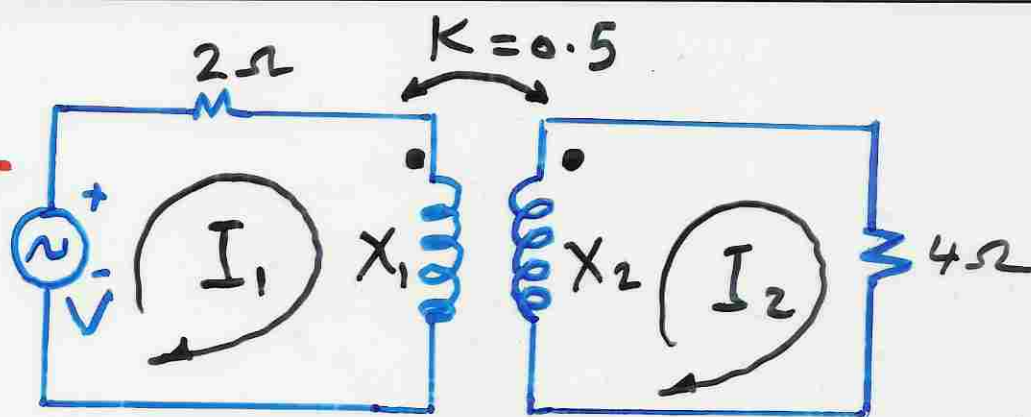


Ex:

$$V = 24 \angle 0^\circ \text{ Volt}$$

$$X_1 = 8 \Omega$$

$$X_2 = 32 \Omega$$



$$X_M = K \sqrt{X_1 \cdot X_2} = 0.5 \sqrt{8 \times 32} = 8 \Omega$$

Find I_1 & I_2 .

$$I_1 = \frac{\begin{vmatrix} 24 & -j8 \\ 0 & 4+j32 \end{vmatrix}}{\begin{vmatrix} 2+j8 & -j8 \\ -j8 & 4+j32 \end{vmatrix}} = \frac{96 + j768}{8 + j64 + j32 - 256 + 64}$$

$$= \underline{3.73 \angle -69.6^\circ \text{ Amp.}}$$

$$I_2 = \frac{\begin{vmatrix} 2+j8 & 24 \\ -j8 & 0 \end{vmatrix}}{\begin{vmatrix} 2+j8 & -j8 \\ -j8 & 4+j32 \end{vmatrix}} = \frac{192 \angle 90^\circ}{207.5 \angle 152.45^\circ}$$

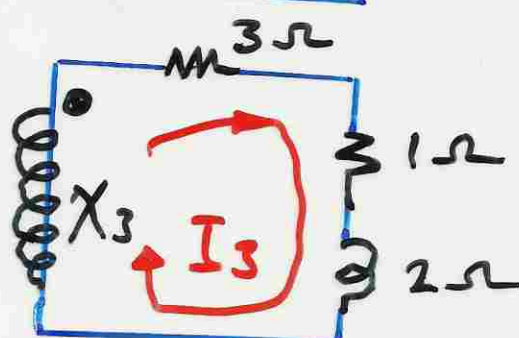
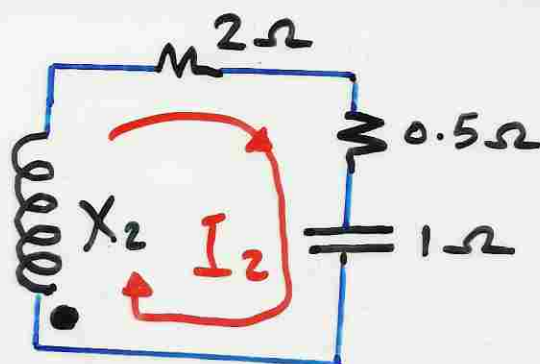
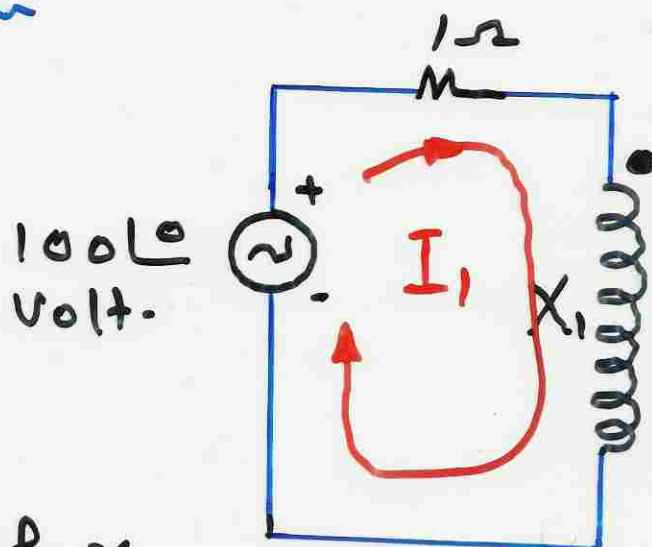
$$= \underline{0.925 \angle -62.45^\circ \text{ Amp.}}$$

By loop

$$24 = (2+j8)I_1 - j8I_2 \quad \text{--- (1)}$$

$$0 = -j8I_1 - (4+j32)I_2 \quad \text{--- (2)}$$

Ex



If $X_1 = 2\Omega$

$$X_2 = 4\Omega$$

$$X_3 = 5\Omega$$

$$X_{12} = 0.5\Omega$$

$$X_{23} = 0.8\Omega$$

$$X_{31} = 0.7\Omega$$

Write the loop currents:

$$100\angle 0 = (1+j2)I_1 + j0.5I_2 - j0.7I_3$$

$$0 = j0.5I_1 + (2.5+j4-j1)I_2 - j0.8I_3$$

$$0 = -j0.7I_1 - j0.8I_2 + (4+j7)I_3$$

Transients in RL & RC Circuit.

Transients are produced when ever:

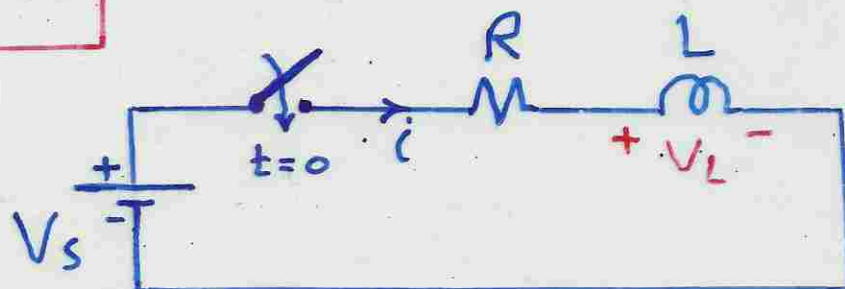
- 1- An apparatus or circuit is suddenly connected to or disconnected from the Supply
- 2- The Circuits are short-circuited.
- 3- There is a sudden change in the applied voltage from one finite value to another.

** Transients take place in electro magnetic circuit (R-L) and electro static (R-C) circuit only.

Transients in R-L circuits

Let i be the resultant current, i_{ss} is the final steady-state current & i_t is the transient current

$$i = i_{ss} + i_t$$



Energy stored in the inductor at time when the switch is closed.

