Q1/ Prove that

$$\cos z = \frac{e^{iz}+e^{-iz}}{2} \text{ and } \sin z \frac{e^{iz}-e^{-iz}}{2i}$$

Sol

$$e^{iz} = \cos z + i\sin z$$

$$e^{-iz} = \cos z - i\sin z$$

by addition

$$e^{iz}+e^{iz} = 2\cos z$$

$$\Rightarrow \cos z = \frac{e^{iz}+e^{-iz}}{2}$$

by substructing

$$e^{iz}-e^{-iz} = 2i\sin z$$

$$\Rightarrow \sin z = \frac{e^{iz}-e^{-iz}}{2i}$$

حلول اسئلة الرياضيات / المرحلة الاولى
الفصل الثاني

Q2/ $\displaystyle\int \frac{dt}{\sqrt{25t^2-9}} \Leftarrow a=3$

Sol/

Let $5t = a\sec\theta \Rightarrow 5t = 3\sec\theta,\ t = \frac{3}{5}\sec\theta$

$5dt = 3\sec\theta.\tan\theta.d\theta$

$\Rightarrow dt = \frac{3}{5}\sec\theta.\tan\theta d\theta$

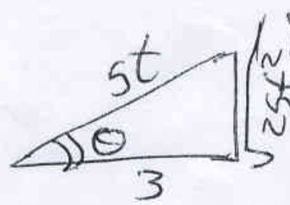
$$= \int \frac{\frac{3}{5}\sec\theta.\tan\theta.d\theta}{\sqrt{25*(\frac{3}{5}\sec\theta)^2-9}} = \int \frac{\frac{3}{\cancel{5}}\sec\theta\tan\theta\, d\theta}{\sqrt{\cancel{25}*\frac{9}{\cancel{25}}\sec^2\theta-9}}$$

$$= \int \frac{\frac{\cancel{3}}{5}\sec\theta\tan\theta}{\cancel{3}\sqrt{\sec^2\theta-1}}d\theta = \int \frac{\frac{1}{5}\sec\theta.\tan\theta.d\theta}{\sqrt{\tan^2\theta}}$$

$$= \frac{1}{5}\int \frac{\sec\theta\cancel{\tan\theta}d\theta}{\cancel{\tan\theta}} = \frac{1}{5}\int \sec\theta\, d\theta$$

$$= \frac{1}{5}\ Ln|\sec\theta+\tan\theta| + C$$

$$= \frac{1}{5}\ Ln\left|\frac{5t}{3}+\frac{\sqrt{25t^2-9}}{3}\right| + C$$



Q3/ Assume $V_1 = i + 2j - K$ and $V_2 = -2i + 2j + 2K$ find area of parallogram and triangle determind by V_1 and V_2.

Sol

$$V = V_1 \times V_2 = \begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} i & j & K \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} - j \begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} + K \begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix}$$

$$= i(4+2) - j\,\overset{0}{\cancel{(2-2)}} + K(2+4)$$

$$= 6i + 6K$$

Area of parallogram $= |V_1 \times V_2| =$

$$= \sqrt{36+36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

Area of triangle $= \dfrac{|V_1 \times V_2|}{2} = \dfrac{6\sqrt{2}}{2}$

$$= 3\sqrt{2}$$

Q4/ B/ Evaluate $\displaystyle\int_0^{\pi/6} \cos\theta \cdot 4^{-\sin\theta}\, d\theta$

Sol

$$4^{-\sin\theta} \longrightarrow 4^{-\sin\theta} \cdot -\cos\theta \cdot \ln 4$$

$$= \int_0^{\pi/6} \cos\theta \cdot 4^{-\sin\theta}\, d\theta * \frac{-\ln 4}{-\ln 4}$$

$$= \frac{-1}{\ln 4}\left[4^{-\sin\theta}\right]_0^{\pi/6}$$

$$= \frac{-1}{\ln 4}\left[4^{-\sin(\frac{\pi}{6})} - 4^{-\sin(0)}\right]$$

$$= \frac{-1}{\ln 4}\left[4^{-\frac{1}{2}} - 4^{0}\right]$$

$$= \frac{-1}{\ln 4}\left[\frac{1}{4^{\frac{1}{2}}} - 1\right] = \frac{-1}{\ln 4}\left[\frac{1}{2} - 1\right]$$

$$(2^2)^{\frac{1}{2}}$$

$$= \frac{-1}{\ln 4} \cdot \frac{-1}{2} = \frac{1}{2\ln 4}$$

Q4/A//

$$\begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 3 & 5 & 2 \\ -1 & 2 & 0 \\ 4 & 5 & H \end{vmatrix} \end{array}$$

$$= 3\begin{vmatrix} 2 & 0 \\ 5 & H \end{vmatrix} - 5\begin{vmatrix} -1 & 0 \\ 4 & H \end{vmatrix} + 2\begin{vmatrix} -1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= 3(2H) - 5(-H) + 2(-5-8)$$

