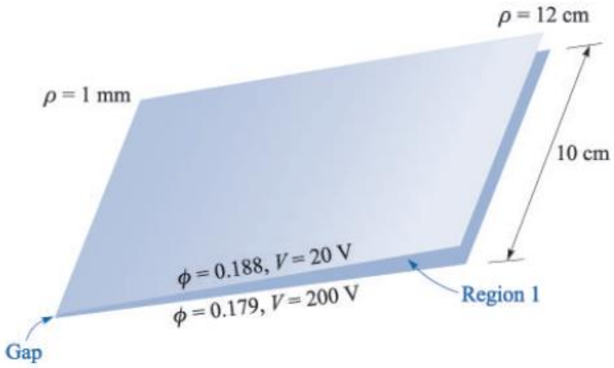
حلول اسئلة الامتحان النهائي للفصل الثاني ٢٠١٦-٢٠١٧ نموذج اول

Note: Answer Only <u>five</u> Questions

Q1) **[12 Marks]**

(a) The two conducting planes illustrated in Figure below are defined by $0.001 < \rho < 0.120$ m, $0 < z < 0.1$ m, $\varphi = 0.179$ and 0.188 rad. The medium surrounding the planes is air. For region 1, $0.179 < \emptyset < 0.188$, neglect fringing and find: (i) $V(\emptyset)$, (ii) $\mathbf{E}(\rho)$, (iii) $\mathbf{D}(\rho)$, (iv) ρ_S.



Solution:

(i) Laplace's equation is now

$$\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \emptyset^2} = 0$$

We exclude $\rho = 0$ and have

$$\frac{d^2 V}{d\emptyset^2} = 0$$

The solution is

$$V = A\emptyset + B$$

and so

$$20 = A(0.188) + B$$
$$200 = A(0.179) + B$$

Subtracting one equation from the other, we find

$$-180 = A(0.188 - 0.179) \ \rightarrow \ A = -20000$$
$$20 = -20000(0.188) + B \ \rightarrow \ B = 3780$$

Finally: $V(\emptyset) = -20000\emptyset + 3780$ V

(ii) $\mathbf{E}(\rho) = -\nabla V = -\frac{1}{\rho}\frac{dV}{d\emptyset}\mathbf{a}_\emptyset = \frac{20000}{\rho}\mathbf{a}_\emptyset$ V/m

(iii) $\mathbf{D}(\rho) = \epsilon_0 \mathbf{E}(\rho) = 8.854 \times 10^{-12} \times \frac{20000}{\rho}\mathbf{a}_\emptyset = \frac{0.177}{\rho}\mathbf{a}_\emptyset \ \mu\text{V/m}$

(iv) $\rho_S = \mathbf{D}\cdot\mathbf{n}|_{\text{surfsce}} = \frac{1.77 \times 10^{-7}}{\rho}\mathbf{a}_\emptyset \cdot \mathbf{a}_\emptyset = \frac{1.77 \times 10^{-7}}{\rho} \ \text{C/m}^2$

(b) Calculate the mutual inductance per unit length between two coaxial solenoids of radius $\rho_1 =$ 1 cm and $\rho_2 = 2$ cm, with $N_1 = 80$ and $N_2 = 100$ turns/cm, respectively. Also, find the inductance per unit length for each coaxial.

Solution:

$$M_{21} = \mu_0 N_1 N_2 \pi \rho_1^2 = M_{12}$$
$$M_{21} = \mu_0 N_1 N_2 \pi \rho_1^2 = 4\pi \times 10^{-7} \times 8000 \times 10000 \times \pi \times 10^{-4} = 31.6 \text{ mH/m}$$
$$L_1 = \mu_0 N_1^2 S_1 = 4\pi \times 10^{-7} \times 8000^2 \times \pi \times 10^{-4} = 25.3 \text{ mH/m}$$
$$L_2 = \mu_0 N_2^2 S_2 = 4\pi \times 10^{-7} \times 10000^2 \times \pi \times 4 \times 10^{-4} = 157.9 \text{ mH/m}$$

Q2) **[12 Marks]**

(a) Two semi-infinite filaments on the z axis lie in the regions $-\infty < z < -1$ and $1 < z < \infty$. Each carries a current I in the $\mathbf{a}_z$ direction. Calculate $\mathbf{H}$ as a function of ρ and $\emptyset$ at $z = 0$.