By Kirchhoff's Voltage Law.

$$V_s = Ri + L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{-Ri + V_s}{L} = -\frac{R}{L} \left(i - \frac{V_s}{R} \right).$$

multiply both sides by ((dt)).

$$\left(\frac{di}{dt} \right) dt = -\frac{R}{L} \left(i - \frac{V_s}{R} \right) dt.$$

OR

$$di = -\frac{R}{L} \left(i - \frac{V_s}{R} \right) dt.$$

The Variable can be separated, we get:

$$\frac{di}{i - \frac{V_s}{R}} = -\frac{R}{L} dt \quad \text{----- ①}$$

integrate both sides of eq(1), using X & y as symbols of integration.

$$\int_{I_0}^{i(t)} \frac{dx}{x - \frac{V_s}{R}} = -\frac{R}{L} \int_0^t dy.$$

I_0 = is the current at $t=0$.

$i(t)$ = is the current at any (t) greater than Zero

$$\ln \frac{i(t) - V_s/R}{I_0 - V_s/R} = -\frac{R}{L} t.$$

OR

$$\frac{i(t) - V_s/R}{I_0 - V_s/R} = e^{-R/L t}$$

OR

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-R/L t}$$

OR in general.

$$\underline{i(t) = i_{ss} + A e^{-t/\tau} \quad \text{----- ②}}$$

Where $A = I_0 - i_{s.s} = I_0 - \frac{V_s}{R}$.

and τ is called (Time Constant). $\tau = \frac{L}{R}$.

When the initial energy in the inductor is zero ($I_0 = 0$), then equation (2) becomes:

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-t/\tau}$$

This equation shows that after the switch has been closed ($t=0$) the current increases exponentially from Zero to a final value of (V_s/R) , by the rate of time constant (τ).

$$\text{Since } V_L = L \frac{di}{dt}$$

\therefore The voltage across an inductor is

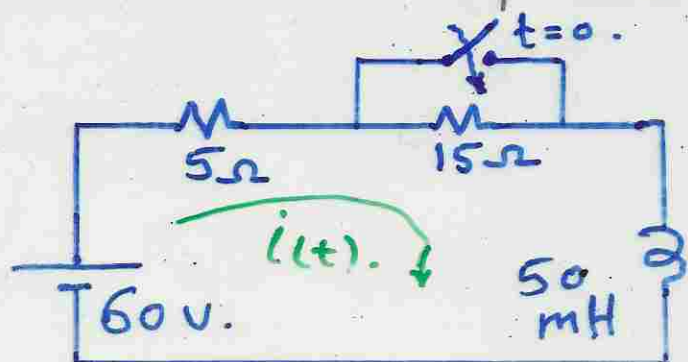
$$V_L = L \left(-\frac{R}{L} \right) \cdot -\frac{V_s}{R} e^{-t/\tau}$$

$$\therefore \underline{V_L = V_s e^{-t/\tau}}$$

Ex: For the circuit shown, the switch is closed at time ($t=0$). Find the current flowing through the circuit as a function of time there after.

$$i = i_{s.s} + A e^{-t/\tau}$$

$$i_{s.s} = \frac{60}{5} = 12 \text{ Amp.}$$



Since $R = 5\Omega$, $L = 50\text{mH}$.

$$\therefore \tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{5} = 1 \times 10^{-2} .$$

$$\therefore i = 12 + A e^{-100t} \quad \text{Amp.}$$

$$\text{at time } t=0 , i_0 = \frac{60}{5+15} = 3 \text{ Amp.}$$

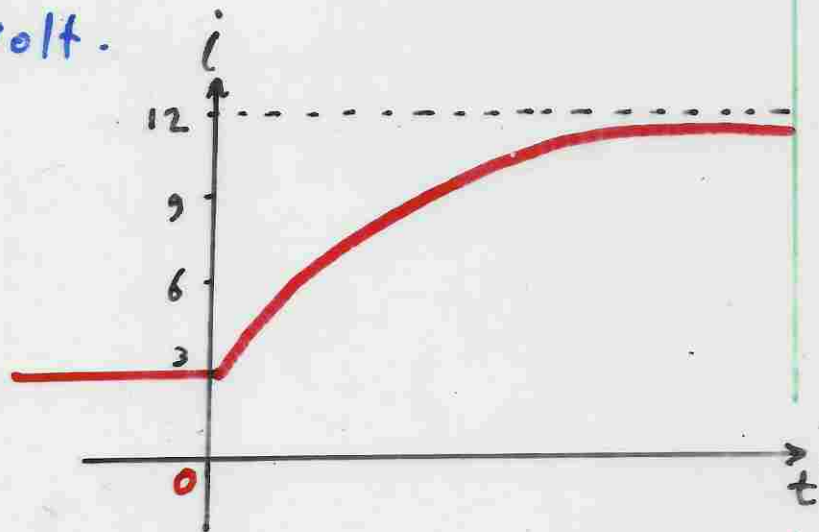
$$\therefore 3 = 12 + A \quad \longrightarrow \quad A = -9 \text{ Amp.}$$

$$\therefore \underline{i = 12 - 9 e^{-100t}} \quad \text{Amp.}$$

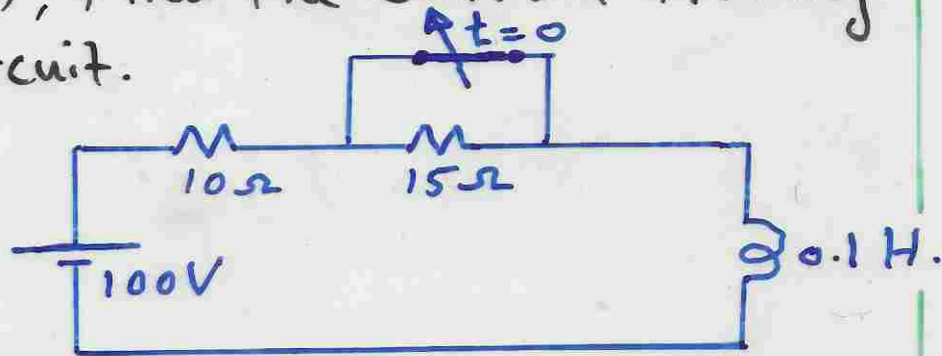
$$V_L = L \frac{di}{dt} .$$

$$= 50 \times 10^{-3} \times -9 \times -100 e^{-100t} .$$

$$= 45 e^{-100t} \quad \text{Volt.}$$



Ex: In the Circuit shown, the Switch is opened at time ($t=0$), Find the current flowing through the Circuit.



