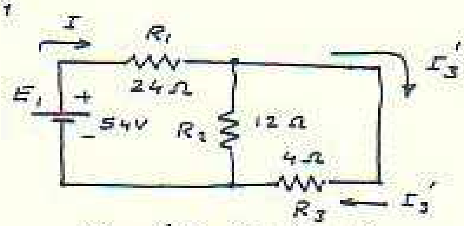
Example: Using the superposition theorem, determine the current through the 4-Ω resistor for the network shown.



Solution: Consider the effect of E_1

$$I = \frac{E_1}{R_T} = \frac{54}{27} = 2\,A$$

Using the current division rule ∴

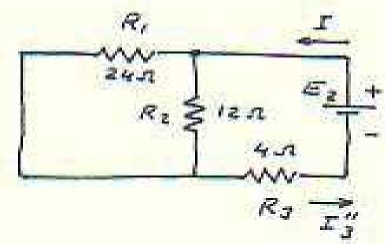


$$\therefore\ I_3' = I\,\frac{R_2}{R_2+R_3}$$
$$= 2\,\frac{12}{12+4} = \underline{1.5\,A}$$

$$R_T = (R_2 /\!/ R_3) + R_1$$
$$(12 /\!/ 4) + 24 = 3 + 24$$
$$= 27\,\Omega$$

* Consider the effect of E_2:

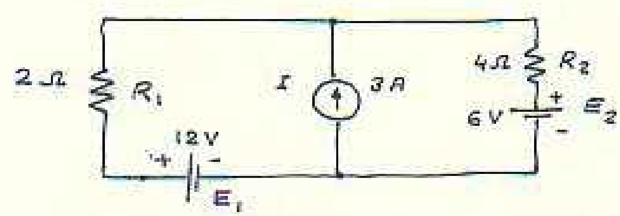
$$I = I_3'' = \frac{E_2}{R_T}$$
$$R_T = (24/\!/12) + 4$$
$$= 8 + 4$$
$$= 12\,\Omega$$



$$\therefore\ I_3'' = \frac{48}{12} = \underline{4\,A}$$

$$\therefore\ I_3 = I_3'' - I_3'$$
$$= 4 - 1.5 = \underline{2.5\,A} \quad (\text{in the direction of } I_3'').$$

Example: Using the superposition theorem, find the current through the 2-Ω resistor of the network shown.

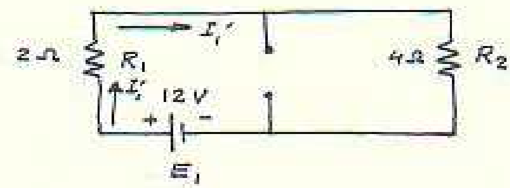


EE4

Solution:

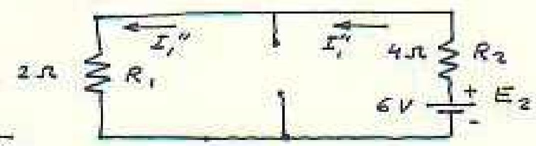
* The effect of E_1

Remove the voltage source E_2 (short circuited) and the current source I (open circuited); the network will be as shown:



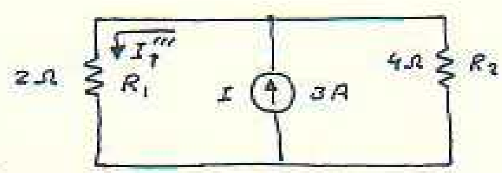
$$\therefore\ I_1' = \frac{E_1}{R_T} = \frac{12}{2+4} = 6\,A$$

* The effect of E_2: removing E_1 & I, the network will be as shown:



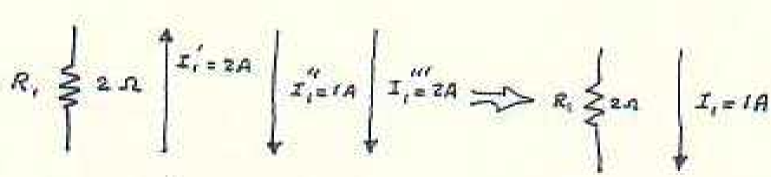
$$\therefore\ I_1'' = \frac{E_2}{R_T} = \frac{6}{2+4} = 1\ A$$

* The effect of I: removing E_1 and E_2, the network will be as shown:



$$\therefore\ I_1''' = I\,\frac{R_2}{R_1+R_2} = (3)\,\frac{4}{4+2} = 2\,A$$

$$\therefore\ I_1 = \underbrace{I_1'' + I_1'''}_{\text{same direction}} - \underbrace{I_1'}_{\text{opposite direction}} \Rightarrow I_1 = 1+2-1 = \underline{1\,A}$$



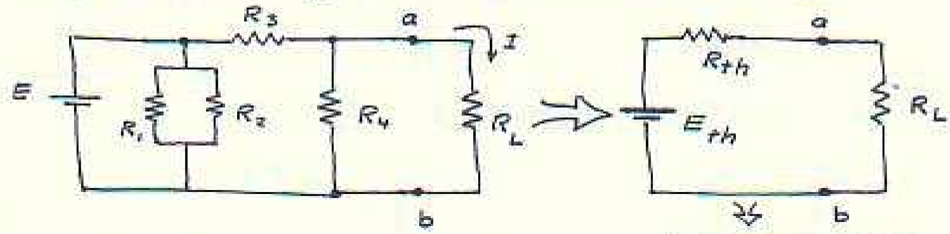
Resulting current in R_2

3.2 Thevenin's Theorem

EE4

Thevenin's Theorem : Thevenins theorem states that " Any two-terminal linear biletaral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

Consider the network shown, it can be replaced by the voltage source E_{th} and the series resistor R_{th} :



* To find I through the resistance R_L $\Rightarrow$ $\boxed{I = \dfrac{E_{th}}{R_{th} + R_L}}$

* Steps to find E_{th} and R_{th} :

STEP 1 : Remove that portion of the network across which the Thevenins equivalent circuit is to be found.

STEP 2 : Mark the terminals of the remaining two-terminal network.

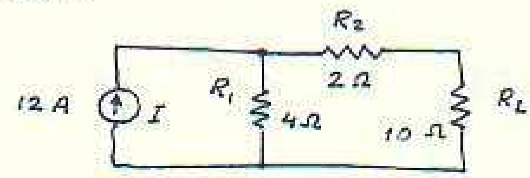
STEP 3 (R_{th}) : Calculate R_{th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuits), and finding the resultant resistance between the two marked terminals.

STEP 4 (E_{th}) : Calculate E_{th} by first returning all sources to their original positions and finding the open circuit voltage between the marked terminals

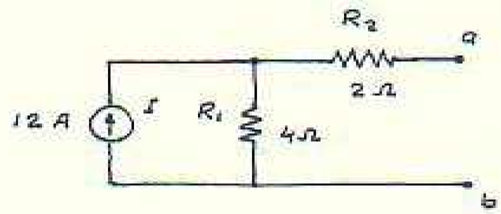
STEP 5 : Draw the Thevenins equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

EE4

Example: Using Thevenins theorem, find the current in the $R_L = 10\,\Omega$ of the network shown.