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

Solution:

One way to do this is to use the field from an infinite line and subtract from it that portion of the field that would arise from the current segment at $-1 < z < 1$, found from the Biot-Savart law. Thus,

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\emptyset - \int_{-1}^{1} \frac{I dz \mathbf{a}_z[\rho \mathbf{a}_\rho - z\mathbf{a}_z]}{4\pi[\rho^2 + z^2]^{3/2}}$$

The integral part simplifies and is evaluated:

$$\int_{-1}^{1} \frac{I dz \mathbf{a}_z[\rho \mathbf{a}_\rho - z\mathbf{a}_z]}{4\pi[\rho^2 + z^2]^{3/2}} = \frac{I\rho}{4\pi}\mathbf{a}_\emptyset \left. \frac{z}{\rho^2\sqrt{\rho^2 + z^2}} \right|_{-1}^{1} = \frac{I}{2\pi\rho\sqrt{\rho^2 + 1}}\mathbf{a}_\emptyset$$

Finally,

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\emptyset - \frac{I}{4\pi\rho\sqrt{\rho^2 + 1}}\mathbf{a}_\emptyset = \frac{I}{2\pi\rho}\left[1 - \frac{1}{\sqrt{\rho^2 + 1}}\right]\mathbf{a}_\emptyset$$

(b) If $\chi_m = 6.95$ for a material and $\mathbf{B} = 10y\mathbf{a}_x + 20x\mathbf{a}_y$ mT. Calculate the following $\mathbf{H}$, μ, μ_r, $\mathbf{M}$, $\mathbf{J}$, $\mathbf{J}_B$, $\mathbf{J}_T$

Solution:

$\mathbf{B} = \mu_0 \mu_r \mathbf{H} = \mu_0(1 + \chi_m)\mathbf{H}$ then

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0(1 + \chi_m)} = \frac{(10y\mathbf{a}_x + 20x\mathbf{a}_y) \times 10^{-3}}{4\pi \times 10^{-7} \times 7.95} = y\mathbf{a}_x + 2x\mathbf{a}_y \text{ kA/m}$$
$$\mu = \mu_0 \mu_r = \mu_0(1 + \chi_m) = 4\pi \times 10^{-7} \times 7.95 = 10\ \mu\text{H/m}$$
$$\mu_r = 1 + \chi_m = 7.95$$
$$\mathbf{M} = \chi_m \mathbf{H} = 6.95 \times (y\mathbf{a}_x + 2x\mathbf{a}_y) = 6.95y\mathbf{a}_x + 13.9x\mathbf{a}_y \text{ kA/m}$$
$$\mathbf{J} = \nabla \times \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = 10^3 \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & 2x & 0 \end{vmatrix} = \mathbf{a}_z \text{ kA/m}^2$$

$$\mathbf{J}_B = \nabla \times \mathbf{M} = \nabla \times (\chi_m \mathbf{H}) = 6.95 \times 10^3 \mathbf{a}_z = 6.95\mathbf{a}_z \text{ kA/m}^2$$

$$\mathbf{J}_T = \nabla \times \frac{\mathbf{B}}{\mu_0} = \frac{1}{\mu_0}\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \frac{10^{-2}}{4\pi \times 10^{-7}}\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & 2x & 0 \end{vmatrix} = 7.95 \mathbf{a}_z \text{ kA/m}^2$$

Or in another way

$$\mathbf{J}_T = \nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J}_T = \nabla \times \frac{\mu_0 \mu_r \mathbf{H}}{\mu_0} = \nabla \times \mu_r \mathbf{H} = \mu_r \mathbf{J} = 7.95 \mathbf{a}_z \text{ kA/m}^2$$