$$i = i_{s.s} + A e^{-t/\tau}$$

$$i_{s.s} = \frac{100}{10+15} = 4 \text{ Amp.}$$

$$R = 25 \Omega, \quad L = 0.1 \text{ H.}$$

$$\therefore \tau = \frac{L}{R} = \frac{0.1}{25}$$

$$\therefore i = 4 + A e^{-250t}$$

at time $t=0$, $i_0 = \frac{100}{10} = 10 \text{ Amp.}$

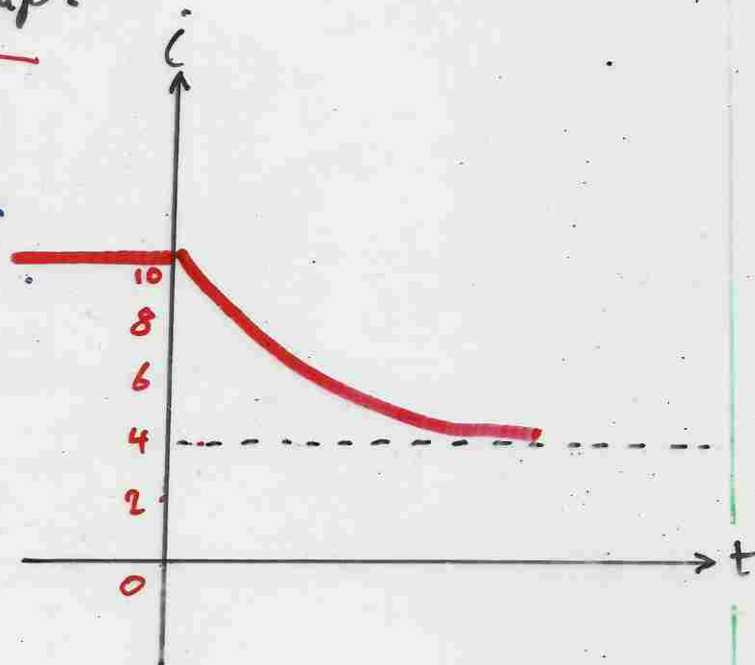
$$\therefore 10 = 4 + A \longrightarrow A = 6 \text{ Amp.}$$

$$\therefore \underline{i = 4 + 6 e^{-250t} \text{ Amp.}}$$

$$V_L = L \frac{di}{dt}$$

$$= 0.1 \times 6 \times -250 e^{-250t}$$

$$= -150 e^{-250t} \text{ Volt.}$$

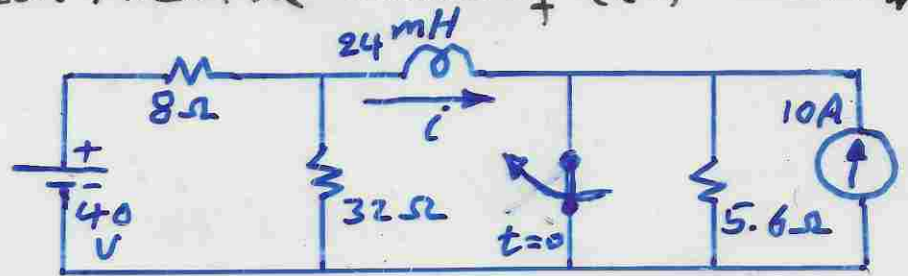


Ex: The switch has been closed for a long time, At $t=0$ the switch is opened. Find the value of $i(t)$ when $t \geq 0$

$$i = i_{ss} + A e^{-t/\tau}$$

$$i_{ss_1} = 10 \frac{5.6}{5.6 + 8 \parallel 32}$$

$$= 4.666 \text{ Amp.}$$



$$i_{ss_2} = \frac{40}{8 + 32 \parallel 5.6} \times \frac{32}{32 + 5.6} = 2.666 \text{ Amp.}$$

$$\therefore i_{ss} = i_{ss_2} - i_{ss_1} = \underline{-2 \text{ Amp.}}$$

$$R = 8 \parallel 32 + 5.6 = 12 \Omega, \quad L = 24 \text{ mH.}$$

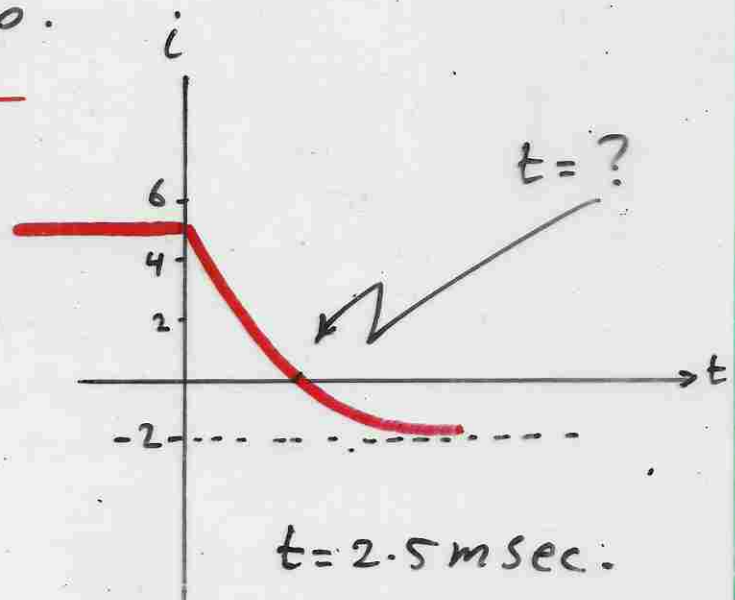
$$\therefore \tau = \frac{L}{R} = \frac{24 \times 10^{-3}}{12} = 2 \times 10^{-3}$$

$$\therefore i = -2 + A e^{-t/\tau} = -2 + A e^{-500t}$$

at time $t=0$ $i_0 = \frac{40}{8} = \underline{5 \text{ Amp.}}$

$$\therefore 5 = -2 + A \longrightarrow A = 7 \text{ Amp.}$$

$$\therefore \underline{i = -2 + 7 e^{-500t} \text{ Amp.}}$$

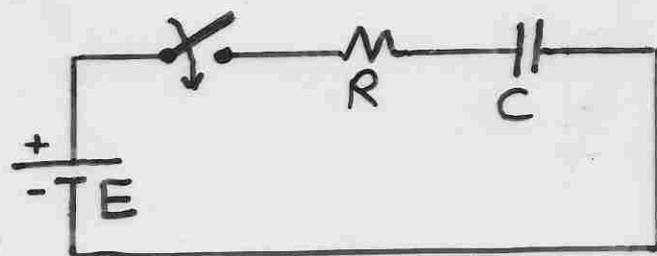


Transients in R-C Circuits :

Let (V_c) be the resultant voltage across the capacitor, $(V_{c.s.s})$ is the steady-state voltage and (V_t) is the transient voltage.

$$\therefore V_c = V_{c.s.s} + V_t$$

$$\underline{V_c = V_{c.s.s} + A e^{-t/\tau}}$$



$$\tau = \text{Time Constant} = R.C.$$

and $V_{c.s.s} = E$ at $t > 0$.

