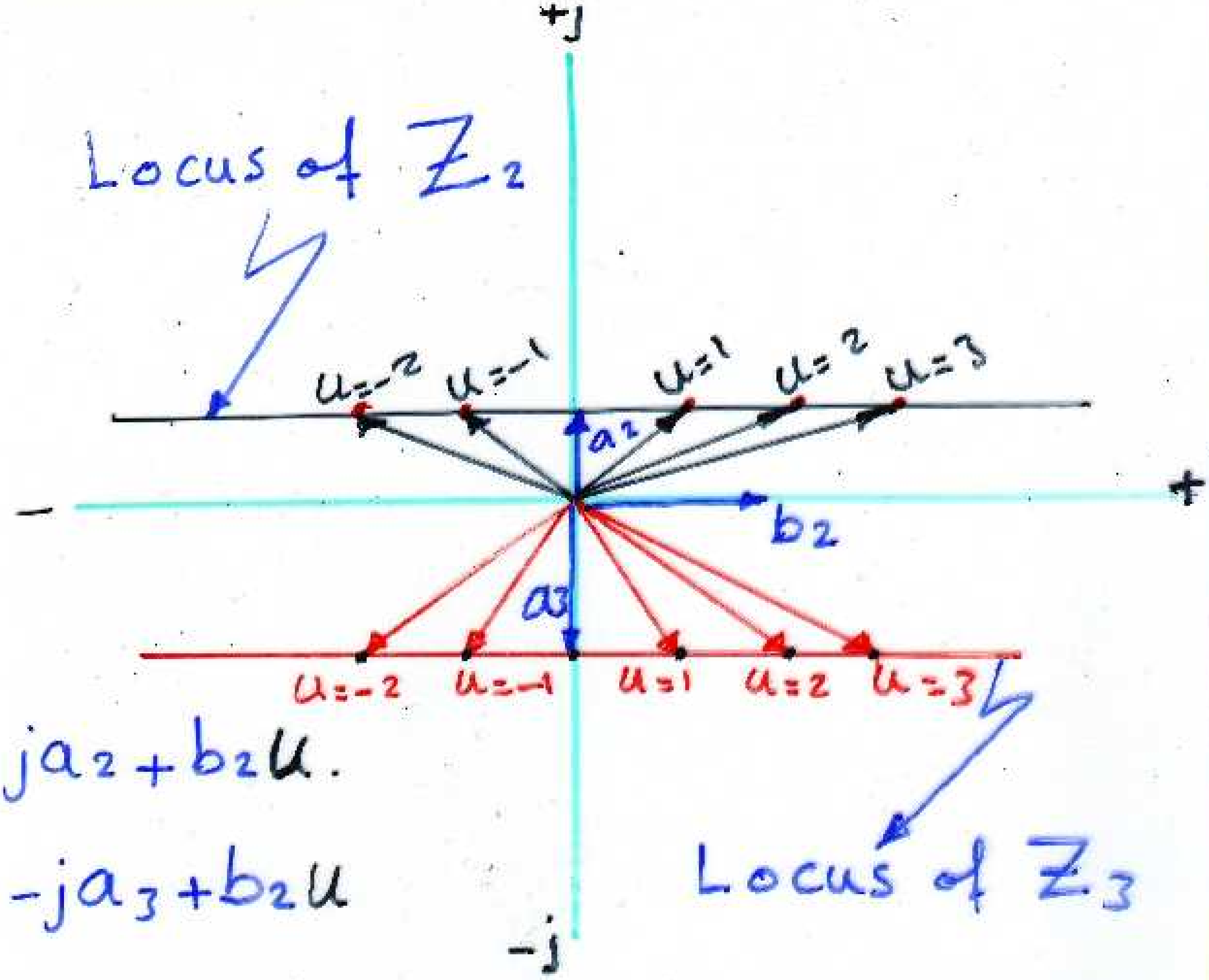


$Z_2 = ja_2 + b_2u.$

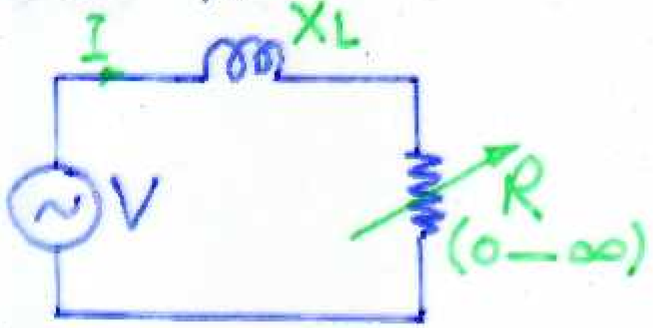
$Z_3 = -ja_3 + b_2u$

These results may be applied directly in circuit analysis for plotting the impedance Loci for simple series circuits or the admittance Loci for simple parallel circuits as one of the parameters is varied. In the impedance diagram, resistance is plotted in the horzontal direction and reactance in the vertical direction.

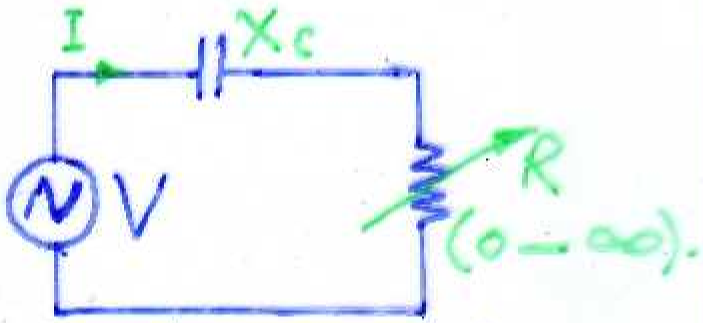
Circle Diagram of a Series Circuit

① Circuit of Constant Reactance With Variable Resistance :-

Consider a circuit having a constant reactance but variable resistance (Varying from $0 \rightarrow \infty$) and supplied with constand Voltage.

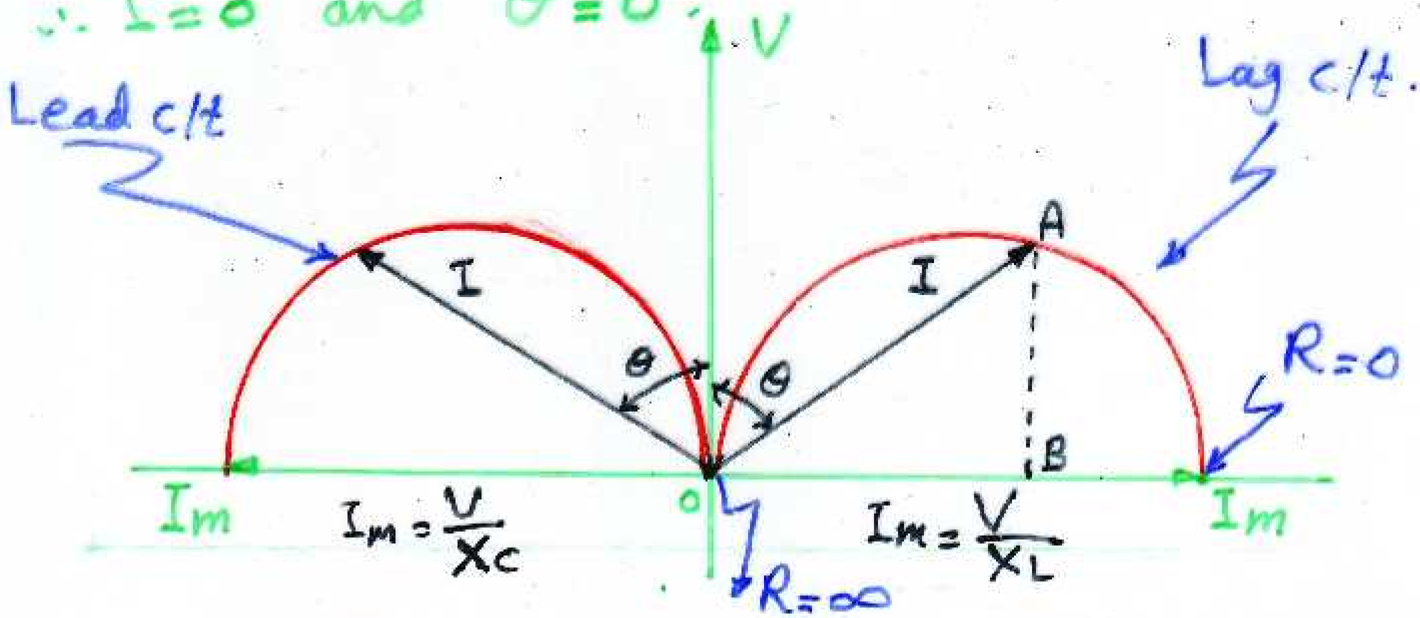


Lagging circuit



Leading Circuit.