Q3) **[12 Marks]**

(a) The permittivity is 5 μH/m in region 1 where $x < 0$, and 20 μH/m in region 2 where $x > 0$. If there is a surface current density $\mathbf{K} = 150\mathbf{a}_y - 200\mathbf{a}_z$ A/m at $x = 0$, and if $\mathbf{H}_1 = 300\mathbf{a}_x - 400\mathbf{a}_y + 500\mathbf{a}_z$ A/m, find: (i) $\mathbf{B}_1$; (ii) $\mathbf{B}_2$; (iii) $\mathbf{H}_2$.

Solution:

(i)

$$\mathbf{B}_1 = \mu_1 \mathbf{H}_1 = 5 \times 10^{-6}(300\mathbf{a}_x - 400\mathbf{a}_y + 500\mathbf{a}_z) = 1.5\mathbf{a}_x - 2\mathbf{a}_y + 2.5\mathbf{a}_z \text{ mT}$$

(ii)

$$\mathbf{B}_{N1} = (\mathbf{B}_1 \cdot \mathbf{a}_{N12})\mathbf{a}_{N12} = [(1.5\mathbf{a}_x - 2\mathbf{a}_y + 2.5\mathbf{a}_z) \cdot \mathbf{a}_x](\mathbf{a}_x) = 1.5\mathbf{a}_z \text{ mT}$$

$$\mathbf{B}_{N1} = \mathbf{B}_{N2} = 1.5\mathbf{a}_x \text{ mT}$$

We next determine the tangential components:

$$\mathbf{B}_{t1} = \mathbf{B}_1 - \mathbf{B}_{N1} = -2\mathbf{a}_y + 2.5\mathbf{a}_z \text{ mT}$$

$$\mathbf{H}_{t1} = \frac{\mathbf{B}_{t1}}{\mu_1} = \frac{(-2\mathbf{a}_y + 2.5\mathbf{a}_z) \times 10^{-3}}{5 \times 10^{-6}} = -400\mathbf{a}_y + 500\mathbf{a}_z \text{ A/m}$$

Thus,

$$\mathbf{H}_{t2} = \mathbf{H}_{t1} - \mathbf{a}_{N12} \times \mathbf{K} = -400\mathbf{a}_y + 500\mathbf{a}_z - \mathbf{a}_x \times (150\mathbf{a}_y - 200\mathbf{a}_z)$$

$$= -400\mathbf{a}_y + 500\mathbf{a}_z - 150\mathbf{a}_z - 200\mathbf{a}_y = -600\mathbf{a}_y + 350\mathbf{a}_z \text{ A/m}$$

$$\mathbf{B}_{t2} = \mu_2 \mathbf{H}_{t2} = 20 \times 10^{-6}(-600\mathbf{a}_y + 350\mathbf{a}_z) = -12\mathbf{a}_y + 7\mathbf{a}_z \text{ mT}$$

Therefore,

$$\mathbf{B}_2 = \mathbf{B}_{N2} + \mathbf{B}_{t2} = 1.5\mathbf{a}_x - 12\mathbf{a}_y + 7\mathbf{a}_z \text{ mT}$$

(iii)

$$\mathbf{H}_2 = \frac{\mathbf{B}_2}{\mu_2} = \frac{(1.5\mathbf{a}_x - 12\mathbf{a}_y + 7\mathbf{a}_z) \times 10^{-3}}{20 \times 10^{-6}} = 75\mathbf{a}_x - 600\mathbf{a}_y + 350\mathbf{a}_z \text{ A/m}$$

(b) Let $\mu = 10^{-5}$ H/m, $\epsilon = 4 \times 10^{-9}$ F/m, $\sigma = 0$, and $\rho_v = 0$. Find k (including units) so that each of the following pairs of fields satisfies Maxwell's equations: (i) $\mathbf{D} = 6\mathbf{a}_x - 2y\mathbf{a}_y + 2z\mathbf{a}_z$ nC/m², $\mathbf{H} = kx\mathbf{a}_x + 10y\mathbf{a}_y - 25z\mathbf{a}_z$ A/m; (ii) $\mathbf{E} = (20y - kt)\mathbf{a}_x$ V/m, $\mathbf{H} = (y + 2 \times 10^6 t)\mathbf{a}_z$ A/m.