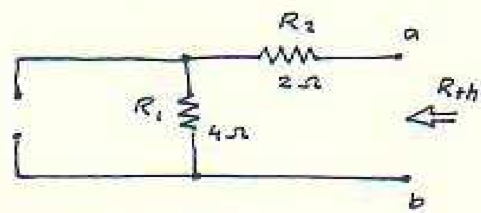


Solution: steps 1 and 2:



step 3: $R_{th} = ?$

Remove the current source I, then calculate R_{th} between the terminals a and b;



$$\therefore R_{th} = R_1 + R_2 = 4 + 2 = 6\,\Omega$$

step 4: $E_{th} = ?$

Return the current source to its original position then determine E_{th} across the open circuit terminals a and b.

$I_2 = 0$

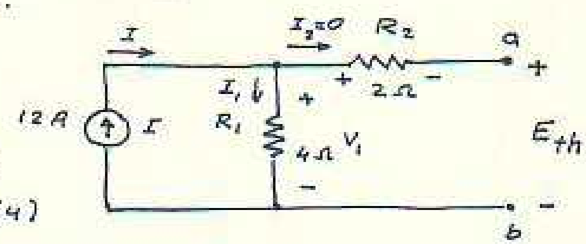
$\Rightarrow I_2 R_2 = 0$

Then

$$E_{th} = I_1 R_1 - I_2 R_2$$

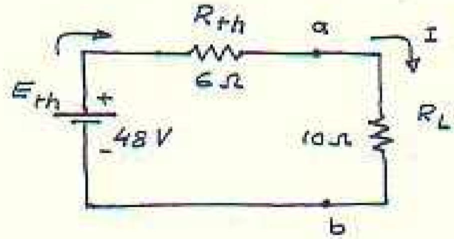
$$= I_1 R_1 = 12(4)$$

$$= 48\ V$$

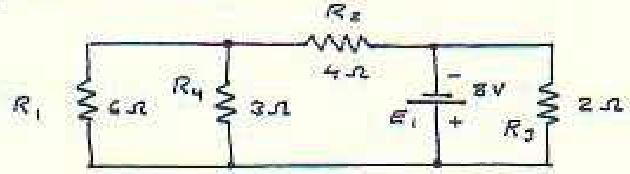


step 5: Draw the Thevenins equivalent circuit representing the network between points a and b with R_L added.

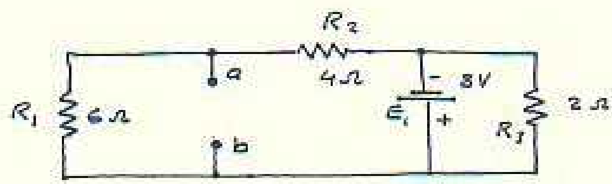
$$\therefore I = \frac{E_{th}}{R_{th} + R_L} = \frac{48}{6 + 10} = 3\ A$$



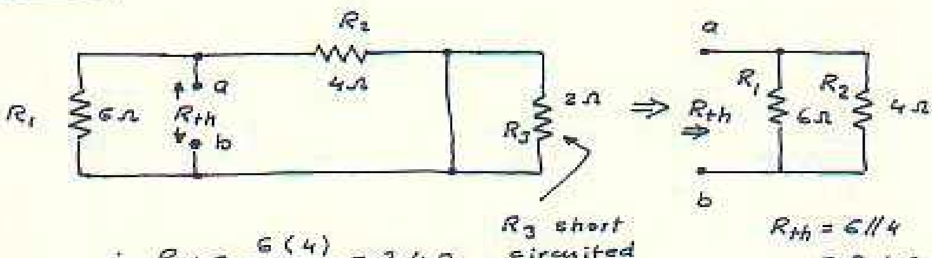
Example: For the circuit shown, find the current in the 3-Ω resistor using Thevenin's theorem.



Solution: steps 1 and 2



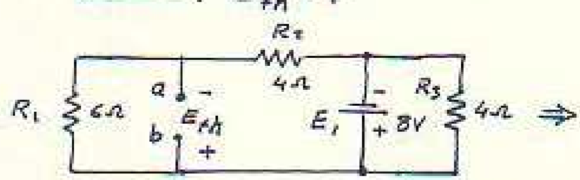
step 3: $R_{th} = ?$



$$\therefore R_{th} = \frac{6(4)}{6 + 4} = 2.4\,\Omega$$

$R_{th} = 6 /\!/ 4$
$= 2.4\,\Omega$

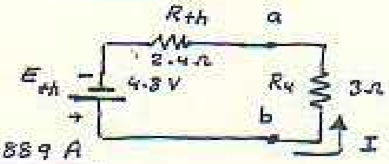
step 4: $E_{th} = ?$



using the voltage divider rule,

$$E_{th} = E_1 \frac{R_1}{R_1 + R_2} = 8\,\frac{6}{6 + 4} = 4.8\ V$$

step 5

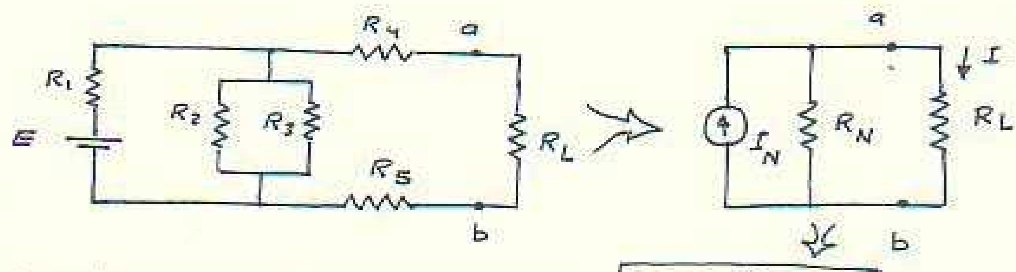


$$\therefore I = \frac{E_{th}}{R_{th} + R_4} = \frac{4.8}{2.4 + 3} = \underline{0.889\ A}$$

3.3. Norton's Theorem

: Norton's theorem states that " Any two terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor."

Consider the network shown ; it can be replaced by the current source I_N and the parallel resistor R_N ;



To find the current through $R_L \Rightarrow$ $\boxed{I = \dfrac{I_N R_N}{R_N + R_L}}$

How to find I_N and R_N

STEP 1

: Remove that portion of the network across which the Norton equivalent circuit is found.

STEP 2

: Mark the terminals of the remaining two-terminal network.

STEP 3 (R_N)

: Calculate R_N by first removing all the sources (voltage sources replaced by short circuits and current sources replaced by open circuits) and then finding the resultant resistance between the two marked terminals.