* If $R = 0$ the $I = \dfrac{V}{X_L}$ or $\dfrac{V}{X_C}$ this current is (Max. Value), and $\theta = 90^\circ$.
* If (R) is increased, then I & θ will decrease, in the limiting case, When $R = \infty$
$\therefore I = 0$ and $\theta = 0$.



* Since $P = V.I.\cos\theta$. (watt).

and $AB = I\cos\theta$. (see fig).

$\therefore$ **AB** represents on a suitable scale the power consumed by the (R_L) circuit.

Condition of Maximum power :-

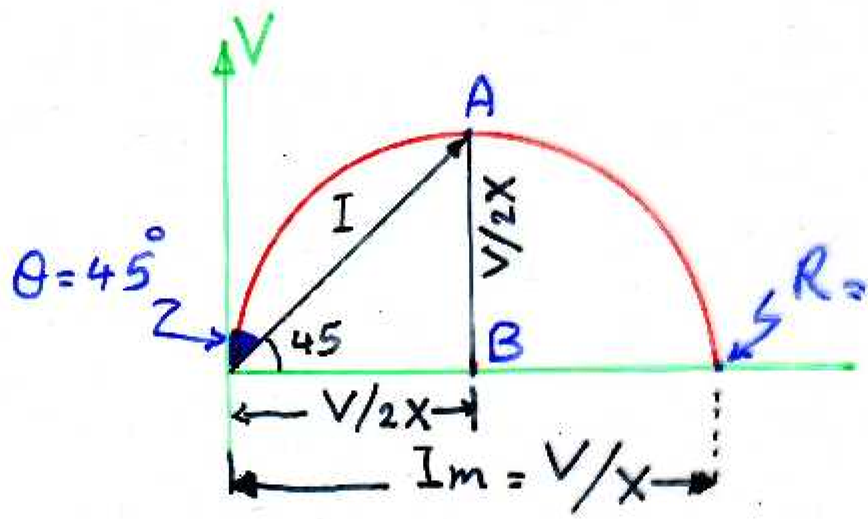
From above equations:

$$P = V \times AB.$$

and maximum power (P_{max}) take place when

$$AB = \frac{I_m}{2} \quad (\text{when } \theta = 45)$$

(when AB becomes as a radius of the circle) as shown.



$$\therefore P_{max} = V \times AB$$

$$= V \times \frac{I_m}{2}.$$

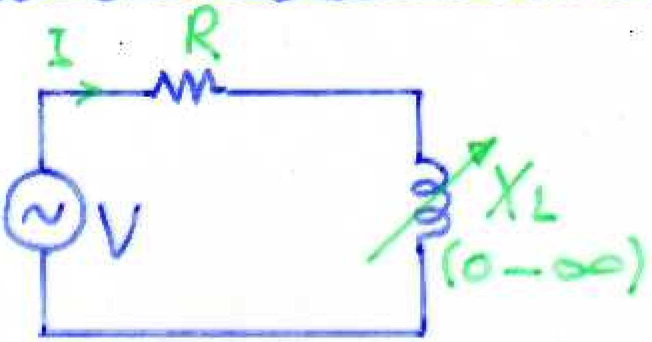
and for R_L circuit $I_m = V/X_L$.

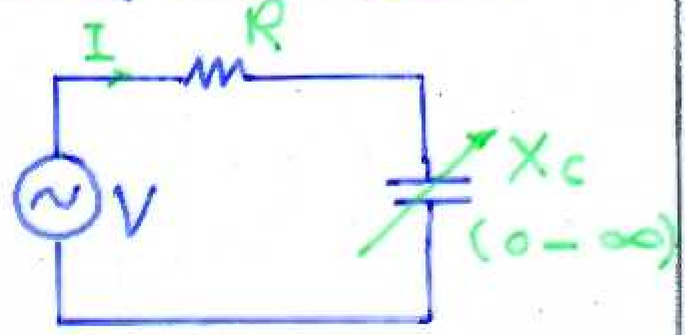
$$\therefore \boxed{P_{max} = \frac{V^2}{2X_L}} = \frac{V^2}{2\omega L}. \quad (\text{watt})$$

& For (R_C) circuit

$$\boxed{P_{max} = \frac{V^2}{2X_C}}. \quad (\text{watt}).$$

② Circuit of constant resistance with variable reactance :-





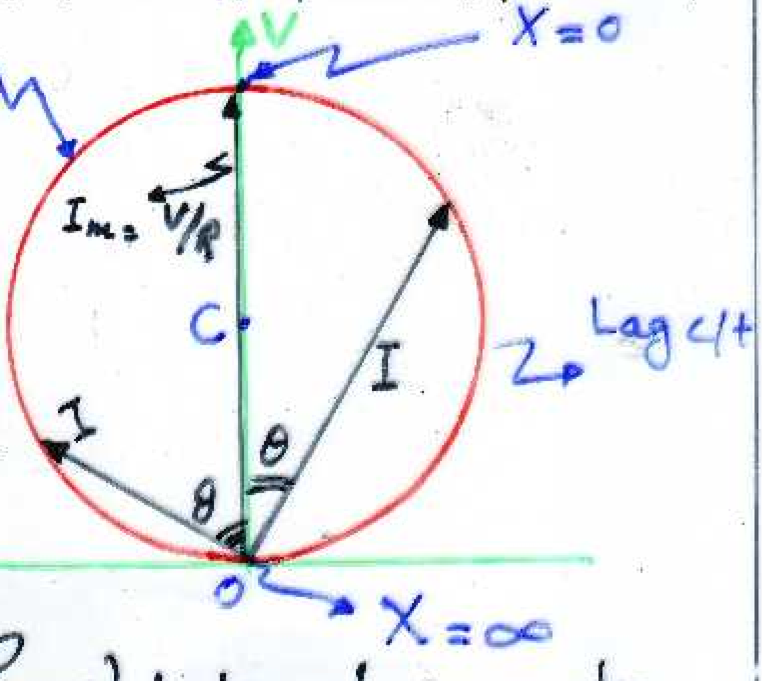
For both circuits

$$I = V / \sqrt{R^2 + X^2} .$$

But when X_L or $X_C = 0$.

∴ The current is max. $\left(I_{max} = \frac{V}{R} \right)$. and the current becomes (Zero) when X_L or X_C becom-es (∞). The end point of the current vector describes a semi-circle with radius :

$OC = V/2R$.



Since $P = V . I . \cos\theta$.

$$= V \left(\frac{V}{R \mp jX} \right) \cos\theta .$$

∵ V & R (constant).