Solution:

(i)

$$\nabla \cdot \mathbf{H} = \rho_v = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = k + 10 - 25$$

but $\rho_v = 0$, then

$k + 10 - 25 = 0 \ \rightarrow k = 15$ A/m² because it has the same unit of ρ_v

(ii) As $\sigma = 0$ then $\mathbf{J} = \sigma\mathbf{E} = 0$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

i.e.

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{H} = \nabla \times \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 20y - kt & 0 & 0 \end{vmatrix} = \mathbf{a}_x$$

$$\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial(\epsilon \mathbf{E})}{\partial t} = \epsilon \frac{\partial \mathbf{E}}{\partial t} = -\epsilon k \mathbf{a}_x$$

Return to Maxwell equation

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

then

$$\mathbf{a}_x = -\epsilon k \mathbf{a}_x$$

Leads to

$$k = -\frac{1}{\epsilon} = -\frac{1}{4 \times 10^{-9}} = -2.5 \times 10^8$$

To find the unit of k return to $\mathbf{E} = (20y - kt)\mathbf{a}_x$ thus kt has the same unit of $\mathbf{E}$ which is V/m. Then unit of k is (V/m)/s or V/(m.s).

Q4) **[12 Marks]**

(a) A current filament carrying 15 A in the $\mathbf{a}_z$ direction lies along the entire z axis. Find $\mathbf{H}$ in rectangular coordinates at $P_B(2, -4, 4)$.

$$\int_{-\infty}^{\infty} \frac{dx}{(ax^2 + bx + c)^{1.5}} = \left[\frac{(4ax + 2b)}{(4ac - b^2)\sqrt{ax^2 + bx + c}}\right]_{-\infty}^{\infty} = \frac{8a}{(4ac - b^2)\sqrt{ax^2 + bx + c}}$$

Solution:

Applying the Biot-Savart Law, we obtain

$$\mathbf{H} = \oint \frac{Id\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \int_{-\infty}^{\infty} \frac{Idz\mathbf{a}_z \times \left[2\mathbf{a}_x - 4\mathbf{a}_y + (4 - z)\mathbf{a}_z\right]}{4\pi(z^2 - 8z + 36)^{3/2}}$$

$$= \int_{-\infty}^{\infty} \frac{Idz\left[4\mathbf{a}_x + 2\mathbf{a}_y\right]}{4\pi(z^2 - 8z + 36)^{3/2}}$$

with $a = 1, b = -8, c = 36$ the integral results

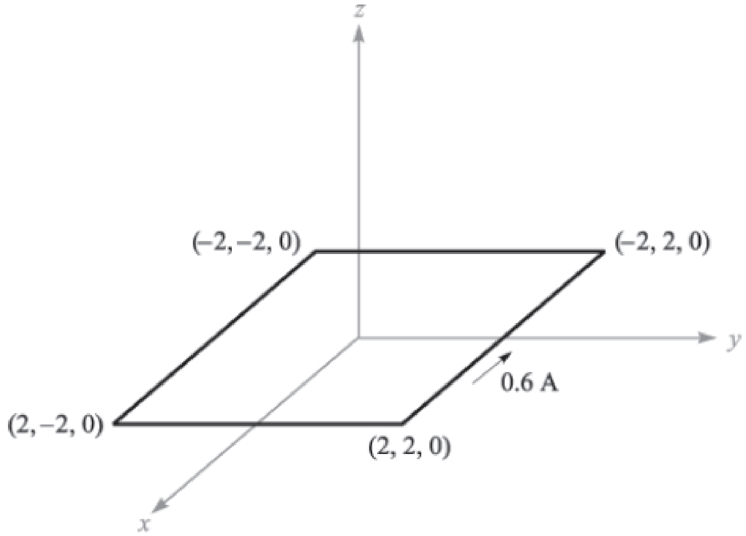
$$\mathbf{H} = \frac{I}{4\pi}\left[\frac{8 \times 1 \times \left(4\mathbf{a}_x + 2\mathbf{a}_y\right)}{(4 \times 1 \times 36 - 64)\sqrt{z^2 - 8z + 36}}\right] = \frac{I}{4\pi}\left[\frac{8\left(4\mathbf{a}_x + 2\mathbf{a}_y\right)}{80\sqrt{z^2 - 8z + 36}}\right] = \frac{I}{40\pi}\left(4\mathbf{a}_x + 2\mathbf{a}_y\right)$$