STEP 4 (I_N)

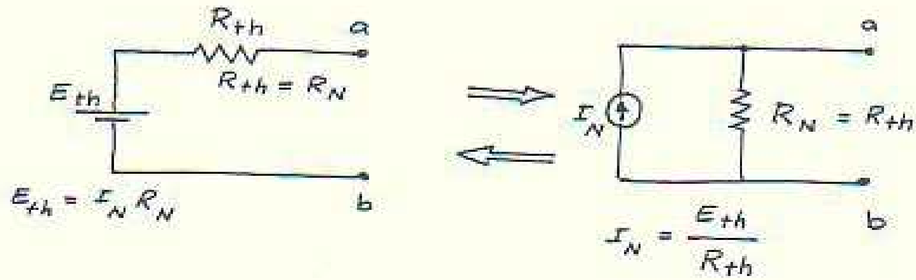
: Calculate I_N by first returning all sources to their original position and then finding the short circuit current between the marked terminals

STEP 5

: Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

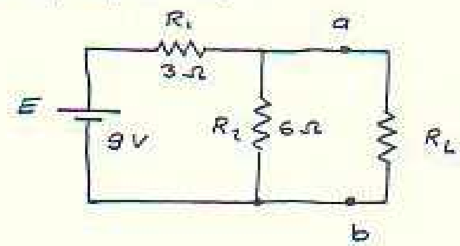
Relation between Norton equivalent circuit and Thevenin's equivalent circuit

: The Norton and Thevenin equivalent circuits can also be found from each other by using the source transformation previously discussed, as shown;



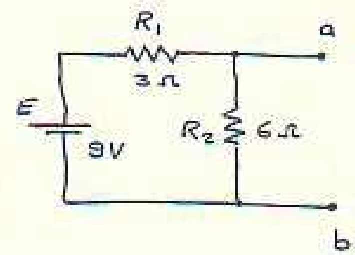
Example

: For the circuit shown, find the Norton equivalent circuit for the network to the left of (a–b).



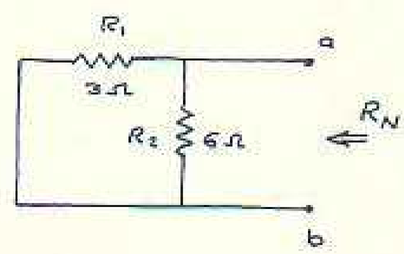
Solution

: steps 1 and 2



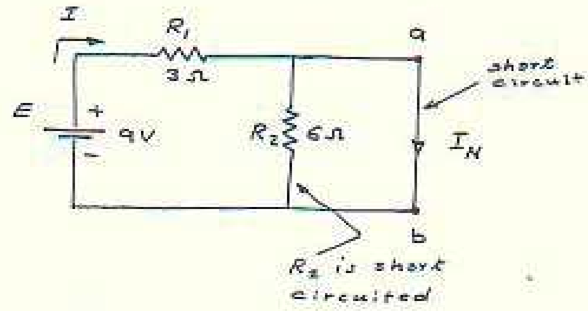
step 3 : $R_N = ?$

$$R_N = R_1 \,//\, R_2 = \frac{3(6)}{3+6} = 2\,\Omega$$

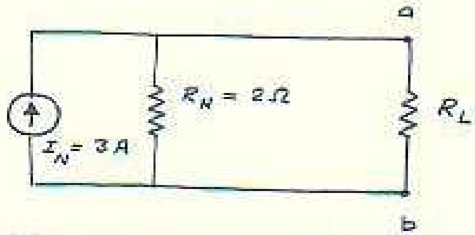


STEP 4 $I_N = ?$

$$I_N = I = \frac{E}{R_1} = \frac{9}{3}$$
$$= 3\ A$$

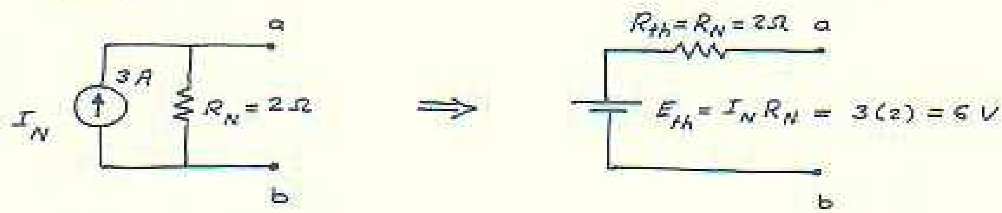


∴ step 5 :

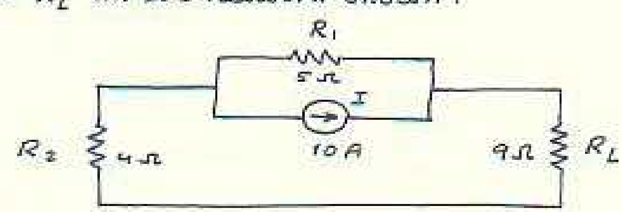


which is the Norton equivalent circuit of the network.

Note : Thevenin's theorem can be determined by Norton's theorem as shown :

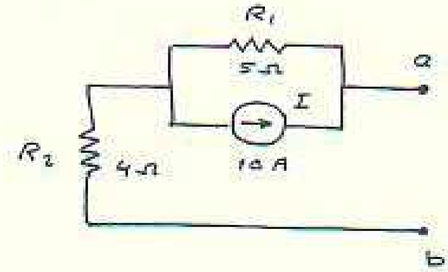


Example : Using Norton theorem find the current through the load resistor R_L in the network shown.

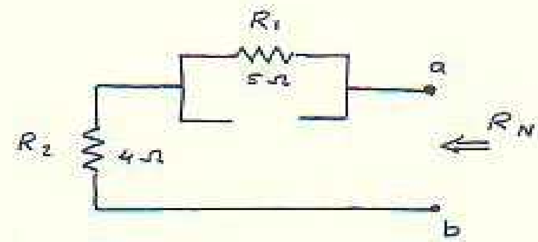


Solution:

step1 and 2

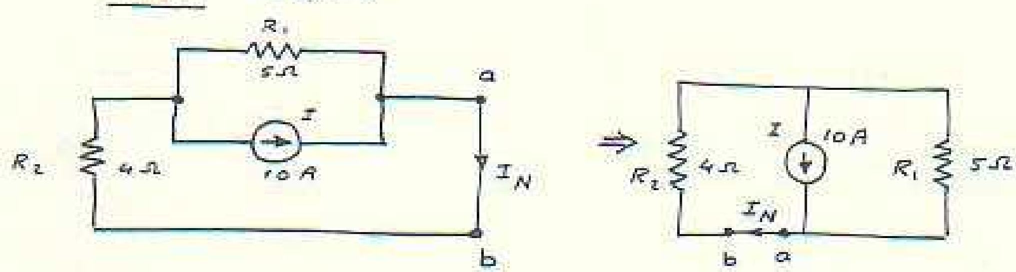


step 3 : $R_N = ?$



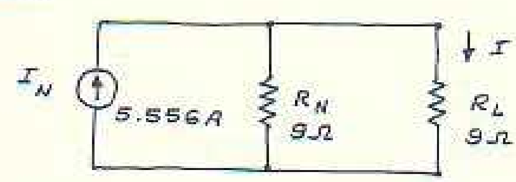
$$
\begin{aligned}
R_N &= R_1 + R_2 \\
&= 5 + 4 \\
&= 9\ \Omega
\end{aligned}
$$