$$= 6H + 5H - 26 \quad = 11H - 26$$

$$\Rightarrow$$

$$\begin{vmatrix} H+1 & 2 \\ 3 & 10 \end{vmatrix} = 10H + 10 - 6$$

$$= 10H + 4$$

$$\Rightarrow \quad 11H - 26 = 10H + 4$$

$$\Rightarrow \boxed{H = 30}$$

Q/ $\int \frac{x^2-7}{x^3-2x^2-3x}\,dx$

$$= \int \frac{x^2-7}{x(x^2-2x-3)}\,dx = \int \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}$$

$$\Rightarrow \frac{A(x-3)(x+1) + Bx(x+1) + Cx\cdot(x-3)}{x(x-3)(x+1)}$$

$$= \frac{Ax^2 - 2Ax - 3A + Bx^2 + Bx + Cx^2 - 3Cx}{x(x-3)(x+1)}$$

$$= \frac{x^2(A+B+C) + x(-2A+B-3C) - 3A}{x(x-3)(x+1)}$$

$A+B+C=1$

$-2A+B-3C=0$

$-3A=-7 \Rightarrow A=\frac{7}{3}$

$A+\cancel{B}+C=1$

$-2A+\cancel{B}-3C=0$

$C = \frac{-6}{4} = \frac{-3}{2}$

$B = \frac{1}{6}$

$$= \int \frac{+\frac{7}{3}}{x}\,dx + \int \frac{1/6\,dx}{(x-3)} + \int \frac{-\frac{3}{2}\,dx}{(x+1)}$$

$$= \frac{7}{3}\,\mathrm{Ln}(x) + \frac{1}{6}\,\mathrm{Ln}(x-3) - \frac{3}{2}\,\mathrm{Ln}(x+1) + C$$

Q/6 Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$ about the y-axis?

Sol/

$$V = \int_1^4 \pi \left[R(y)\right]^2 dy$$

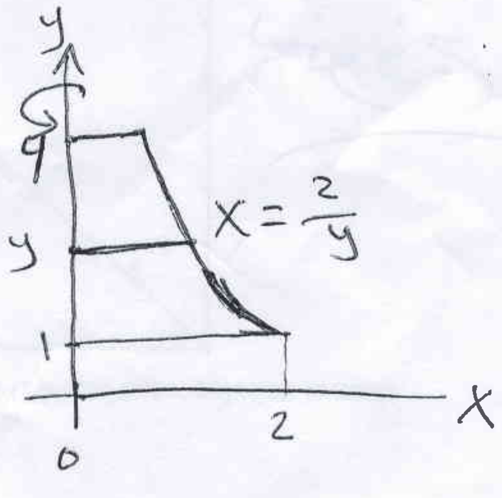
$$V = \int_1^4 \pi \cdot \left(\frac{2}{y}\right)^2 dy$$

$$V = \int_1^4 \pi \frac{4}{y^2} dy$$

$$= 4\pi \int_1^4 \frac{1}{y^2} dy = 4\pi \int_1^4 y^{-2} dy$$

$$V = 4\pi \left[\frac{-1}{y}\right]_1^4$$

$$= 4\pi \left[\frac{-1}{4} + 1\right] = \cancel{4}\pi \frac{3}{\cancel{4}} = 3\pi$$



Q3// Find A^{-1} if $A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 2 & -3 & 1 \\ 0 & 1 & 4 \\ -1 & 2 & 3 \end{bmatrix} \end{array}$

sol/ D

) $D = 2\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 4 \\ -1 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = 2(3-8) + 3(0+4) + 0(D+1)$

) $|A| = 3$

$1 = +1\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & 4 \\ -1 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = (3-8) - (0+4) + (0+1)$

$= -5 - 4 + 1$ (السطر الأول)

$= -1\begin{vmatrix} -3 & 1 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} = -(-9-2) + 1(6+1) - 1(4-3)$

$= 11 + 7 - 1$ (الصف الثاني)

$A_3 = +1\begin{vmatrix} -3 & 1 \\ 1 & 4 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = (-12-1) - (8-0) + (2+0)$

$= -13 - 8 + 2$ (الصف الثالث)

$cof(A) = \begin{bmatrix} -5 & -4 & 1 \\ 11 & 7 & -1 \\ -13 & -8 & 2 \end{bmatrix}$

(3) $adj = cof^T(A) = \begin{bmatrix} -5 & 11 & -13 \\ -4 & 7 & -8 \\ 1 & -1 & 2 \end{bmatrix}$

(4) $A^{-1} = \dfrac{adj(A)}{|A|} = \dfrac{1}{3}\begin{bmatrix} -5 & 11 & -13 \\ -4 & 7 & -8 \\ 1 & -1 & 2 \end{bmatrix}$

and