with I =15 A

$$\mathbf{H} = \frac{15}{40\pi}\left(4\mathbf{a}_x + 2\mathbf{a}_y\right) = \frac{3}{4\pi}\left(2\mathbf{a}_x + \mathbf{a}_y\right) = 0.477\mathbf{a}_x + 0.239\mathbf{a}_y \text{ A/m}$$

(b)

If $\mathbf{B} = x\mathbf{a}_x + y^2\mathbf{a}_y - 2z\mathbf{a}_z$, find the total force on the rectangular loop shown in Figure below.



Solution:

First, note that in the plane $z = 0$, the z component of the given field is zero, so will not contribute to the force.

$$\mathbf{F} = \int_{\text{Loop}} Id\mathbf{L} \times \mathbf{B}$$

With $I = 0.6$

$$\mathbf{F} = \int_{-2}^{2} 0.6dx\mathbf{a}_x \times \left(x\mathbf{a}_x + y^2|_{y=-2}\mathbf{a}_y\right) + \int_{-2}^{2} 0.6dy\mathbf{a}_y \times \left(x|_{x=2}\mathbf{a}_x + y^2\mathbf{a}_y\right)$$

$$+ \int_{2}^{-2} 0.6dx\mathbf{a}_x \times \left(x\mathbf{a}_x + y^2|_{y=2}\mathbf{a}_y\right) + \int_{2}^{-2} 0.6dy\mathbf{a}_y \times \left(x|_{x=-2}\mathbf{a}_x + y^2\mathbf{a}_y\right)$$

$$= \int_{-2}^{2} 0.6(4)dx\mathbf{a}_z + \int_{-2}^{2} 0.6(2)dy(-\mathbf{a}_z) + \int_{2}^{-2} 0.6(4)dx\mathbf{a}_z + \int_{2}^{-2} 0.6(-2)dy(-\mathbf{a}_z)$$

$$= 2.4\mathbf{a}_z \int_{-2}^{2} dx - 1.2\mathbf{a}_z \int_{-2}^{2} dy + 2.4\mathbf{a}_z \int_{2}^{-2} dx + 2.4\mathbf{a}_z \int_{2}^{-2} dy = 9.6\mathbf{a}_z - 4.8\mathbf{a}_z - 9.6\mathbf{a}_z - 4.8\mathbf{a}_z$$

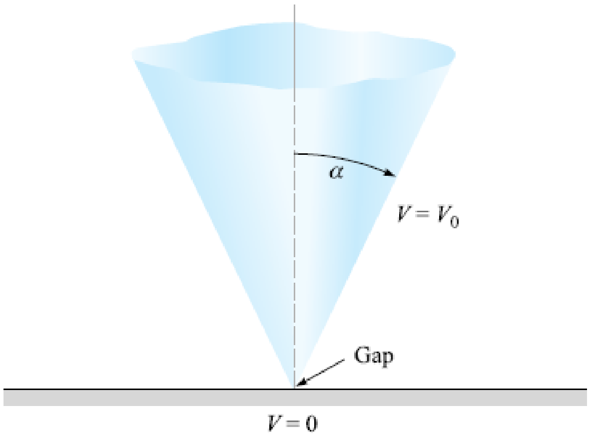
$$= -9.6\mathbf{a}_z \text{ N}$$

Q5) **[12 Marks]**

By using Laplace's equations to derive a formula to calculate the capacitance of the cone shown in the Figure below where $\theta = \alpha$ at V_0 and the plane $\theta = \pi/2$ at $V = 0$.