step 4 $I_N = ?$



$$
\begin{aligned}
\therefore\ I_N &= I \cdot \frac{R_1}{R_1 + R_2} \\
&= 10\,\frac{5}{5+4} \\
&= 5.556\ A
\end{aligned}
$$

step 5

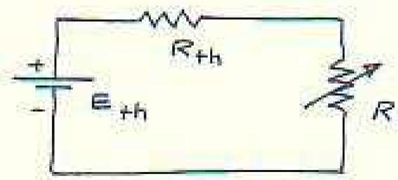


$$\therefore\ I = \frac{I_N}{2} = 2.778\ A$$

3.4 Maximum Power Transfer Theorem

: The maximum power transfer theorem states the following:

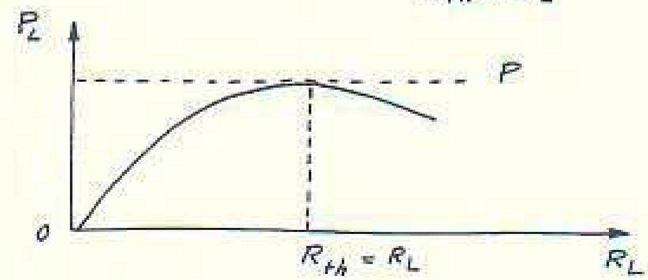
" A load will recieve maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thevenin resistance of the network as seen by the load."



For maximum power transfer $\Rightarrow$ $\boxed{R_{th} = R_L}$

$$I = \frac{E_{th}}{R_{th} + R_L}$$

$$P_L = I^2 R_L = \left(\frac{E_{th}}{R_{th} + R_L} \right)^2 R_L$$



$\therefore$ $P_{L\,max}$ $\Rightarrow$ at $R_{th} = R_L$

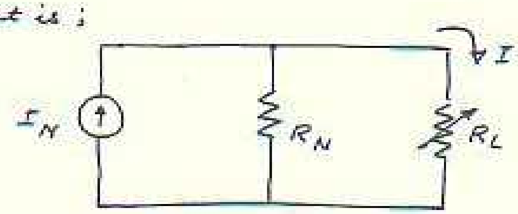
$$P_{L\,max} = \left(\frac{E_{th}}{2 R_{th}} \right)^2 R_{th} = \frac{E_{th}^2}{4 R_{th}}$$

$$\therefore \boxed{P_{L\,max} = \frac{E_{th}^2}{4 R_{th}}}$$

* When dealing with Norton equivalent circuit, maximum power transfer takes place when:

$$\boxed{R_N = R_L}$$

That is ;



Max. power transfer at

$$R_N = R_L$$

$$P_L = I^2 R_L$$

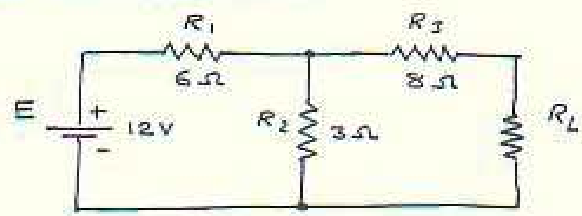
$$= \left(I_N \cdot \frac{R_N}{R_N + R_L}\right) \cdot R_L$$

$$\therefore \; P_{L_{max}} = \left(I_N \cdot \frac{R_N}{2R_N}\right)^2 R_N$$

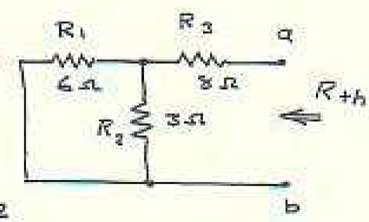
$$= \frac{I_N^2 R_N}{4}$$

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}}$$

Example: For the network shown, determine the value of R_L for maximum power transfer, and calculate the power delivered under these conditions.



Solution:

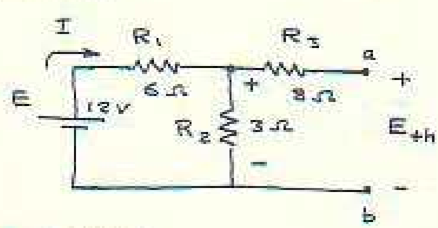


$$* \quad R_{th} = (R_1 // R_2) + R_3$$

$$= \frac{6(3)}{6+3} + 8$$

$$= 10\,\Omega$$

$\therefore$ For max. power the value of $R_L = R_{th} = 10\,\Omega$

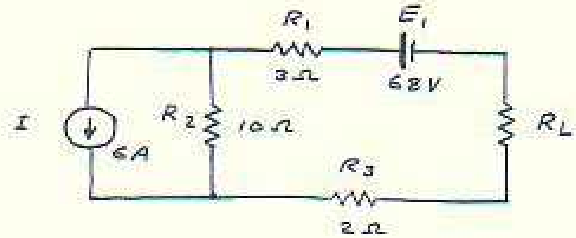


$$* \quad E_{th} = \frac{E \cdot R_2}{R_1 + R_2}$$

$$= \frac{12(3)}{6+3} = 4V$$

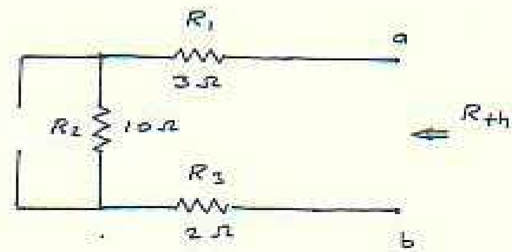
$$\therefore \; P_{L_{max}} = \frac{E_{th}^2}{4R_{th}} = \frac{(4)^2}{4(10)} = \underline{0.4\,W}$$

Example: Find the value of R_L in the network shown, for maximum power to R_L and determine the maximum power.