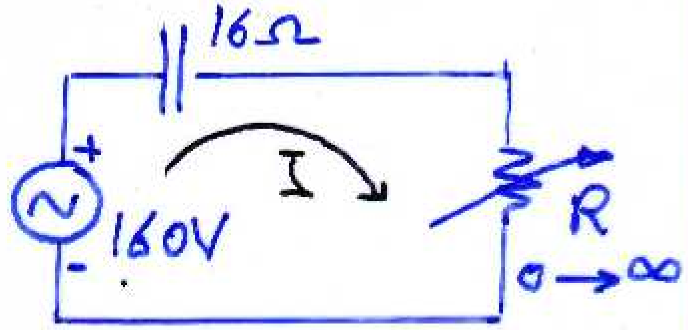
∴ Maximum power (P_{max}) takeplace only when $(X = 0) \longrightarrow \theta = 0$.

∴ $\boxed{P_{max} = \frac{V^2}{R} = V . I_{max}}$ (watt).

For both R-L & R-C.

Ex

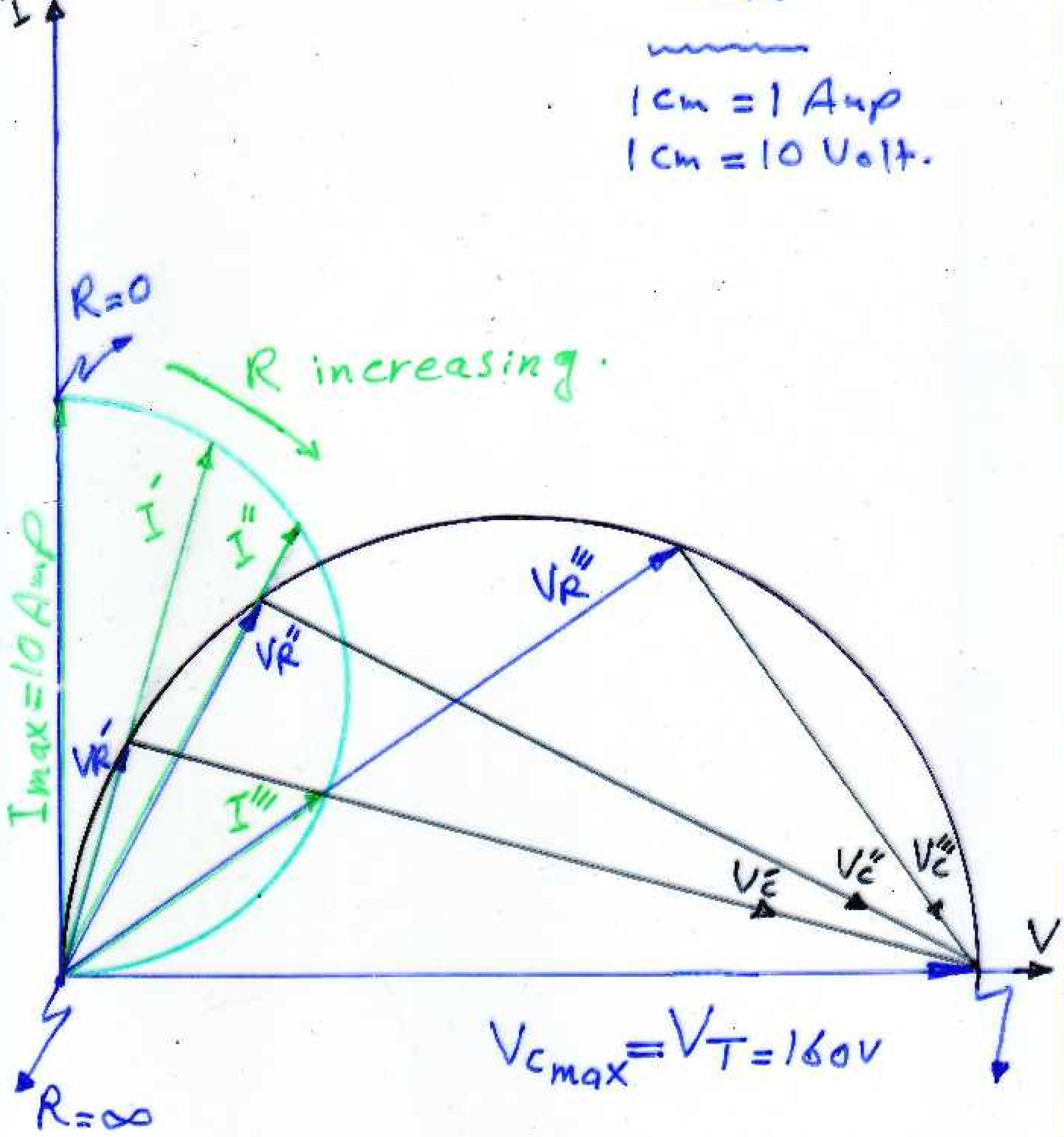
For the Circuit draw the Locus diagram if (R) Varies from (0 to ∞).



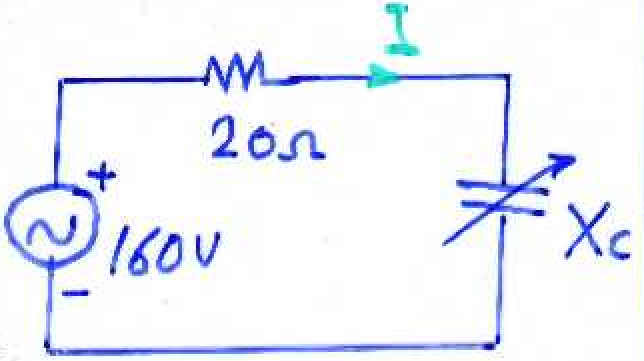
Scale:

1 cm = 1 Amp

1 cm = 10 Volt.