$$\therefore V_c = E + A e^{-t/Rc}$$

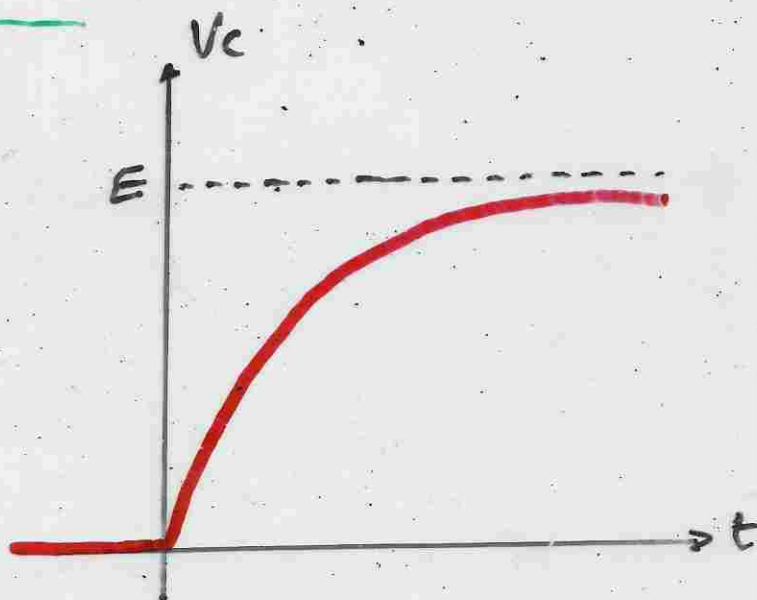
at time $t=0$ $V_c(0) = 0$

$$\therefore 0 = E + A \rightarrow A = -E \text{ volt.}$$

$$\therefore \underline{V_c = E - E e^{-t/Rc}}$$

Since $i = C \frac{dv}{dt}$

$$\therefore \underline{i = \frac{E}{R} e^{-t/Rc}}$$

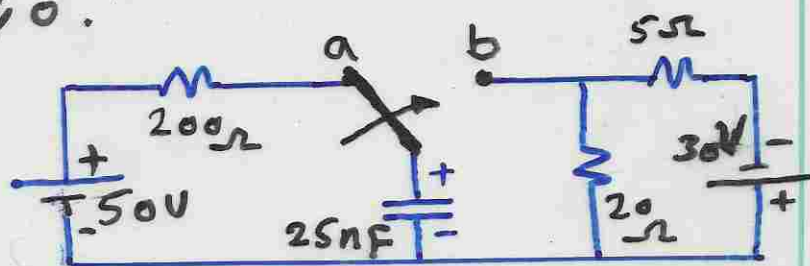


Ex: The switch in the circuit has been in position (a) for a long time. At time $t=0$, the switch is moved to position (b).

1. Find $V_c(t)$ for $t \geq 0$.

2. " $i(t)$ for $t \geq 0$.

$$V_c = V_{c,ss} + A e^{-t/\tau}$$



$$V_{c,ss} = -\frac{30}{20+5} \times 20 = -24 \text{ Volt.}$$

$$\tau = R.C$$

$$R = 20 \parallel 5 = 4 \Omega \quad \therefore \tau = 4 \times 25 \text{ nF} = 100 \times 10^{-9}$$

$$\therefore V_c = -24 + A e^{-10^7 t} \text{ volt.}$$

$$\text{at time } t=0 \quad V_c(0) = 50 \text{ Volt.}$$

$$\therefore 50 = -24 + A \rightarrow A = 74 \text{ Volt.}$$

$$\therefore \underline{V_c = -24 + 74 e^{-10^7 t}}$$

$$i = C \frac{dV_c}{dt}$$

$$= 25 \times 10^{-9} \times 74 \times (-10^7) e^{-10^7 t}$$

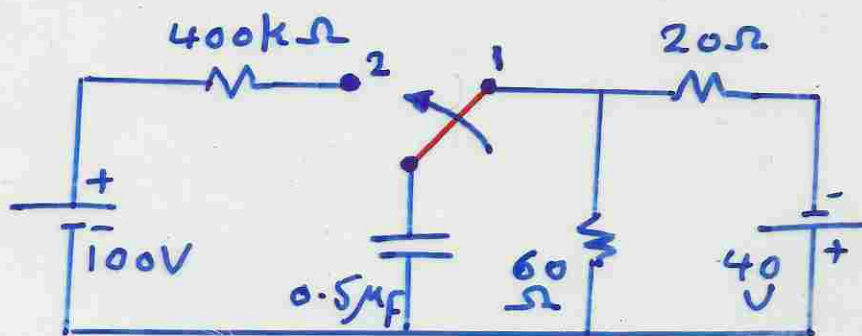
$$= \underline{-18.5 e^{-10^7 t}} \text{ Amp.}$$

Ex: The switch in the circuit shown has been in position (1) for a long time. At time $t=0$, the switch is moved to (2).

a- Find $V_C(t)$ & $i(t)$ for $t \geq 0$.

b- How Long after the switch is in position (2) the capacitor voltage passing through Zero

c- plot $V_C(t)$ & $i(t)$.



a) $V_C = V_{C_{ss}} + A e^{-t/\tau}$
 $V_{C_{ss}} = 100 \text{ V}$

$$\tau = R \cdot C = 400 \times 10^3 \times 0.5 \times 10^{-6} = 0.2$$

$$\therefore V_C = 100 + A e^{-5t}$$

at time $t=0$, $V_C(0) = -\frac{40}{60+20} \cdot 60 = -30 \text{ V}$.

$$\therefore -30 = 100 + A \longrightarrow A = -130 \text{ Volt}$$

$$\therefore \underline{V_C = 100 - 130 e^{-5t} \text{ Volt}}$$

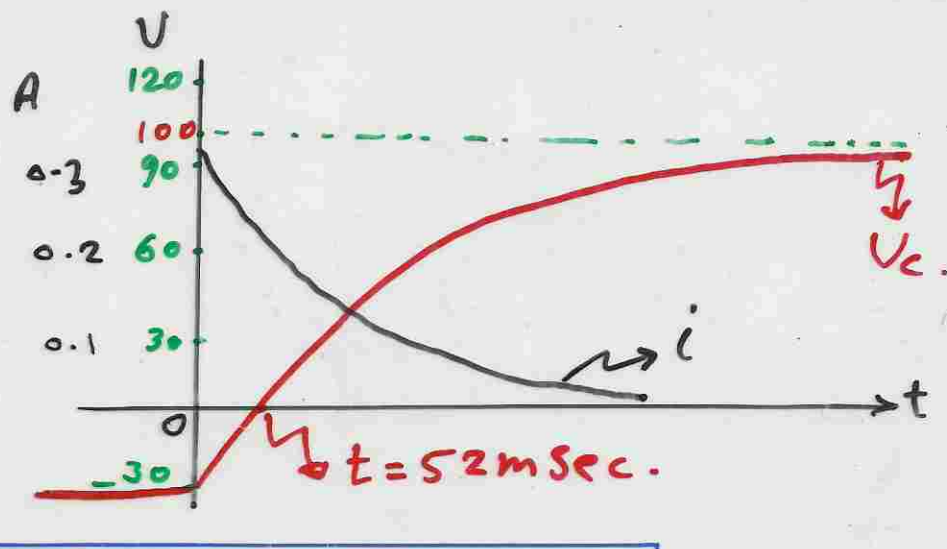
$$i = C \frac{dV_C}{dt} = 0.5 \times 10^{-6} \times 130 \times 5 e^{-5t}$$