Solution:

* R_{th}:

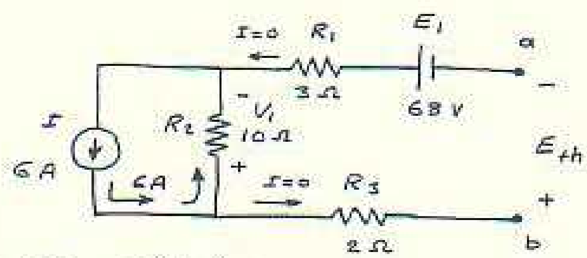


$R_{th} = 3 + 10 + 2$
$= 15\ \Omega$

∴ For max power transfer

$$R_L = R_{th} = 15\ \Omega$$

* E_{th}:



$E_{th} = E_1 + V_1$
$= E_1 + I R_1 = 68 + 6(10)$
$= 128\ V$

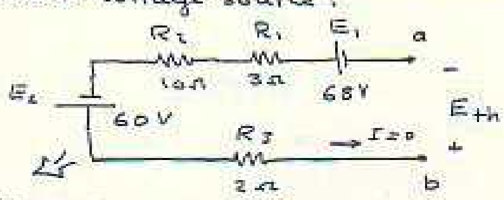
$$\therefore P_{L_{max}} = \frac{E_{th}^2}{4R_{th}} = \frac{(128)^2}{4(15)} = 273.07\ W$$

OR

$R_{th} = R_1 + R_2 + R_3$
$= 3 + 10 + 2$
$= 15\ \Omega$

$R_L = R_{th} = 15\ \Omega$ for max. power

The current source can be converted into a voltage source:



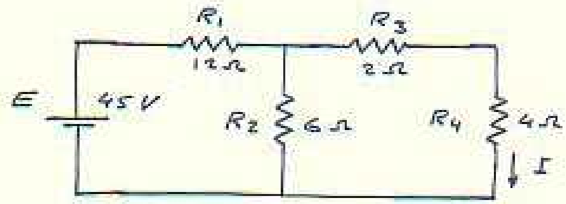
$E_{th} = 68 + 60 = 128\ V$

3.5 Reciprocity Theorem

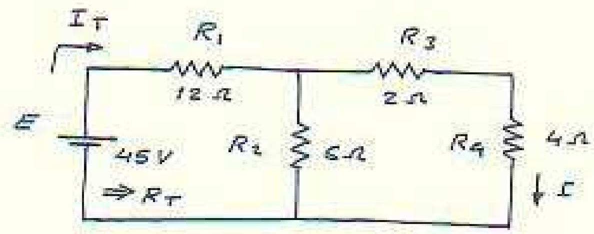
: The reciprocity theorem is applicable only to a single-source networks. The theorem states that:

> " The current I in any branch of a network due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured."

Example : For the network shown, determine the current I. Is the reciprocity theorem satisfied ?



Solution :



$$I_T = \frac{E}{R_T}$$

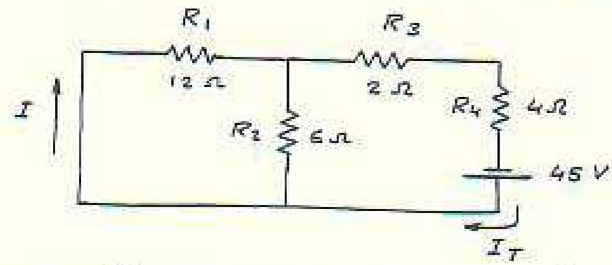
$$= \frac{45}{15} = \underline{3\ A}$$

$$R_T = [(\underbrace{R_3 + R_4}_{\substack{2+4 \\ 6}}) // \underbrace{R_2}_{6}] + R_1$$

$$= 3 + 12 = 15\ \Omega$$

$$\boxed{\therefore I = \frac{3}{2} = 1.5\ A}$$

To check the reciprocity, place E in the branch of the current I, and calculate the current in the branch where E was originally exist.



$R_T = (R_1 // R_2) + R_3 + R_4$

$= \dfrac{12(6)}{12+6} + 2 + 4 = 4 + 2 + 4$

$\therefore R_T = 10\ \Omega$

$I_T = \dfrac{E}{R_T}$

$= \dfrac{45}{10}$

$\therefore I_T = 4.5\ A$

Finding I ?

$$I = I_T \frac{R_2}{R_1 + R_2} = 4.5 \frac{6}{12+6}$$

$$\therefore I = 1.5\ A$$

Since $I = 1.5\ A$

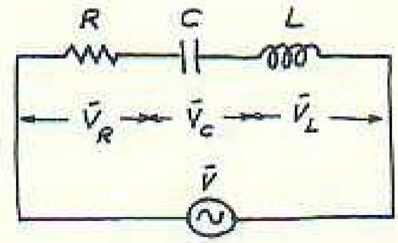
$\therefore$ The reciprocity theorm is satisfied.

8. Resonance in AC Circuits

EEB

8.1 Resonance in Series AC Circuits

: Consider the circuit shown;

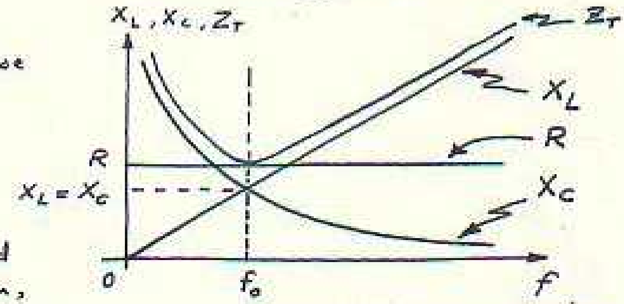


* we have;

$$\bar{Z}_T = R + jX_L - jX_C$$

* At certain frequency (f_o), in the frequency response, we have:

$$\boxed{X_L = X_C}$$



Graphical Representation of Resonance.

* This frequency (f_o) is called the resonance frequency. Then, at this frequency:

$$\boxed{\bar{Z}_T = R} \Rightarrow \text{and hence;}$$

$$\boxed{\bar{V} = \bar{V}_R = \bar{I}R}$$

* The frequency at which resonance takes place, can be obtained as:

$$X_L = X_C$$

$$\omega_o L = \frac{1}{\omega_o C} \Rightarrow \omega_o^2 = \frac{1}{LC} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$\boxed{\therefore \; f_o = \frac{1}{2\pi\sqrt{LC}}}$$

f_o is in (Hz) if L is in Henry and C in Farad.

* Some points to remember (when an R-L-C circuit in resonance):

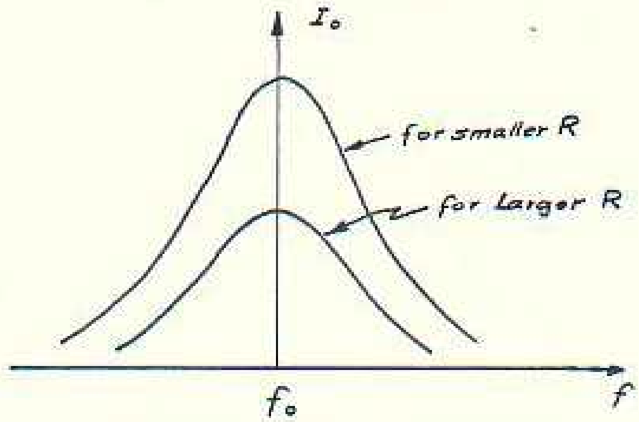
٧ نقاط لازم نتذكرها .