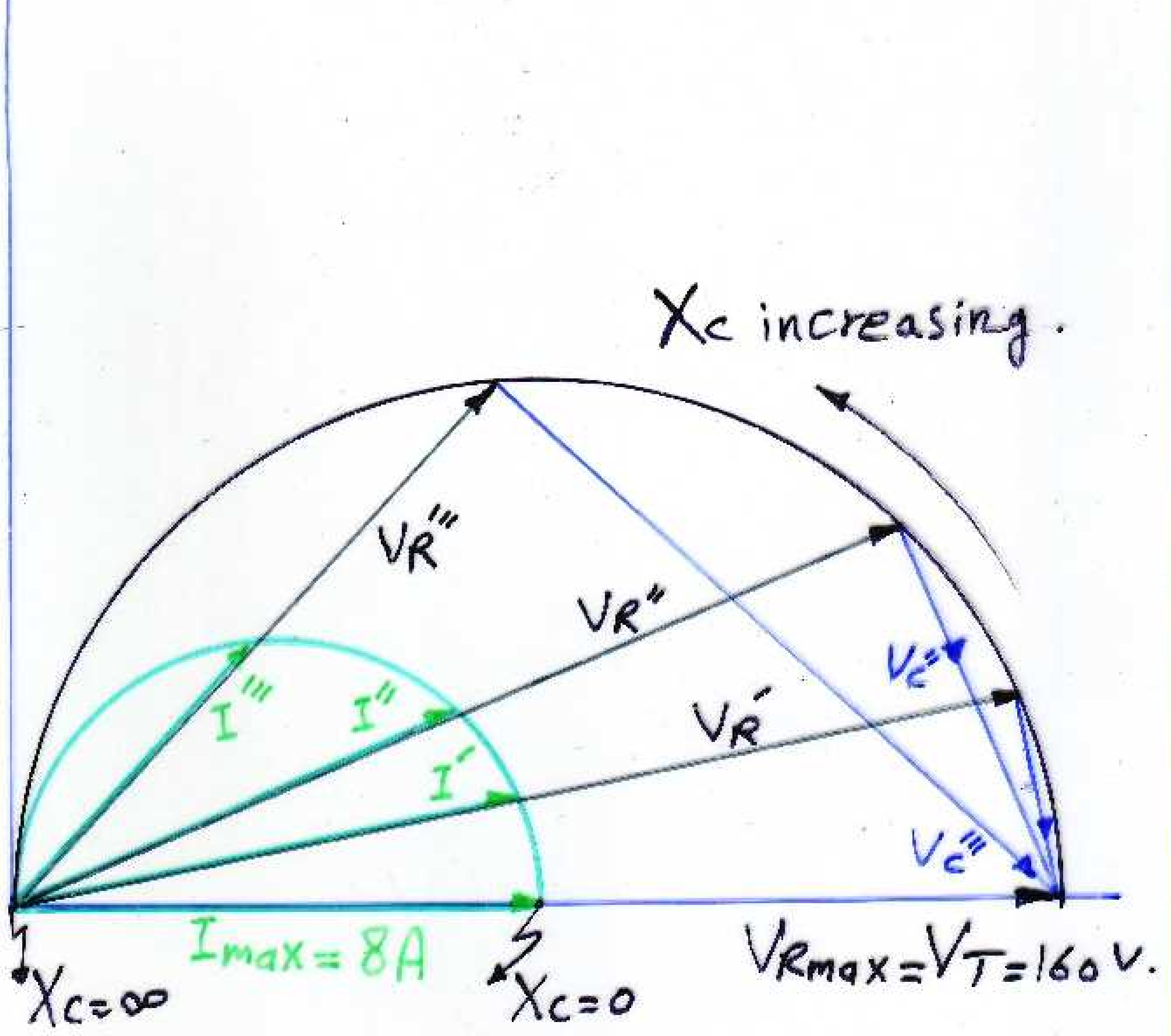


Ex: For the circuit draw the Locus diagram of Voltages and current if X_c varies from $(0 \longrightarrow \infty)$.

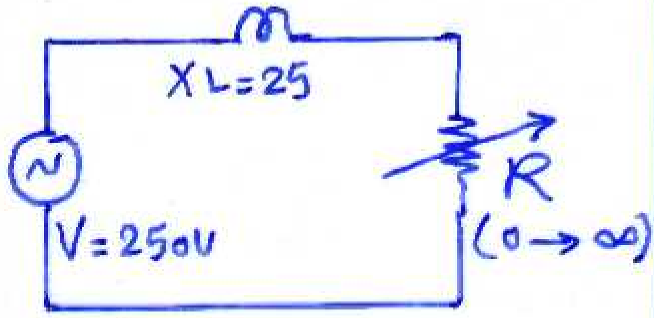


Scale:

1 cm = 1 Amp.
1 cm = 10 Volt.



Ex: For the Circuit, Draw the Locus diagram, then find the (R) and (P.f) at max. Power, If the P.f (0.866) for this Condition find (I, Power and the resistance).



$I_{max} = \frac{250}{25} = 10\ Amp$

(When $R = 0$).

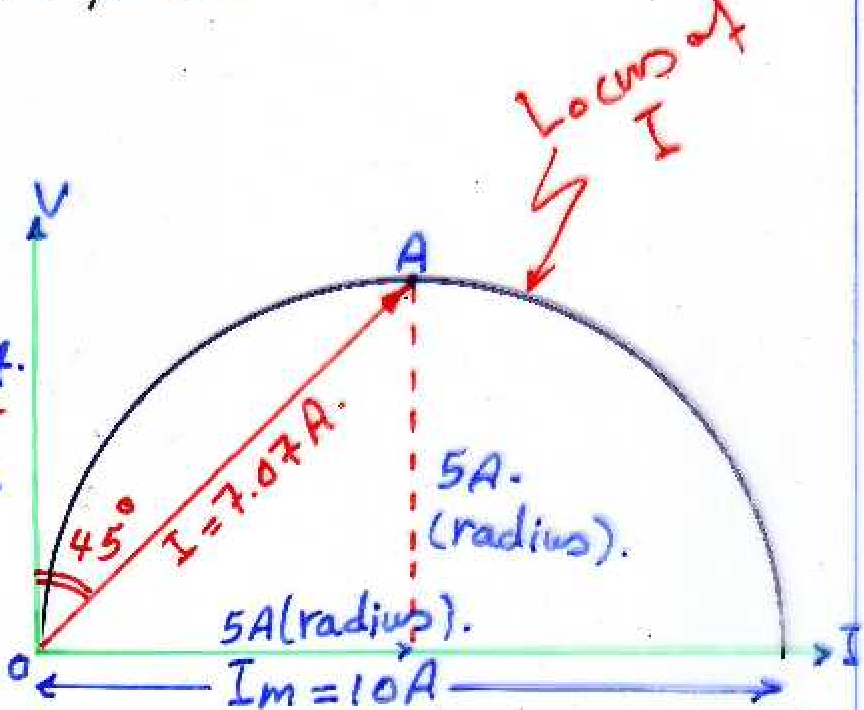
$\therefore P_{max} = V\frac{I_m}{2} = 1250\ Watt.$

The current (OA) from fig = 7.07 Amp.

$P.f = \cos 45 = 0.707.$

Under this Condition

$R = X_L = 25\ \Omega$



*When $P.f = 0.866$

$\theta = 30°$.

From the Locus

$I = OA = 5\ Amp.$

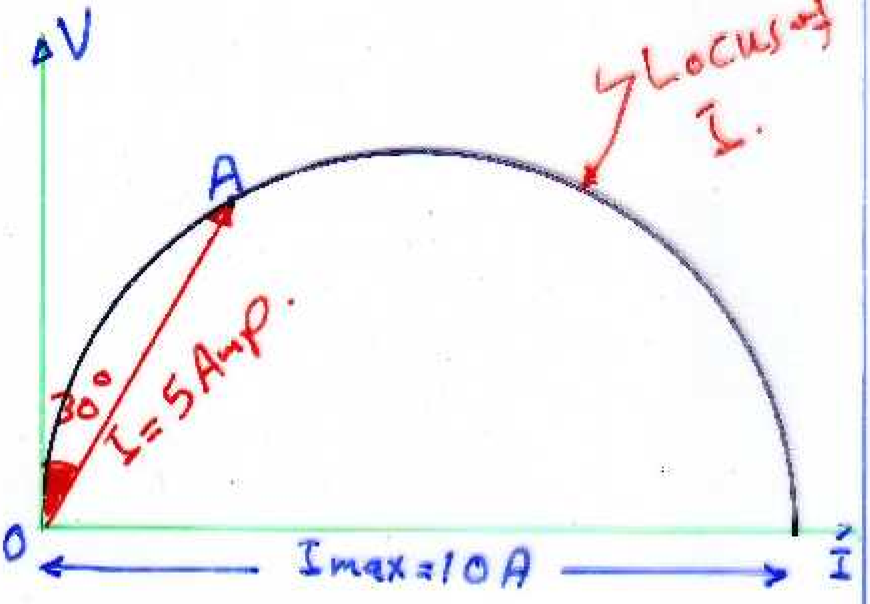
$\therefore P = V.I.\cos\theta$

$= 250 \times 5 \times 0.866$

$= 1082.5\ Watt.$

$= I^2.R.$

$\therefore R = \frac{1082.5}{(5)^2} = 43.3\ \Omega.$

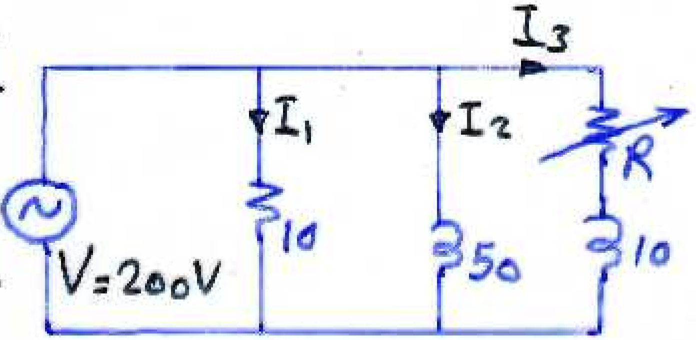


Check

$\theta = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{25}{43.3}$

$= 30°.$

Ex: For the circuit, draw the Locus diagram of currents, find the value of the max. Power supplied.