$$= \underline{0.325 e^{-5t} \text{ mAmp}}$$

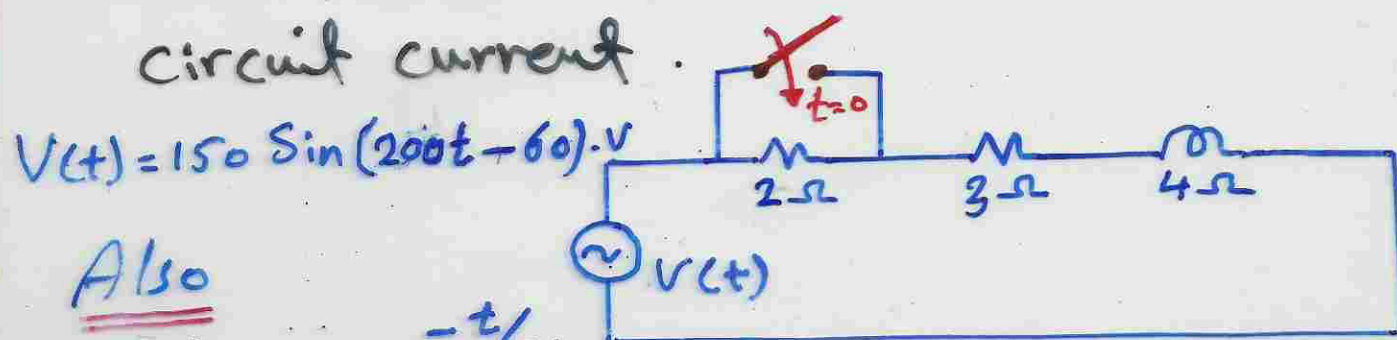
b) $0 = 100 - 130 e^{-5t} \quad \therefore 100 = 130 e^{-5t}$

$$\ln \frac{100}{130} = -5t \quad \therefore -0.262 = -5t$$

$$\therefore t = 0.052 \text{ sec} \quad \therefore \underline{t = 52 \text{ msec}}$$



Ex: For the A-c circuit shown, find the circuit current.



Also

$$i = i_{ss} + A e^{-t/\tau}$$

$$I_{mss} = \frac{150 \angle -60}{3 + j4} = 30 \angle -113.13 \text{ Amp.}$$

$$\therefore i_{ss} = 30 \sin(200t - 113.13) \text{ Amp.}$$

$$\frac{R}{L} = \frac{R \cdot \omega}{\omega L} = \frac{3 \times 200}{4} = 150.$$

$$\therefore i = 30 \sin(200t - 113.13) + A e^{-150t}$$

At time $t = 0$ $i(0) =$

$$I_m = \frac{150 \angle -60}{5 + j4} = 23.42 \angle -98.66 \text{ Amp.}$$

$$\therefore i_{BT} = 23.42 \sin(200t - 98.66).$$

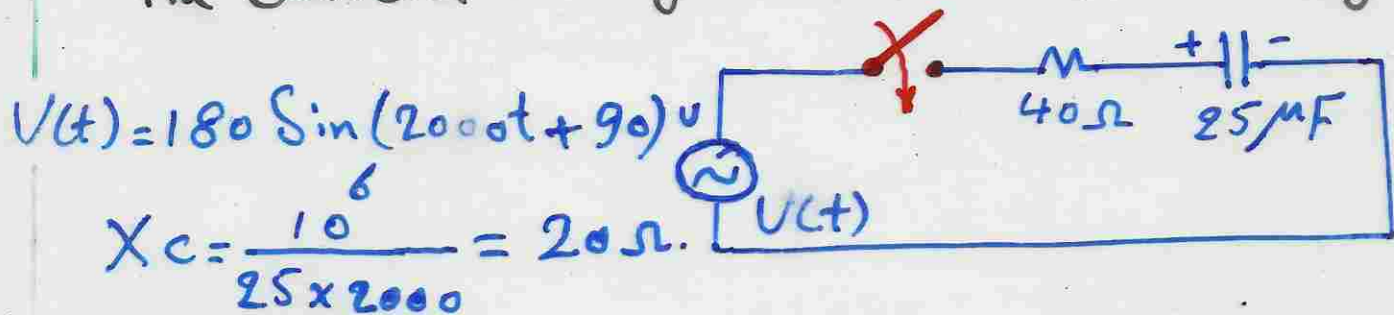
$$\therefore i(0) = 23.42 \sin(-98.66) = -23.15 \text{ Amp.}$$

$$\therefore -23.15 = 30 \sin(-113.13) + A$$

$$\therefore A = 4.438 \text{ Amp.}$$

$$\therefore i = 30 \sin(200t - 113.13) + 4.438 e^{-150t} \text{ Amp.}$$

Ex: For the Circuit Shown, the Capacitor had a charge of $q = 1250 \times 10^{-6}$ coulomb. Find the current through the circuit after closing switch.



$$I_{m,ss} = \frac{180 \angle 90^\circ}{40 - j20} = 4.02 \angle 116.5^\circ \text{ Amp.}$$

$$\therefore i_{ss} = 4.02 \sin(2000t + 116.5^\circ) \text{ Amp.}$$

$$\therefore V_{c,ss} = 80.4 \sin(2000t + 26.5^\circ) \text{ volt.}$$

$$\therefore V_C = 80.4 \sin(2000t + 26.5^\circ) + A e^{-t/\tau}$$

$$\tau = R.C = 40 \times 25 \times 10^{-6} = 10^{-3}$$

$$V_C = \frac{q}{C} = \frac{1250 \times 10^{-6}}{25 \times 10^{-6}} = 50 \text{ Volt.}$$

$$\therefore \text{at time } t=0 \quad V_C(0) = 50 \text{ volt.}$$

$$\therefore 50 = 80.4 \sin 26.5^\circ + A \rightarrow A = 14 \text{ volt.}$$

$$\therefore V_C = 80.4 \sin(2000t + 26.5^\circ) + 14 e^{-1000t} \text{ V}$$

$$\text{But } i = C \frac{dV_C}{dt}$$

$$\therefore \dot{i} = 25 \times 10^{-6} \left[80.4 \times 2000 \cos(2000t + 26.5) - 14000 e^{-1000t} \right]$$

$$\therefore \dot{i} = \underline{4.02 \cos(2000t + 26.5) - 0.35 e^{-1000t}} \quad \text{Amp}$$

OR

$$\dot{i} = \dot{i}_{ss} + A e^{-t/\tau}$$

$$\dot{i}_{ss} = 4.02 \sin(2000t + 116.5).$$

$$\therefore \dot{i} = 4.02 \sin(2000t + 116.5) + A e^{-1000t}$$

at time $t=0$ $\dot{i}(0)$.

$$V_T(0) = R \dot{i}(0) + V_C(0) \longrightarrow \text{K.V.L.}$$

$$180 = 40 \cdot \dot{i}(0) + 50$$

$$\therefore \dot{i}(0) = 3.25 \text{ Amp.}$$

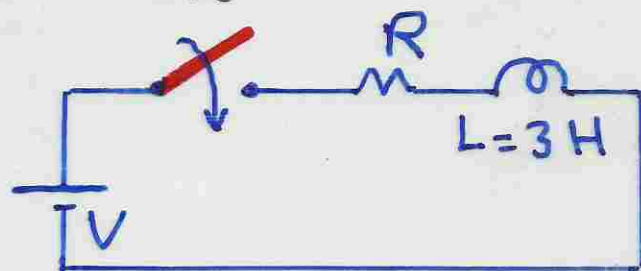
$$\therefore 3.25 = 4.02 \sin 116.5 + A$$

$$\therefore A = 3.25 - 3.6 = -0.35 \text{ Amp.}$$

$$\therefore \dot{i} = \underline{4.02 \sin(2000t + 116.5) - 0.35 e^{-1000t}} \quad \text{Amp.}$$

Ex: For the Circuit, When the switch was closed at time $t=0$, the voltage across the coil is (50V). and then dropped to (15V), after a time period of 0.03 Sec. Find : a- The resistance (R). b- The current i after 0.015 Sec. from switch closure.