- The overall (total) reactance of the circuit is zero (ie, $X_L - X_C = 0$).
- The circuit impedance is minimum (ie, $Z_T = R$).
- Circuit current is maximum, ($\bar{I}_o = \frac{V}{Z_o} = \frac{V}{R}$).
- Circuit power factor angle is $0°$ $\Rightarrow$ ie, the power factor = 1.
- At resonance $\omega^2 LC = 1$.
- The quality factor Q_o (at resonance) = $\tan\phi = \frac{\omega_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

current at Resonance

* The Resonance Curve (Frequency Response) :

⇨

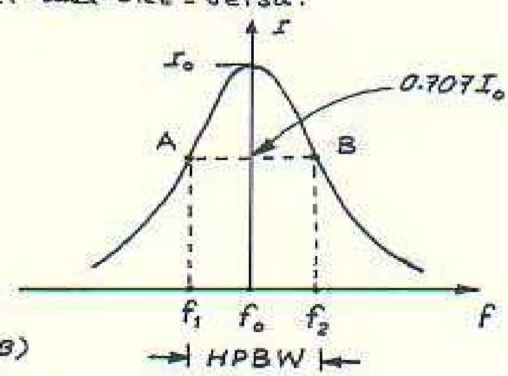


Typical Frequency Response (Resonance Curve) for a series R-L-C circuit in Resonance.

$$\text{Bandwidth} = \Delta\omega = \omega_2 - \omega_1 \Rightarrow \boxed{\Delta f = f_2 - f_1}$$

The narrower the bandwidth, the higher the selectivity of the circuit and vice-versa.

$$P_A = P_B = \left(\frac{I_o}{\sqrt{2}}\right)^2 R = \frac{I_o^2 R}{2} = \frac{P}{2}$$



* Bandwidth is often called (HPBW) to denote the bandwidth at which half the power takes place.

* Also this bandwidth is called (-3dB) bandwidth for the same reason but in a logarithmic scale. (-3dB comes from 10 log 0.5).

In Summary : For an R.L.C circuit in resonance, the following remarks regarding the points A and B in the frequency response:

- Current is $\frac{I_o}{\sqrt{2}} = 0.707\, I_o$
- Impedance is $\sqrt{2}R$ or $\sqrt{2}Z_o$
- $P_A = P_B = \frac{P_o}{2}$
- The circuit phase angle is $\phi = \pm 45^\circ$
- The quality factor $= Q = \tan\phi = \tan 45^\circ = 1$
- $HPBW = f_2 - f_1$

* How to find f_2 and f_1

* At lower half power frequencies ; $\omega_1 < \omega_o \Rightarrow \omega_1 L < \frac{1}{\omega_1 C}$ and $\phi = 45^\circ$.

$$\therefore \frac{1}{\omega_1 C} - \omega_1 L = R \quad \Rightarrow \quad \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

putting $\frac{\omega_o}{Q_o} = \frac{R}{L}$ and $\omega_o^2 = \frac{1}{\sqrt{LC}}$ in the last equation, then

$$\therefore \omega_1^2 + \frac{\omega_o}{Q_o}\omega_1 - \omega_o^2 = 0$$

The positive solution of the above equation is

$$\omega_1 = \omega_o\left[\sqrt{1+\frac{1}{4Q_o^2}} - \frac{1}{2Q_o}\right]$$

* Similarly at the upper half power frequencies $\omega_2 > \omega_o$, the positive solution for ω_2 will be as:

$$\omega_2 = \omega_o\left[\sqrt{1+\frac{1}{4Q_o^2}} + \frac{1}{2Q_o}\right]$$

* for Larger values of Q_o (typically greater than 10), the factor $\frac{1}{4Q_o^2}$ becomes negligible compared to 1, then:

$$\omega_1 = \omega_0\left(1 - \frac{1}{2Q_0}\right) = \omega_0\left(1 - \frac{1}{2\frac{\omega_0 L}{R}}\right) = \omega_0 - \frac{\omega_0}{2\frac{\omega_0 L}{R}}$$

$$\therefore \omega_1 = \omega_0 - \frac{R}{2L}$$

$$\Rightarrow \boxed{f_1 = f_0 - \frac{R}{4\pi L}}$$

$$\boxed{\omega_1 = \omega_0\left(1 - \frac{1}{2Q_0}\right)}$$

the lower frequency limit.

and ;

Similarly

$$\boxed{\omega_2 = \omega_0\left(1 + \frac{1}{2Q_0}\right)}$$

the upper frequency limit.

$$\boxed{f_2 = f_0 + \frac{R}{4\pi L}}$$

$$(A)\ \boxed{\therefore HPBW = f_2 - f_1 = \frac{R}{2\pi L}} \Rightarrow \boxed{HPBW = f_0/Q_0}$$

Q- Factor of a resonant series circuit

: There are different ways to derive (or to define) the quality factor Q_0 of a series resonant circuit.

* It is given by the voltage magnification produced in the circuit at resonance ;

we have : $\bar{I}_0 = \dfrac{\bar{V}}{R}$ at resonance

$$\therefore Q_0 \Rightarrow \text{voltage magnification is} = \frac{V_{L0}}{V}$$

$$\Rightarrow \frac{V_{L0}}{V} = \frac{I_0 X_{L0}}{I_0 R} = \frac{\text{Reactive Power}}{\text{Active Power}} = \frac{X_{L0}}{R} = \underline{\underline{\frac{\omega_0 L}{R}}}$$

$$= \frac{\text{Reactance}}{\text{Resistance}}$$

OR

$$Q_0 = \text{voltage magnification} = \frac{V_{C0}}{V} = \frac{I_0 X_{C0}}{I_0 R} = \frac{X_{C0}}{R}$$

$$= \frac{\text{Reactive Power}}{\text{Active Power}} = \underline{\underline{\frac{1}{\omega_0 C R}}}$$

** The quality factor can also be defined as:

الطاقة المخزنة في الملف $= \frac{1}{2}LI^2$

$$Q_0 = 2\pi \frac{\text{Maximum Energy stored}}{\text{Energy dissipated per cycle}}$$

$$= 2\pi \cdot \frac{\frac{1}{2}LI^2}{I_0^2 R \cdot T_0} = 2\pi\left[\frac{\frac{1}{2}L(\sqrt{2}I_0)^2}{I_0^2 R\left(\frac{1}{f_0}\right)}\right]$$

$$\boxed{\therefore Q_0 = \frac{I_0^2 (2\pi f_0) L}{I_0^2 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$$

*** OR, the quality factor (Q_o at resonance), can be obtained as:

we have: $f_o = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow \omega_o = \dfrac{1}{\sqrt{LC}}$

and $Q_o = \dfrac{\omega_o L}{R} = \dfrac{\frac{1}{\sqrt{LC}} \cdot L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$$\boxed{\therefore\ Q_o = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

NOTE: Higher values of the Q-factor mean not only higher voltage magnification but also mean high selectivity of the tuning circuit (resonant circuit).