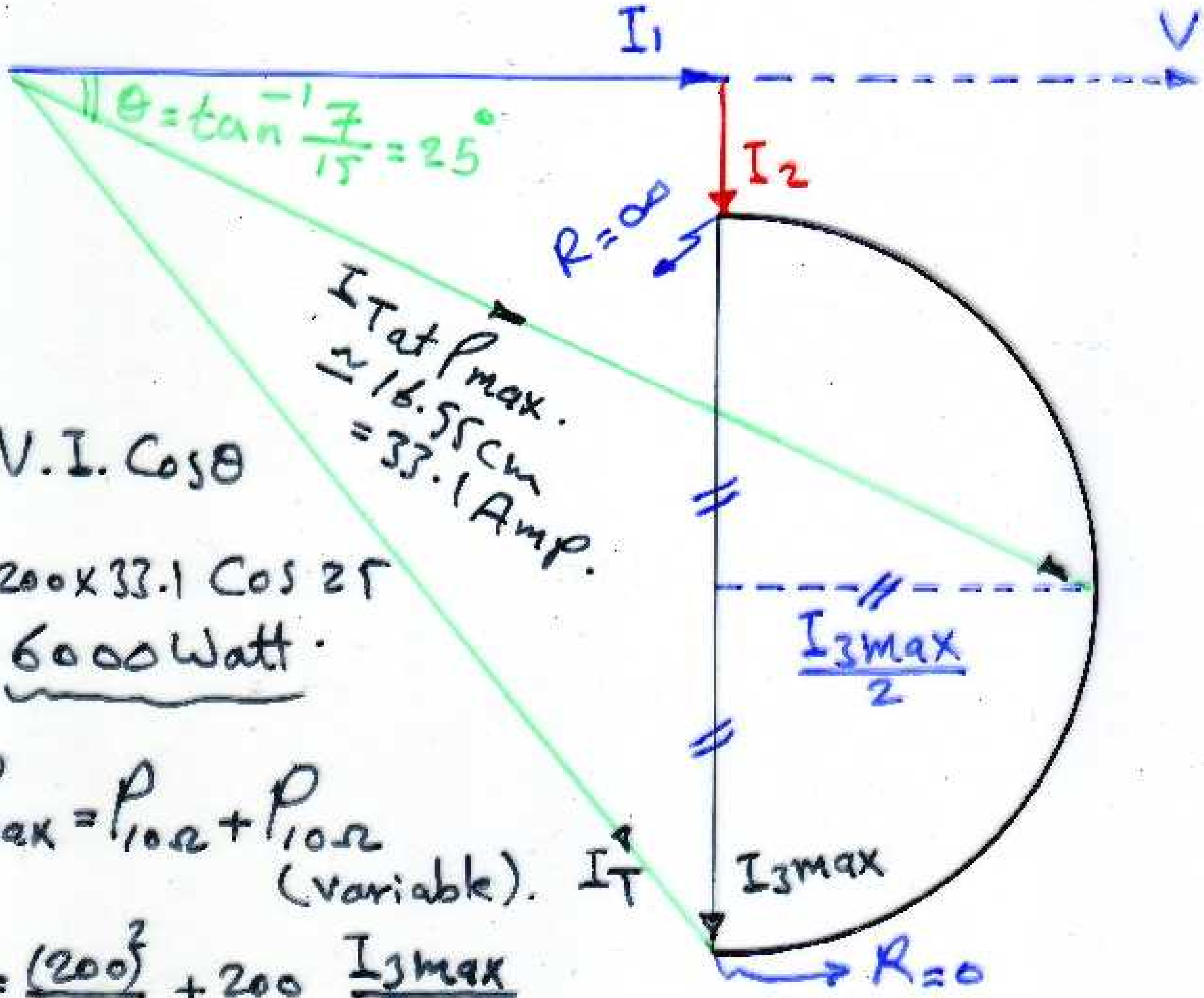


Scale:
1 cm = 2 Amp.

$I_1 = \frac{200}{10} = 20A.$

$I_2 = \frac{200}{50} = 4A.$

$I_{3\,max} = \frac{200}{10} = 20A.$ (when $R = 0$).



$\therefore P_{max} = V.I.\cos\theta$

$= 200 \times 33.1 \cos 25$

$= \underline{6000\ Watt}.$

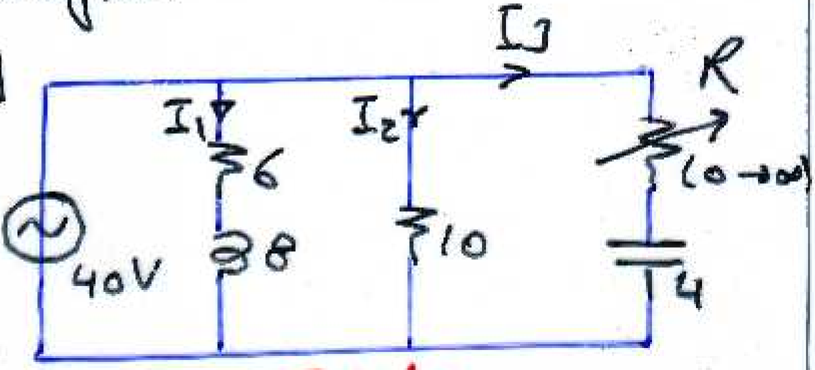
OR $P_{max} = P_{10\Omega} + P_{10\Omega}$ (variable).

$= \frac{(200)^2}{10} + 200\ \frac{I_{3max}}{2}$

$= 4000 + 200\ \frac{20}{2} = \underline{6000\ Watt}.$

OR $P_{max} = \frac{(200)^2}{10} + \left(\frac{200}{\sqrt{10^2+10^2}}\right)^2 \times 10 = \underline{6000\ Watt}$

Ex Draw the Locus diagram of the circuit, then and state wether it is possible to obtain resonance or not, then find (R at resonance).



Scale
1 cm = 1 Amp.