Use $\left\{\int \frac{d\theta}{\sin\theta} = \ln\left(\tan\frac{\theta}{2}\right) + C\right\}$

Solution:



$$\frac{1}{r^2 \sin\theta} \frac{\mathrm{d}}{d\theta}\left(\sin\theta \frac{dV}{d\theta}\right) = 0$$

We exclude $r = 0$ and $\theta = 0$ or π and have

$$\sin\theta \frac{dV}{d\theta} = A$$

The second integral is then

$$V = \int \frac{A}{\sin\theta} d\theta + B$$

But

$$\int \frac{d\theta}{\sin\theta} = \ln\left(\tan\frac{\theta}{2}\right) + C$$

then

$$V = A\ln\left(\tan\frac{\theta}{2}\right) + B \qquad (1)$$

but $V = 0$ at $\theta = \pi/2$ and $V = V_0$ at $\theta = \alpha$, $\alpha < \pi/2$. We obtain

$$V\left(\theta = \frac{\pi}{2}\right) = 0 \;\text{ and }\; V(\theta = \alpha) = V_0$$

Sub in (1)

$$0 = A\ln\left(\tan\frac{\pi}{4}\right) + B \;\rightarrow\; 0 = A(0) + B \;\rightarrow B = 0$$

$$V_0 = A\ln\left(\tan\frac{\alpha}{2}\right) + B \;\rightarrow\; V_0 = A\ln\left(\tan\frac{\alpha}{2}\right) \;\rightarrow A = \frac{V_0}{\ln\left(\tan\frac{\alpha}{2}\right)}$$

Substituting values of A and B in (1) then,

$$V = V_0 \frac{\ln\left(\tan\frac{\theta}{2}\right)}{\ln\left(\tan\frac{\alpha}{2}\right)}$$

$$\mathbf{E} = -\nabla V = -\frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta = -\frac{1}{r}\frac{dV}{d\theta}\mathbf{a}_\theta$$

$$= -\frac{V_0}{r\ln\left(\tan\frac{\alpha}{2}\right)}\frac{d\left[\ln\left(\tan\frac{\theta}{2}\right)\right]}{d\theta}\mathbf{a}_\theta = -\frac{V_0}{r\ln\left(\tan\frac{\alpha}{2}\right)}\cdot\frac{1}{\tan\frac{\theta}{2}}\cdot\left(\sec^2\frac{\theta}{2}\right)\left(\frac{1}{2}\right)\mathbf{a}_\theta$$

$$= -\frac{V_0}{r\ln\left(\tan\frac{\alpha}{2}\right)}\cdot\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\cdot\left(\frac{1}{\cos^2\frac{\theta}{2}}\right)\left(\frac{1}{2}\right)\mathbf{a}_\theta = -\frac{V_0}{r\ln\left(\tan\frac{\alpha}{2}\right)}\cdot\frac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\left(\frac{1}{2}\right)\mathbf{a}_\theta$$

$$= -\frac{V_0}{r\ln\left(\tan\frac{\alpha}{2}\right)}\cdot\frac{1}{\sin\theta}\mathbf{a}_\theta$$

The surface charge density on the cone is then ($\rho_S = D_N|_{\theta=\alpha} = \epsilon\mathbf{E}$)

$$\rho_S = \frac{-\epsilon V_0}{r\sin\alpha\ln\left(\tan\frac{\alpha}{2}\right)}$$

producing a total charge Q,

$$Q = \int_S \rho_S\, dS = \frac{-\epsilon V_0}{r\sin\alpha\ln\left(\tan\frac{\alpha}{2}\right)}\int_0^\infty\int_0^{2\pi}\frac{r\sin\alpha}{r}d\emptyset dr = \frac{-2\pi\epsilon V_0}{\ln\left(\tan\frac{\alpha}{2}\right)}\int_0^\infty dr$$