We have:



$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$V_L = L \frac{di}{dt} = V e^{-\frac{R}{L}t}$$

at time $t=0$, $V_L(0) = V = 50V$.

$\therefore V = 50$ volt.

at time $t = 0.03$ Sec.

$$15 = 50 e^{-\frac{R}{3} \times 0.03}$$

$$\ln \frac{15}{50} = -\frac{R}{3} \times 0.03$$

$$-1.2 = -0.01 R$$

$$\therefore R = \underline{\underline{120 \Omega}}$$

$$\therefore i = \frac{50}{120} - \frac{50}{120} e^{-40t}$$

$$= 0.42 - 0.42 e^{-40t}$$

at time $t = 0.015$ Sec.

$$i = 0.42 - 0.42 e^{-40 \times 0.015}$$

$$= 0.42 - 0.23 = \underline{\underline{0.19 \text{ Amp.}}}$$

OR $i = \underline{\underline{190 \text{ mA}}}$.

"The use of Laplace Transform in Transient analysis"

The Laplace transform of a function $f(t)$ is given by the expression :-

$$\underline{\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt. \quad \text{----- (1)}}$$

Where the symbol $\mathcal{L}\{f(t)\}$ is read as:

The Laplace transform of $f(t)$.

The Laplace transform of $f(t)$ is also denoted by the notation $F(s)$.

$$F(s) = \mathcal{L}\{f(t)\}.$$

This equation emphasizes that once the integral in eq(1) has been evaluated, the resulting expression is a function of s .

** In transient analysis, (t) represents the time domain and since the exponent of (e) in the integral of eq(1) must be dimensionless, (s) must have the dimension of reciprocal time or frequency.

** The Laplace transform - transforms the problem from the time domain to the frequency domain.

Laplace transform of Some Simple function

* Let $f(t) = \underline{A}$ where A is a Constant.

$$\therefore \mathcal{L}\{A\} = \int_0^{\infty} A e^{-st} dt = -\frac{A}{s} e^{-st} \Big|_0^{\infty} = \underline{\frac{A}{s}}$$

* Let $f(t) = \underline{e^{-at}}$.

$$\begin{aligned} \therefore \mathcal{L}\{e^{-at}\} &= \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt \\ &= -\frac{1}{s+a} e^{-(s+a)t} \Big|_0^{\infty} = \underline{\frac{1}{s+a}} \end{aligned}$$

Similar procedure is used with other functions.

« An Abbreviated table of Laplace transform Pairs »

$f(t)$	$F(s)$
A	$\frac{A}{s}$
t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
$t e^{-at}$	$\frac{1}{(s+a)^2}$
$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$

« Some Laplace transform Operations »

* Multiplication by a Constant.

$$\text{if } \mathcal{L}\{f(t)\} = F(s).$$

$$\text{Then } \mathcal{L}\{A f(t)\} = A F(s).$$

* Addition & Subtraction.

$$\text{if } \mathcal{L}\{f_1(t)\} = F_1(s), \quad \mathcal{L}\{f_2(t)\} = F_2(s).$$

$$\mathcal{L}\{f_3(t)\} = F_3(s).$$

$$\text{Then: } \mathcal{L}\{f_1(t) + f_2(t) - f_3(t)\} = F_1(s) + F_2(s) - F_3(s)$$

* Differentiation.

$$\text{if } \mathcal{L}\{f(t)\} = F(s).$$

$$\text{Then: } \mathcal{L}\left\{\frac{df(t)}{dt}\right\} = s F(s) - f(0).$$

* Integration.

$$\text{if } \mathcal{L}\{f(t)\} = F(s).$$

$$\text{Then: } \mathcal{L}\left\{\int f(t)\right\} = \frac{F(s)}{s} + \frac{f(0)}{s}.$$

In Circuits with non-Zero initial Conditions the S -domain equivalent Circuits for each element are developed and a complete equivalent Circuit using the S -domain is built.

* Resistance (R).

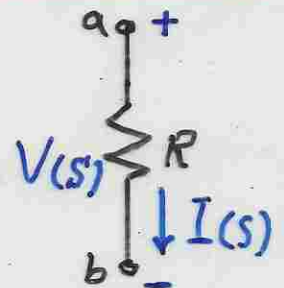
$$V = R i.$$

$$V(s) = R I(s).$$

Time domain



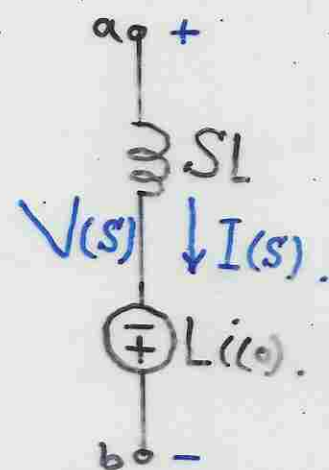
S -domain.



* Inductance (L).

$$V_L = L \frac{di}{dt}.$$

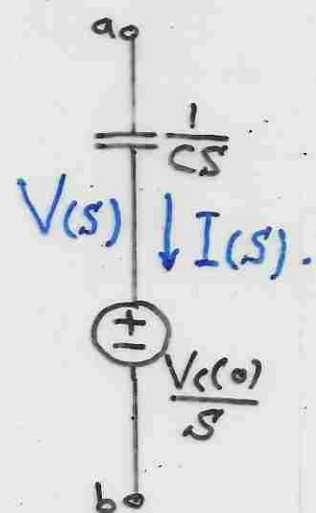
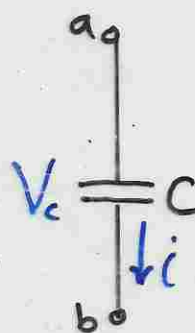
$$V(s) = sL I(s) - L i(0).$$



* Capacitance (C).

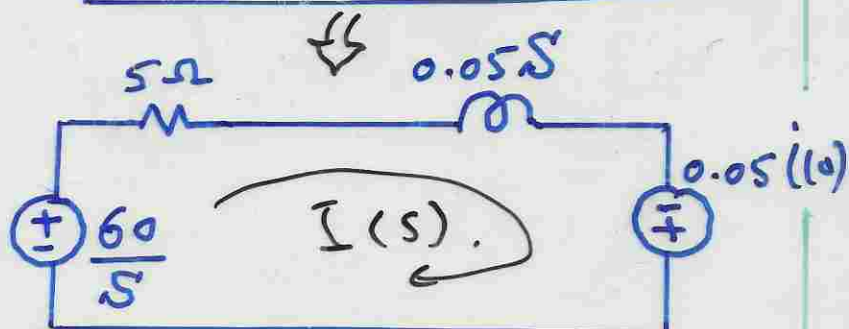
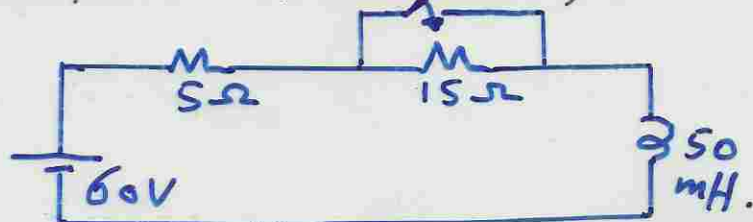
$$V_C = \frac{1}{C} \int i dt.$$

$$V(s) = \frac{I(s)}{sC} + \frac{V_C(0)}{s}$$



Ex: Use the Laplace transform to find $i_L(t)$ & $V_L(t)$ for $t \geq 0$.