* $I_1 = \frac{40}{6+j8} = 4\angle{-57.13}\ A.$

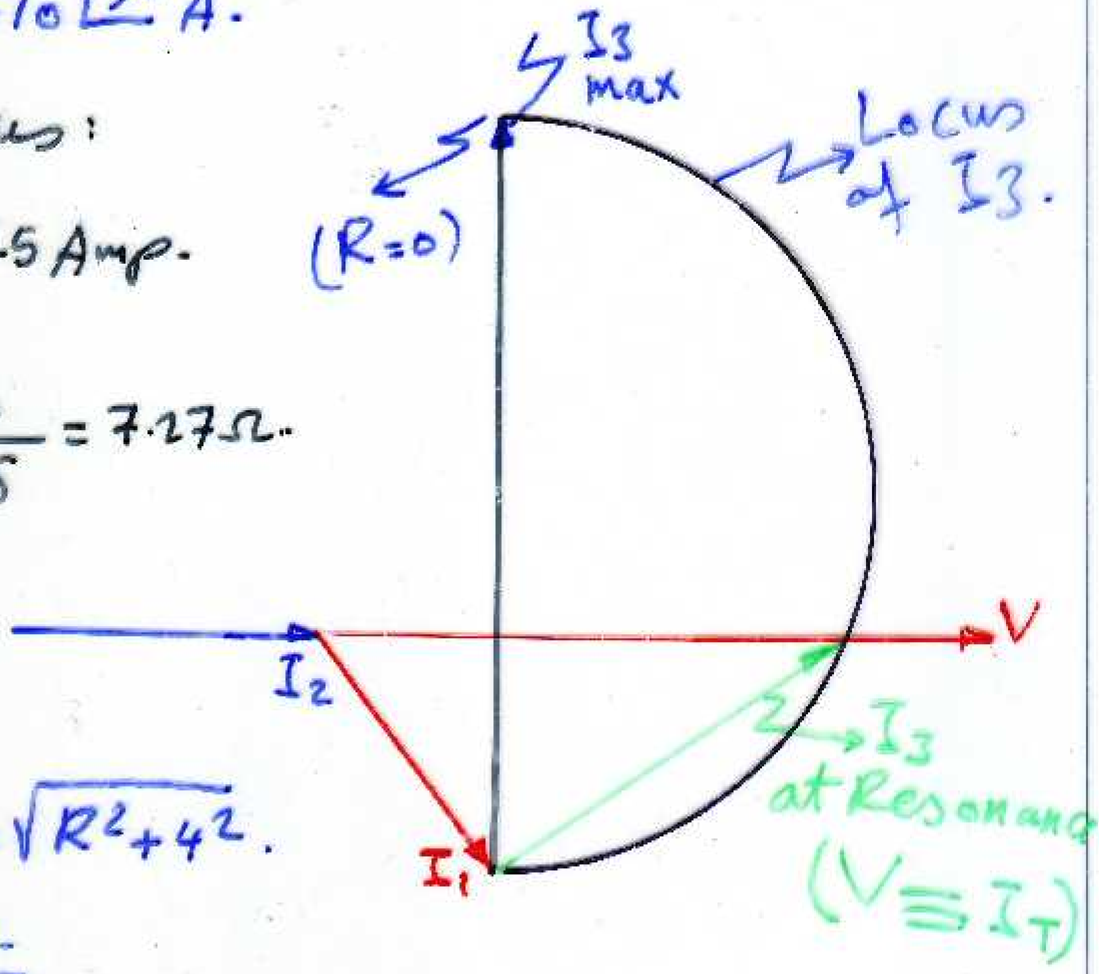
$I_2 = \frac{40}{10} = 4\ A.$

$I_{3\,max} = \frac{40}{-j4} = 10\angle{90^\circ}\ A.$

from the Locus:

I_3 at resonance $= 5.5$ Amp.

$\therefore Z_{Res} = \frac{40}{5.5} = 7.27\,\Omega.$



$\therefore Z_R = 7.27 = \sqrt{R^2 + 4^2}.$

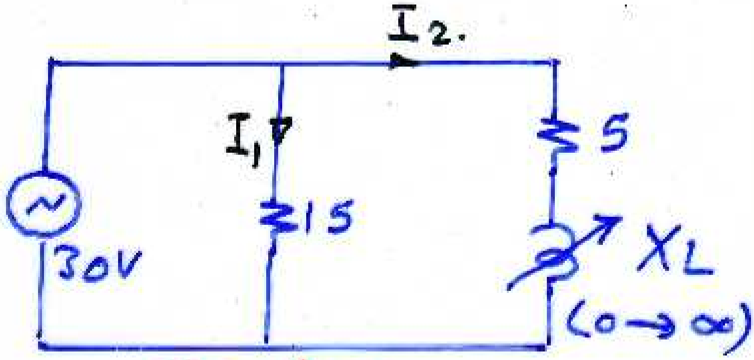
$\therefore R = 6\,\Omega.$ at Resonance.

check your Result.

Ex: Draw the Locus diagram for voltage and current.

$I_1 = \frac{30}{15} = 2\ Amp.$

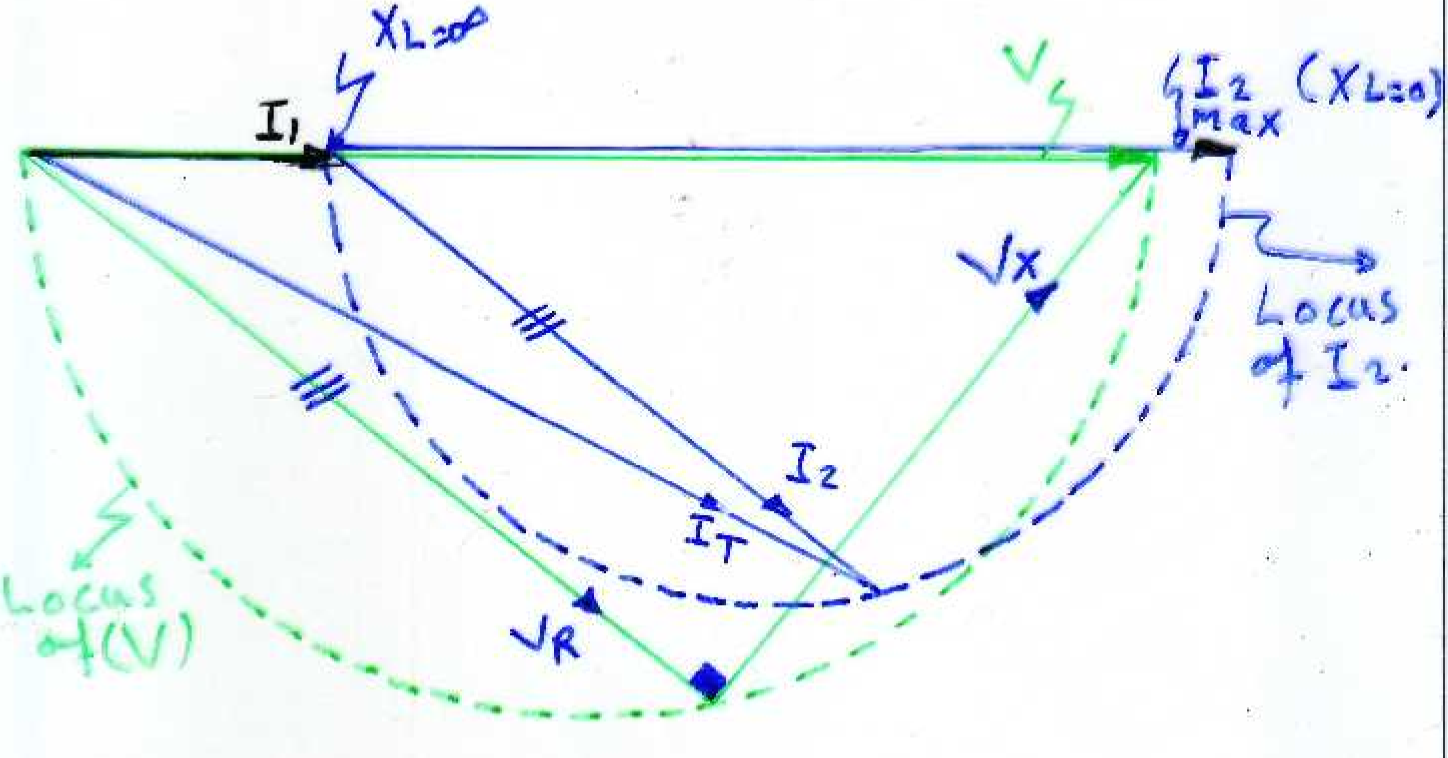
$I_{2\,max} = \frac{30}{5} = 6\ Amp$



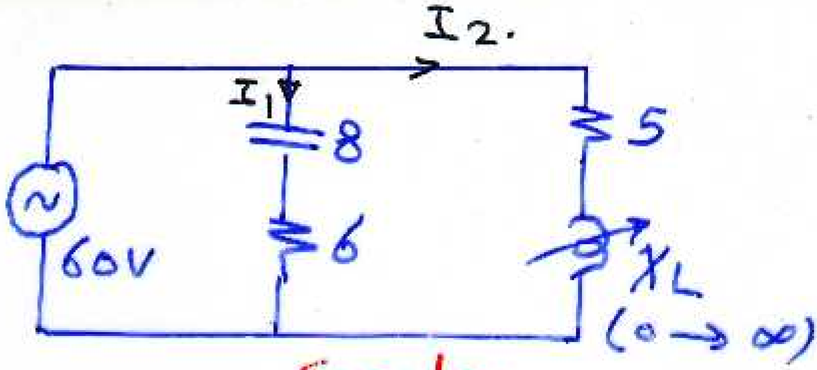
Scale:

1 cm = 0.5 Amp.