$\therefore$ In Summary

$$Q_o = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{X_{Lo}\,X_{Co}}{R}}$$

$$= \frac{f_o}{HPBW} = \frac{f_o}{f_2 - f_1}$$

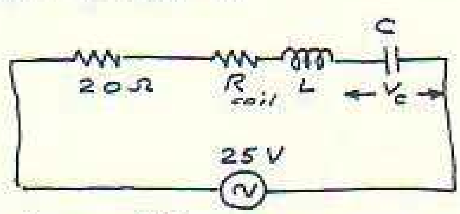
Example: A 20 Ω resistor is connected in series with an inductor, a capacitor and an ammeter across a 25 V supply with variable frequency. When the frequency is 400 Hz, the current is at its maximum value of 0.5 A, and the potential difference across the capacitor is 150 V. Calculate:

(a). the capacitance of the capacitor.
(b). the resistance and the inductance of the inductor.

Solution: $\because$ the current is maximum $\Rightarrow$ $\therefore$ the cct is in resonance

$$\therefore X_c = V_c / I_o = 150/0.5 = 300\ \Omega$$

(a). $X_c = \dfrac{1}{2\pi f_o C} \Rightarrow C = \dfrac{1}{2\pi f X_c}$



$\therefore\ C = \dfrac{1}{2\pi(400)(300)} = 1.3\ \mu F$

(b). $X_L = X_C = 300\ \Omega = 2\pi f_o L \Rightarrow L = \dfrac{300}{2\pi(400)} = 0.119\ H$

* Now R_{coil}? $\Rightarrow$

at resonance Z_T = Resistance of the circuit

$$\therefore \frac{V}{I_o} = R_T$$

$$\therefore \frac{25}{0.5} = 20 + R_{coil}$$

$$\therefore R_{coil} = 50 - 20 = \underline{30\,\Omega}$$

Example: An RLC circuit consists of a series resistance of $1\,k\Omega$, an inductance of 100 mH, and a capacitor of 10 pF. If a voltage of 100 V is applied across the combination, find:

(a). the resonance frequency.
(b). Q-factor of the circuit.
(c). the half-power points.
(d). The half-power bandwidth of the resonance frequency response.

Solution:

(a). The resonance frequency $f_o = ?$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-12}}}$$
$$= \underline{159\ kHz}$$

(b). The quality factor of the circuit $Q_o = ?$

$$Q_o = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000}\sqrt{\frac{100\times10^{-3}}{10\times10^{-12}}}$$

$$\therefore Q_o = \underline{\underline{100}}$$

(c). The half-power points $\Rightarrow f_1 = ?$ & $f_2 = ?$

$$* \ f_1 = f_o - \frac{R}{4\pi L} = (159\times10^{3}) - \frac{1000}{4\pi\times100\times10^{-3}}$$

$$\therefore f_1 = \underline{\underline{158.2\ kHz}}$$

OR

ملاحظة: يمكن حساب HPBW كما يلي :

$$HPBW = f_2 - f_1$$
$$= f_o + \frac{R}{4\pi L} - f_o + \frac{R}{4\pi L}$$
$$\boxed{\therefore \Rightarrow HPBW = \frac{R}{2\pi L}}$$

وبذلك نحصل على النتيجة نفسها .

$$* \ f_2 = f_o + \frac{R}{4\pi L} = (159\times10^{3}) + \frac{1000}{4\pi\times100\times10^{-3}}$$

$$\therefore f_2 = 159.8\ kHz$$

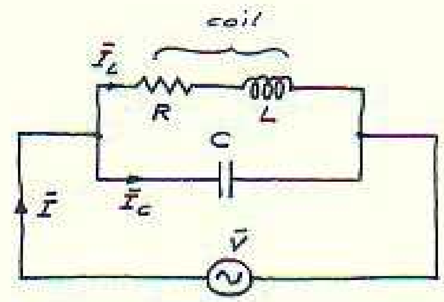
(d). The half power bandwidth HPBW = ?

$$HPBW = f_2 - f_1 = 159.8 - 158.2 = \underline{1.6\ kHz}$$

8.2 Resonance in Parallel Circuits

Consider the parallel RLC circuit shown;

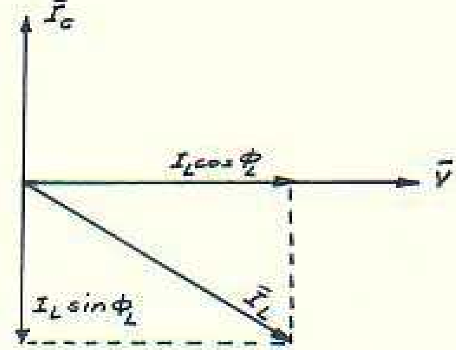
* For this circuit, the resulting phasor diagram can be obtained as shown in the Fig. below



* The CONDITION of resonance for this circuit takes place, when the two reactive components of the line current are EQUAL. This means that:

$$\bar{I}_c = \bar{I}_L \sin\phi_L$$

$$\boxed{\text{or} \quad \bar{I}_c - I_L \sin\phi_L = 0}$$

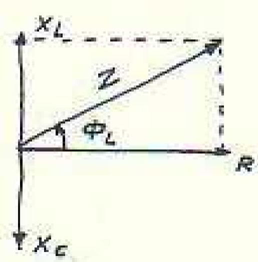


* In terms of impedance, at resonance:

$$Z\sin\phi_L = X_L = X_c$$

$$\Rightarrow X_L = X_c \quad \Rightarrow \quad X_L \cdot X_c = Z^2$$

$$\omega_o L \cdot \frac{1}{\omega_o C} = Z^2$$



$$\left.\begin{array}{l} \dfrac{\omega_o L}{\omega_o C} = Z^2 \\ \text{But } \bar{Z} = R + jX_L \Rightarrow Z^2 = R^2 + X_L^2 \end{array}\right\} \Rightarrow \frac{\omega_o L}{\omega_o C} = R^2 + X_L^2$$

$$\therefore \frac{L}{C} = R^2 + (2\pi f_o)^2 L^2 \quad \Rightarrow (2\pi f_o)^2 L^2 = \frac{L}{C} - R^2$$

$$\boxed{\therefore \quad f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

* يلاحظ انه عندما تكون المقاومة R صغيرة جداً
فإن: $f_o = \frac{1}{2\pi\sqrt{LC}}$ كما في حالة رنين دوائر التوالي .