$$i(0) = \frac{60}{20} = 3 \text{ A.}$$



$$\frac{60}{s} + 0.05 \times 3 = I(s) (5 + 0.05s)$$

$$\therefore \frac{60 + 0.15s}{s} = I(s) (5 + 0.05s)$$

$$\therefore I(s) = \frac{60 + 0.15s}{s(5 + 0.05s)} = \frac{1200 + 3s}{s(100 + s)}$$

$$= \frac{1200}{s(100 + s)} + \frac{3}{(100 + s)}$$

$$\therefore i(t) = \mathcal{L}^{-1} I(s) = \frac{1200}{100} (1 - e^{-100t}) + 3e^{-100t}$$

$$= 12 - 12e^{-100t} + 3e^{-100t} = \underline{12 - 9e^{-100t} \text{ Amp.}}$$

$$V_L(s) = sL I(s) - L i(0)$$

$$= 0.05s \left(\frac{1200 + 3s}{s(100 + s)} \right) - 0.15$$

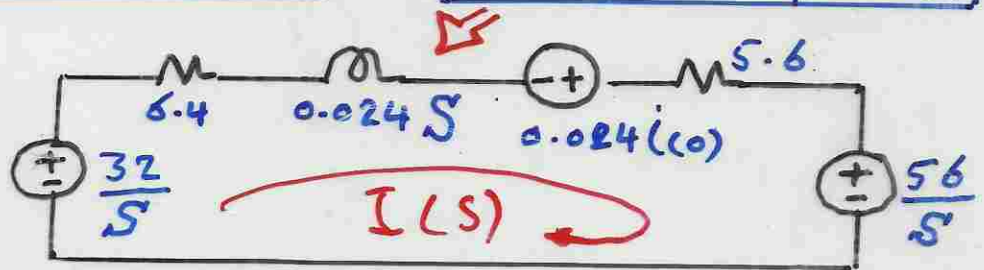
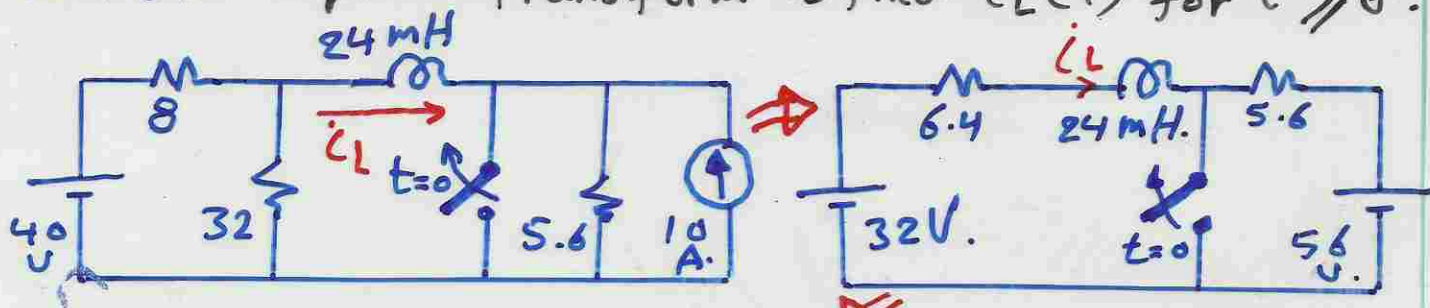
$$= \frac{60 + 0.15s}{100 + s} - 0.15 = \frac{60 + 0.15s - 15 - 0.15s}{100 + s}$$

$$= \frac{45}{100 + s}$$

$$\therefore V_L(t) = \mathcal{L}^{-1} V_L(s) = \underline{45e^{-100t} \text{ V.}}$$

$$\text{OR: } V_L = L \frac{di}{dt} = 45e^{-100t} \text{ V.}$$

Ex: Use Laplace transform to find $i_L(t)$ for $t \geq 0$.



$$i_L(0) = \frac{32}{6.4} = 5 \text{ Amp.}$$

$$\frac{32}{s} + 0.12 - \frac{56}{s} = I(s) (12 + 0.024 s).$$

$$\frac{-24 + 0.12 s}{s} = I(s) (12 + 0.024 s).$$

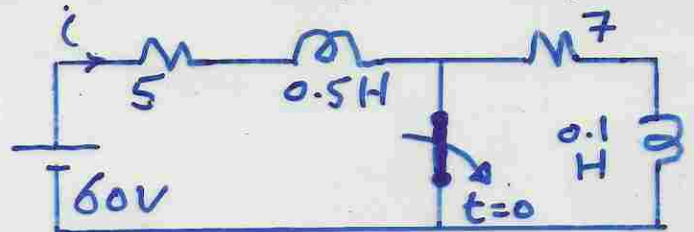
$$\therefore I(s) = \frac{-24 + 0.12 s}{s (12 + 0.024 s)}.$$

$$= \frac{-1000 + 5 s}{s (500 + s)}.$$

$$= \frac{-1000}{s (500 + s)} + \frac{5}{(500 + s)}.$$

$$\begin{aligned} \therefore i_L(t) &= \mathcal{L}^{-1} I(s) = \frac{-1000}{500} (1 - e^{-s00t}) + 5 e^{-s00t} \\ &= -2 + 2 e^{-s00t} + 5 e^{-s00t} \\ &= \underline{-2 + 7 e^{-s00t}} \text{ Amp.} \end{aligned}$$

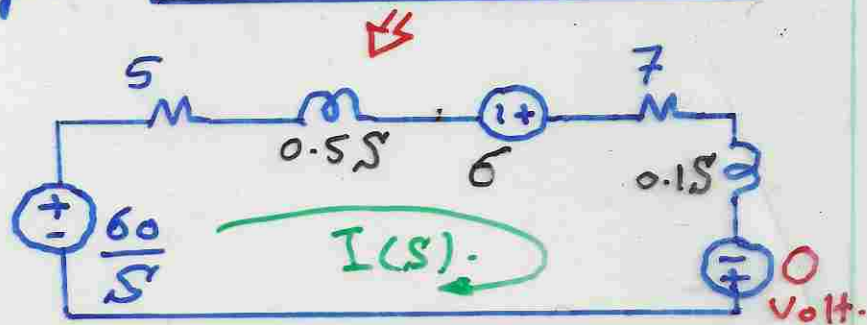
Ex: Use the Laplace transform to find $i(t)$ at $t > 0$.