1 cm = 2 Volt.



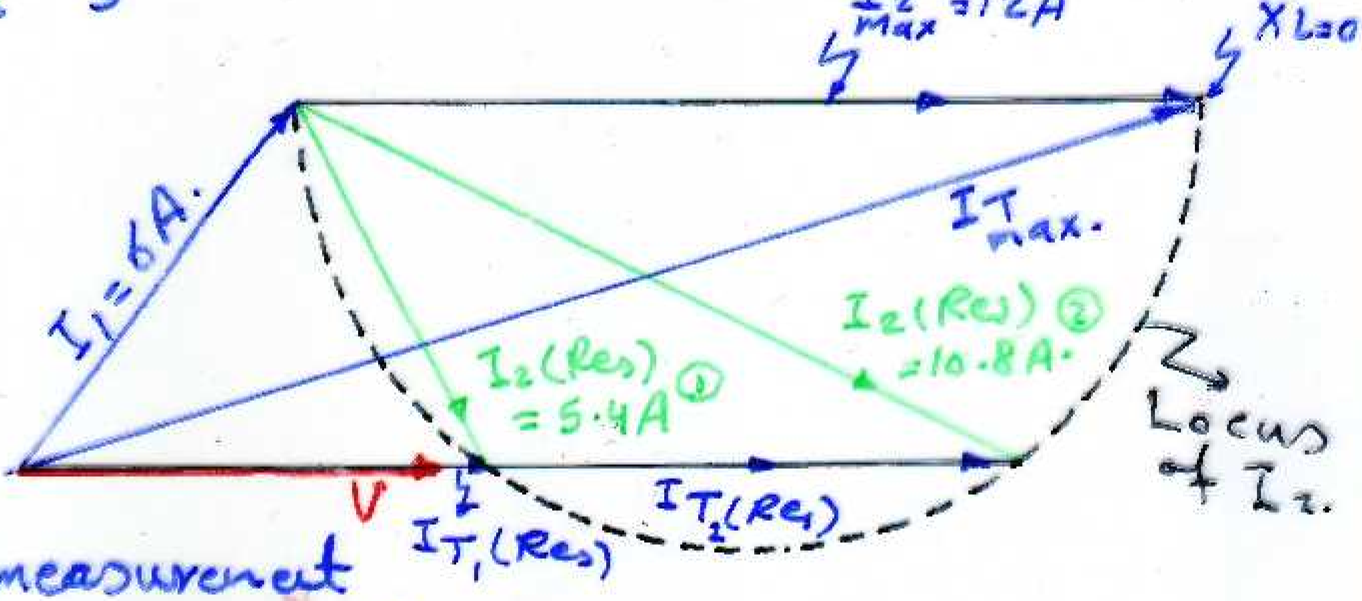
Ex: Draw the Locus diagram, find the X_L at resonance.



$I_1 = \frac{60}{6-j8} = 6 \angle 53.13$ A.

$I_{2\,max} = \frac{60}{5} = 12A$

Scale: 1 cm = 1 Amp.



by measurement

$I_2(Res) = 5.4A \longrightarrow 5.4$ Cm

①

$\therefore Z_{2\,res} = \frac{60}{5.4} = 11.11\Omega$.

$\therefore X_{L2} = \sqrt{(11.11)^2 - 25} = 10\Omega$.

$I_2(Res) = 10.8A \longrightarrow 10.8$ Cm

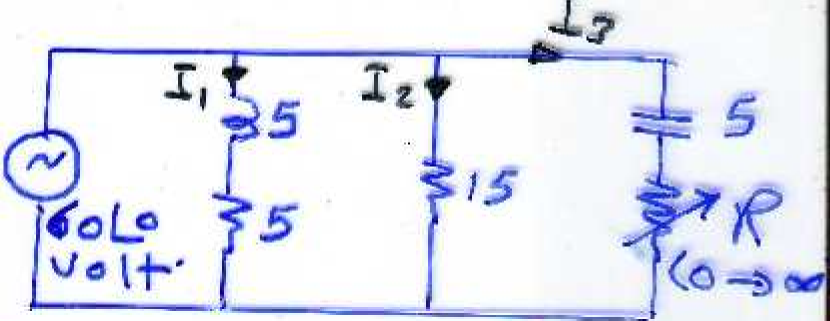
②

$\therefore Z_{2\,res} = \frac{60}{10.8} = 5.55\Omega$.

$\therefore X_{L2} = \sqrt{(5.55)^2 - 25} = 2.4\Omega$.

Check your results

Ex: Draw the Locus diagram and the (R at resonance).

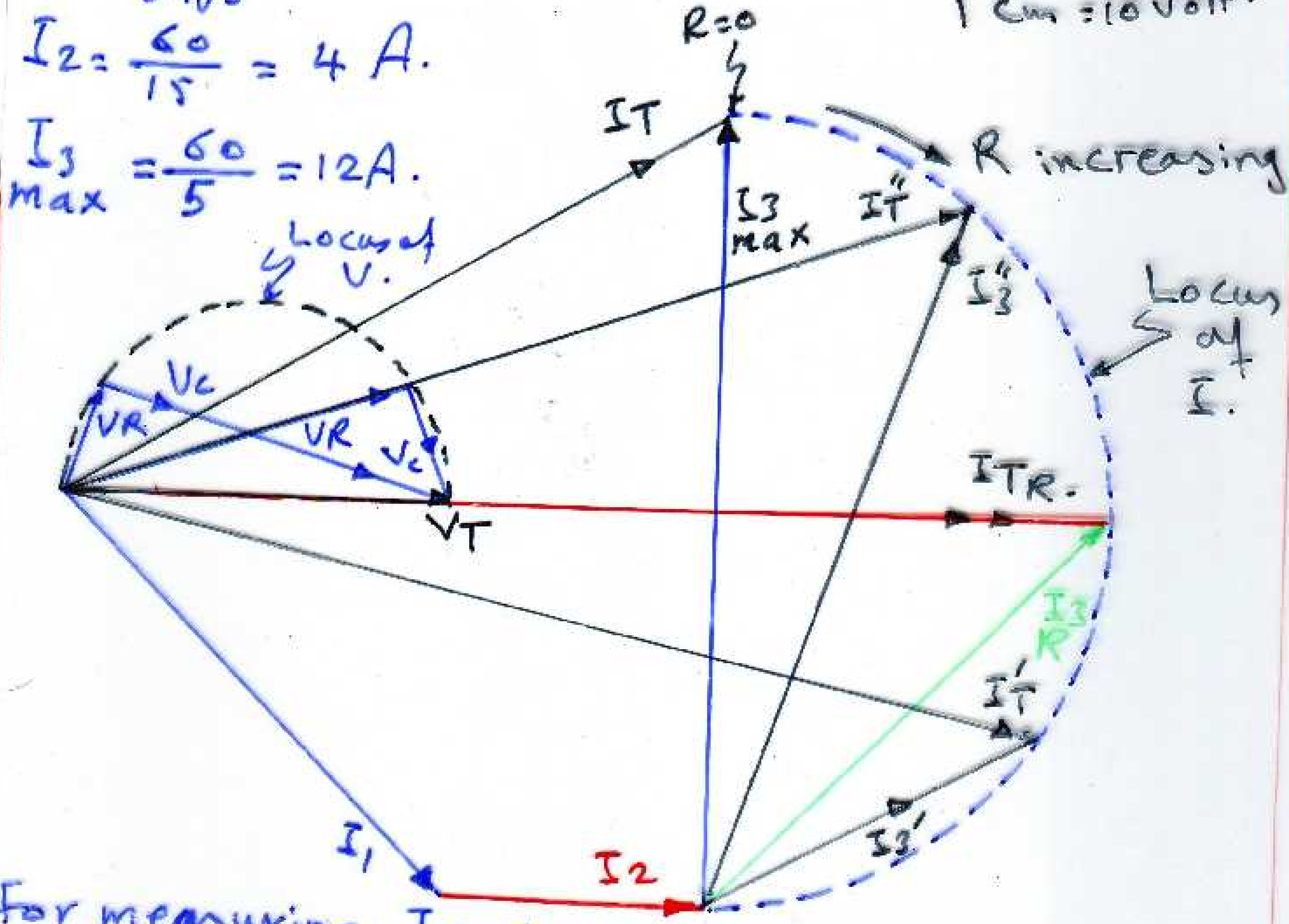


$$I_1 = \frac{60\angle 0}{5+j5} = 8.48\angle{-45}\ A.$$

1 cm = 1 Amp
1 cm = 10 volt.