This leads to an infinite value of charge and capacitance, and it becomes necessary to consider a cone of finite size. Our answer will now be only an approximation because the theoretical equipotential surface is $\theta = \alpha$, a conical surface extending from $r = 0$ to $r = \infty$, whereas our physical conical surface extends only from $r = 0$ to, say, $r = r_1$. The approximate capacitance is

$$C = \frac{|Q|}{V_0} = \frac{2\pi\epsilon r_1}{\ln\left(\cot\frac{\alpha}{2}\right)}$$

Q6) **[12 Marks]**

(a) The magnetic field intensity is given in a certain region of space as

$$\mathbf{H} = \frac{x+2y}{z^2}\mathbf{a}_y + \frac{2}{z}\mathbf{a}_z \text{ A/m}$$

(i) Find **J**. (ii) Evaluate both sides of Stokes' theorem for the field **H** to find the total current passing through the surface $z = 4$, $1 < x < 2$, $3 < y < 5$, in the $\mathbf{a}_z$ direction.

Solution:

(i) $\mathbf{J} = \nabla\times\mathbf{H} = -\frac{\partial H_y}{\partial z}\mathbf{a}_x + \frac{\partial H_y}{\partial x}\mathbf{a}_z = \frac{2(x+2y)}{z^3}\mathbf{a}_x + \frac{1}{z^2}\mathbf{a}_z$

(ii) $I = \int\int \mathbf{J} \cdot d\mathbf{S} = \int\int \mathbf{J}|_{z=4} \cdot \mathbf{a}_z \, dxdy = \int_3^5 \int_1^2 \frac{1}{4^2} dxdy = \frac{1}{8}$ A

the other side of Stokes' theorem This involves two integrals of the y component of **H** over the range $3 < y < 5$. Integrals over x, to complete the loop, do not exist since there is no x component of **H**. We have

$$I = \oint \mathbf{H} \cdot d\mathbf{L} = \int_3^5 \frac{2 + 2y}{16} dy + \int_5^3 \frac{1 + 2y}{16} dy = \frac{1}{8}(2) - \frac{1}{16}(2) = \frac{1}{8} \text{ A}$$

(b) Express the value of **H** in at $P(\rho, \emptyset, \; 0)$ in the field of a coax, centered on the z axis, with $a = 0.3$, $b = 0.5$, $c = 0.7$, $I = 5$A in the $\mathbf{a}_z$ direction in the center conductor (i) $\rho = 0.2$, (ii) $\rho = 0.4$, (iii) $\rho = 0.6$, (iv) $\rho = 0.8$.

Solution:

(i) $\rho = 0.2$ As $\rho < a$

$$\mathbf{H} = \frac{I\rho}{2\pi a^2} \mathbf{a}_\emptyset = \frac{5 \times 0.2}{2\pi(0.3)^2} \mathbf{a}_\emptyset = \frac{1}{2\pi(0.09)} \mathbf{a}_\emptyset = 1.77\mathbf{a}_\emptyset \text{ A/m}$$

(ii) $\rho = 0.4$ As $a < \rho < b$

$$\mathbf{H} = \frac{I}{2\pi\rho} = \frac{5}{2\pi(0.4)} = 1.99\mathbf{a}_\emptyset \text{ A/m}$$

(ii) $\rho = 0.6$ As $b < \rho <$c

$$\mathbf{H} = \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right) = \frac{5}{2\pi(0.6)}\left(\frac{0.7^2 - 0.6^2}{0.7^2 - 0.5^2}\right) = 0.72\mathbf{a}_\emptyset \text{ A/m}$$

(ii) $\rho = 0.8$ As $\rho > c$

$$\mathbf{H} = 0 \text{ A/m}$$