$$i(0) = \frac{60}{5} = 12 \text{ Amp.}$$

$$i_{L_1}(0) = 12 \text{ Amp.}$$

$$i_{L_2}(0) = 0.$$



$$\frac{60}{s} + 6 = I(s) (12 + 0.6s).$$

$$\frac{60 + 6s}{s} = I(s) (12 + 0.6s).$$

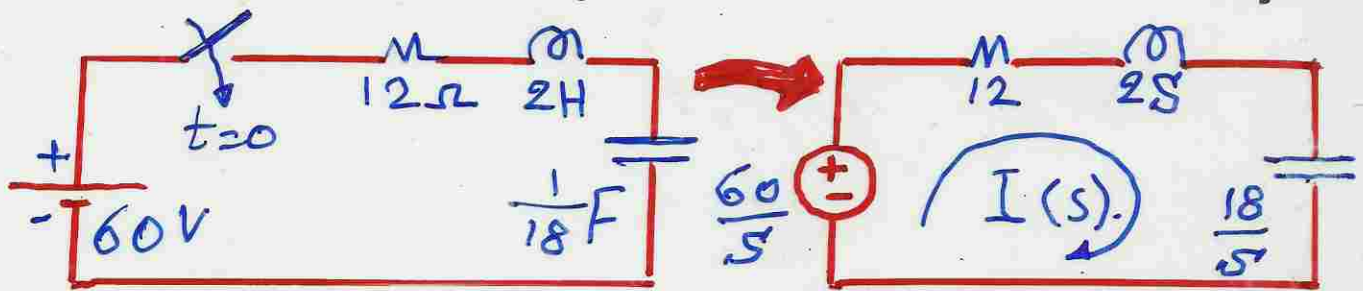
$$\therefore I(s) = \frac{60 + 6s}{s(12 + 0.6s)} = \frac{100 + 10s}{s(20 + s)}.$$

$$= \frac{100}{s(20 + s)} + \frac{10}{(20 + s)}.$$

$$\therefore i(t) = \mathcal{L}^{-1} I(s) = \frac{100}{20} (1 - e^{-20t}) + 10 e^{-20t} \\ = 5 - 5 e^{-20t} + 10 e^{-20t}$$

$$\underline{i(t) = 5 + 5 e^{-20t} \text{ Amp.}}$$

Ex: Use the Laplace transform to find the current through the circuit at $t \geq 0$?



$$I(s) = \frac{V(s)}{Z(s)}$$

$$= \frac{60/s}{12 + 2s + 18/s} = \frac{60/s}{\frac{12s^2 + 2s^3 + 18}{s}}$$

$$= \frac{60}{2s^2 + 12s + 18}$$

$$= \frac{30}{s^2 + 6s + 9}$$

$$= \frac{30}{(s+3)^2}$$

$$\therefore i(t) = \mathcal{L}^{-1} I(s)$$

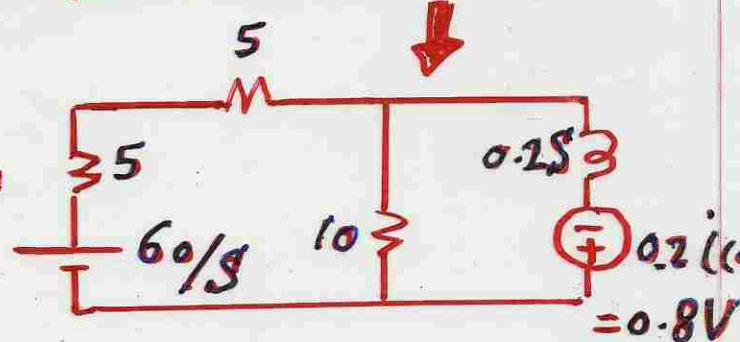
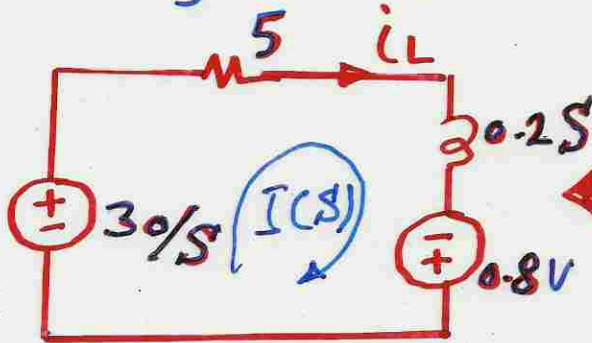
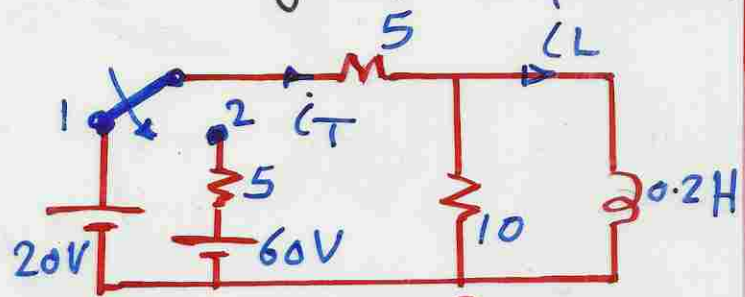
$$= 30te^{-3t} \text{ Amp.}$$

30te^{-3t}

EX: Find i_L & V_L & i_T by using Laplace Transf.

at $t=0$

$$i_L(0) = \frac{20}{5} = 4 \text{ Amp.}$$



$$I_L(s) = \frac{V(s)}{Z(s)} = \frac{30/s + 0.8}{5 + 0.2s} = \frac{30 + 0.8s}{s(5 + 0.2s)}$$

$$= \frac{150 + 4s}{s(25 + s)} = \frac{150}{s(25 + s)} + \frac{4}{(25 + s)}$$

$$\therefore i_L = \mathcal{L}^{-1} I(s) = \frac{150}{25} (1 - e^{-25t}) + 4e^{-25t} \text{ Amp.}$$

$$= 6 - 6e^{-25t} + 4e^{-25t} = \underline{\underline{6 - 2e^{-25t} \text{ A}}}$$

$$\text{and } \underline{\underline{V_L = L \frac{di}{dt} = 0.2(-2)(-25)e^{-25t} = \underline{\underline{10e^{-25t} \text{ V}}}}}$$

OR: $V_L(s) = sL I(s) - L i(0)$

$$= \frac{150 + 4s}{s(25 + s)} \times 0.2s - 0.2 \times 4$$

$$= \frac{30 + 0.8s}{(25 + s)} - 0.8 = \frac{30 + 0.8s - 20 - 0.8s}{(25 + s)}$$

$$\therefore V_L(s) = \frac{10}{25 + s}$$

$$\therefore V_L(t) = \mathcal{L}^{-1} V_L(s) = \underline{\underline{10e^{-25t} \text{ Volt}}}$$

$$\text{Since } V_L(s) = \frac{10}{25+s} = V_{10\Omega}.$$

$$\therefore i_{10\Omega} = \frac{1}{25+s} \text{ Amp.}$$

$$\therefore i_T = i_L(s) + i_{10}(s).$$

$$= \frac{150+4s}{s(25+s)} + \frac{1}{25+s}.$$

$$= \frac{150+4s+s}{s(25+s)} = \frac{150+5s}{s(25+s)}.$$

$$\therefore i_T = \mathcal{L}^{-1}(i_T(s)) = \frac{150}{25} (1 - e^{-25t}) + 5e^{-25t}$$
$$= \underline{\underline{6 - e^{-25t}}} \text{ Amp.}$$