$$I_2 = \frac{60}{15} = 4\ A.$$

$$I_{3\,max} = \frac{60}{5} = 12\ A.$$



For measuring I_3 at resonance

$$= 8.48\ \text{cm} = 8.48\angle 45\ \text{Amp}.$$

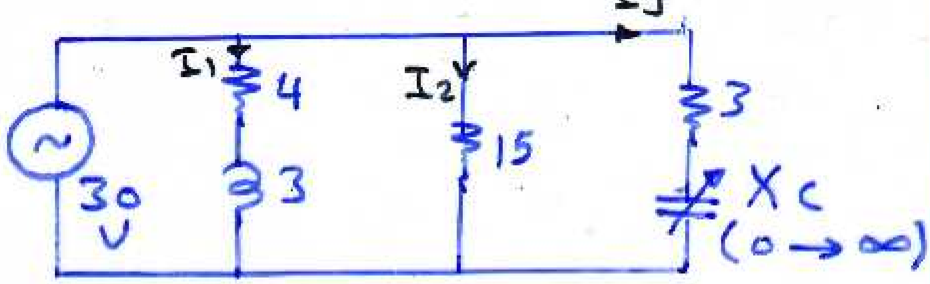
$$\therefore Z_{at\,Res} = \frac{60}{8.48} = 7.07\,\Omega.$$

$$\therefore 7.07 = \sqrt{5^2 + R^2}$$

$$\therefore R = 5\,\Omega$$

check your result.

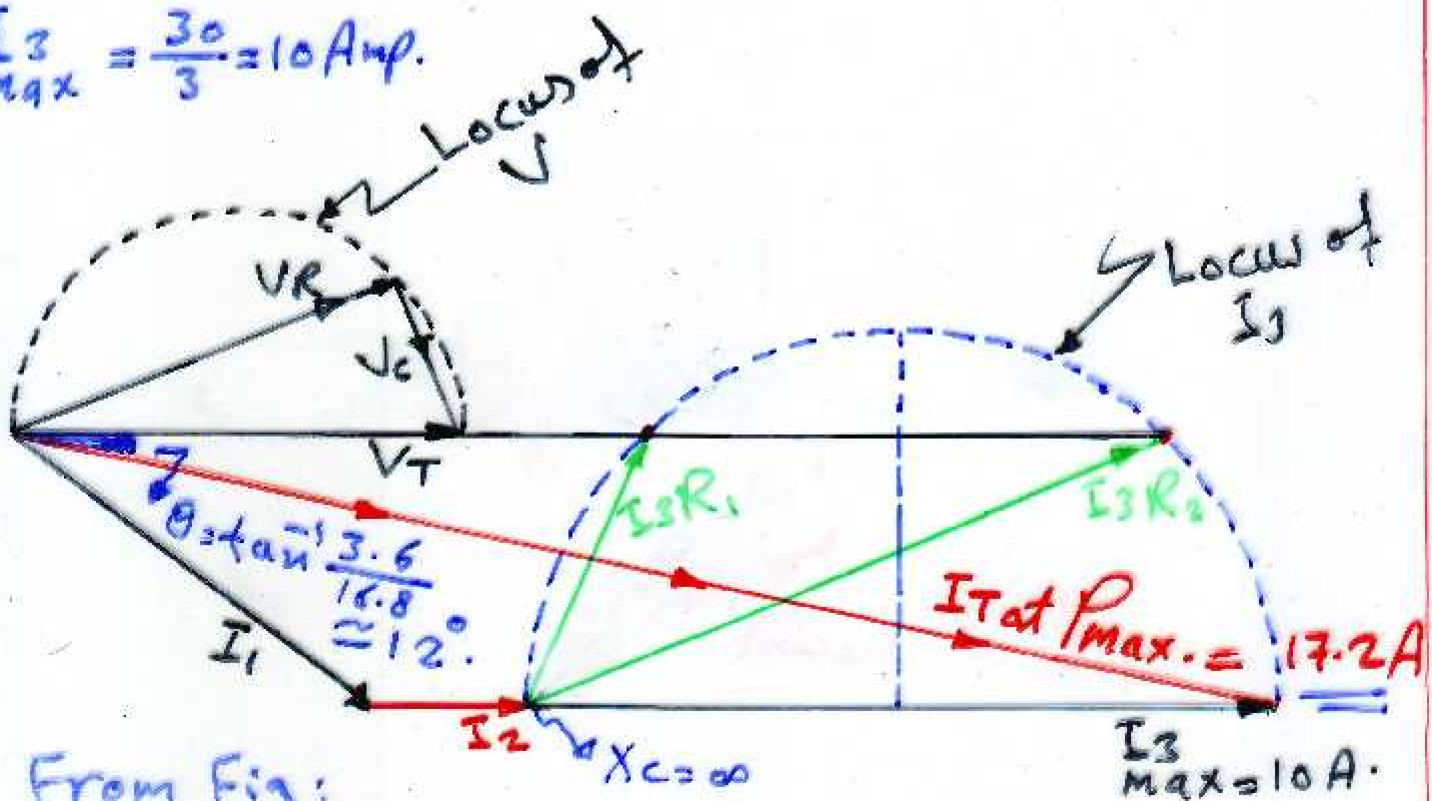
Ex: Find (X_c) at resonance and (X_c) at max. power and the Value of Power max.



$$I_1 = \frac{30}{4+j3} = 6 \angle{-36.86}\ A.$$

$$I_2 = 2\ Amp.$$

$$I_{3\,max} = \frac{30}{3} = 10\ Amp.$$



From Fig:

$I_{3R_1} = 3.9$ Cm $= 3.9A$ $\longrightarrow$ $Z_{3R_1} = \frac{30}{3.9} = 7.7\Omega$.

& $I_{3R_2} = 9.2$ Cm $= 9.2A$ $\longrightarrow$ $Z_{3R_2} = \frac{30}{9.2} = 3.26\Omega$.

$$\therefore 7.7 = \sqrt{3^2 + X_{c_1}^2} \longrightarrow X_{c_1} = 7.08\Omega.$$

$$\& \quad 3.26 = \sqrt{3^2 + X_{c_2}^2} \longrightarrow X_{c_2} = 1.27\Omega.$$

Check the result.

(X_C) at max. power $= 0$.

$$P_{max} = P_{4\Omega} + P_{15\Omega} + P_{3\Omega\,max}\ (X=0).$$

$$= (6)^2 \times 4 + (2)^2 \times 15 + (10)^2 \times 3$$

$$= 144 + 60 + 300 = \underline{504} \text{ watt}.$$

OR $P_{max} = V.I.\cos\theta.$

$$= 30 \times 17.2 \cos(12)$$

$$= \underline{504.7} \text{ watt}.$$

(Same result).

OR $P_{max} = 144 + 60 + V.I_{max}$

$$= 144 + 60 + 30 \times 10 = \underline{504} \text{ watt}.$$