* Current at Resonance

Since the net reactive components of the current, at resonance, is zero, then;

$$\bar{I}_o - \bar{I}_L \sin\phi_L = 0$$

Thus the resultant current at resonance is only the real component (see the phasor diagram) which is:

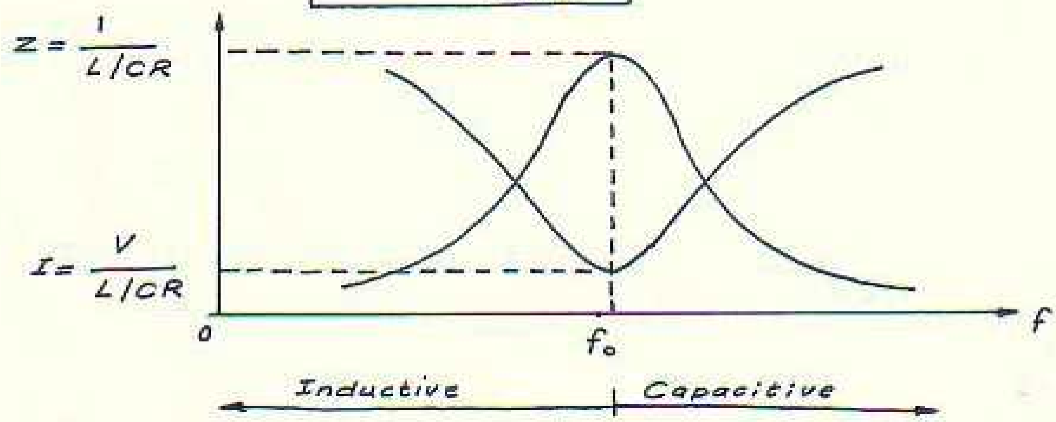
$$\boxed{I_o = I_L \cos\phi_L} \quad \Rightarrow \bar{I}_L = \frac{\bar{V}}{Z}$$

$$\& \quad \cos\phi_L = \frac{R}{Z}$$

$$\therefore \quad I_o = I_L \cos\phi_L = \frac{VR}{Z^2}$$

$$\text{but} \quad Z^2 = \frac{L}{C} \quad \Rightarrow \quad I_o = \frac{VR}{L/C} = \frac{V}{L/CR}$$

$$\boxed{\therefore \; I_o = \frac{V}{L/CR}} \quad \text{current at resonance}$$

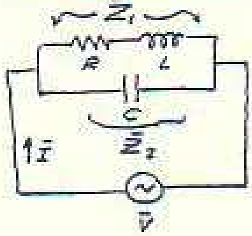


The impedance $\left[Z = \frac{1}{L/CR} \right]$ is called sometimes the effective impedance OR "the dynamic impedance".

طريقة أخرى لاستنتاج f_o وهي بالطريقة السابقة

Alternative Method

using the admittance :



$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L} = \frac{R}{R^2 + X_L^2} - j\,\frac{X_L}{R^2 + X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = j\,\frac{1}{X_C}$$

$\bar{Y}_T = G + jB$

$$\therefore\ \bar{Y}_T = \bar{Y}_1 + \bar{Y}_2$$

$$\therefore\ \bar{Y}_T = \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

(*) In general, any circuit to be in resonance, the imaginary part (j-component) of the circuit impedance or admittance is zero.

Thus ; for our circuit mentioned earlier ;

$$\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0$$

$$\Rightarrow \frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C} \qquad \Rightarrow X_L \cdot X_C = R^2 + X_L^2 = Z^2$$

وهي نفس النتيجة التي تم الحصول عليها سابقا.

* Talking in terms of Suceptance for parallel circuits, the net susceptance is zero at resonance condition. ($B = 0$).

It may be noted that at resonance, the admittance is equal to the conductance (G)

Points to Remember

: The following points about parallel resonance should be noted and compared with those about series resonance. At resonance :

- Net succeptance is zero ($B_T = 0$).
- The admittance equal to the conductance.
- Reactive component of the line current is zero.
- Dynamic impedance $= L/CR$
- Line current at resonance is minimum and equal $\frac{V}{L/CR}$ but it is phase with the applied voltage.
- Power factor of the circuit is unity.

Bandwidth of the Parallel Resonant Circuit: The bandwidth of a parallel circuit is defined in the same way as that for a series circuit. This circuit also has upper and lower half-power frequencies where power dissipated is half of that at resonant frequency.

Quality Factor of a Parallel Circuit: It is defined as the current magnification (in the C-branch or in the coil branch) with respect to the line current drawn from the supply. This means that:

$$Q\text{-factor at resonance} \Rightarrow Q_o = \frac{\bar{I}_c}{\bar{I}}$$

$$\bar{I}_c = \frac{\bar{V}}{X_c} = \frac{\bar{V}}{1/\omega C} = \omega C \bar{V}$$

and

$$\bar{I} = \frac{\bar{V}}{L/CR}$$

$$\therefore Q_o = \frac{\omega C V}{\frac{V}{L/CR}} = \omega C V \times \frac{L/CR}{V}$$

$$\boxed{\therefore Q_o = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R}}$$

which is the same as that for series circuits.

$$\therefore Q_o = \tan\phi$$

Other expressions relating Q_o can be used in parallel circuits as had been used for series circuits.

Example: A capacitor is connected in parallel with a coil having $L = 5.52$ mH and $R = 10\ \Omega$, to a 100 V, 50 Hz supply. Calculate the value of the capacitance for which the current taken from the supply is in phase with the applied voltage.

Solution: Since the current taken from the supply is in phase with the applied voltage $\Rightarrow$ The circuit is in resonance.

Then, at resonance;

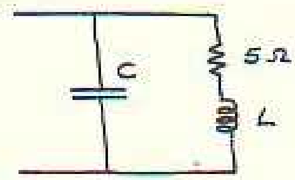
$$Z^2 = \frac{L}{C} \quad \text{or} \quad C = L/Z^2$$

$$X_L = 2\pi f L = 2\pi(50)\, 5.52 \times 10^{-3} = 1.734\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow \quad \therefore Z^2 = 10^2 + 1.734^2 \Rightarrow Z = 10.1\ \Omega$$

$$\therefore C = \frac{L}{Z^2} = \frac{5.52 \times 10^{-3}}{(10.1)^2} = 54.6\ \mu F$$

Example: Calculate the impedance of the parallel-tuned circuit shown at a frequency of 500 kHz and fo a bandwidth of operation equal to 20 kHz. The resistance of the coil is 5 Ω.



Solution: $HPBW = \frac{R}{2\pi L} = 20\times10^{3}$ Hz

$$\therefore L = \frac{5}{2\pi(20\times10^{3})} = 39\ \mu H$$

$$f_o = 500\times10^{3}\ Hz = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{39\times10^{-6}C} - \frac{5^2}{(39\times10^{-6})^2}}$$

$$\therefore C = 2.6\times10^{-9}\ F = 2.6\ nF$$

$$Z = \frac{L}{CR} = \frac{39\times10^{-6}}{2.6\times10^{-9}\times5} = 3\ k\Omega.$$

ملاحظة : في حال اضافة اي عنصر الى الدائرة المذكورة في المثال ، فلا بد من
اشتقاق العلاقة الخاصة بـ (f_o) كما تعلمنا سابقاً .