اجوبة امتحان مادة الرياضيات / المرحلة الثانية

الدور الاول ٢٠١٦ - ٢٠١٧

Q1/ⓐ $y'' - 3y' + 2y = x e^{3x} + 1$

$y_g = y_h + y_p$

$y_h = c_1 e^{x} + c_2 e^{2x}$ $\quad (r_1 = +1 \,,\, r_2 = +2)$

$y_p = (Ax + B)e^{3x} + C$ (ونشتق ونعوض لإيجاد C, B, A)

or $\quad y_p = u_1 v_1 + u_2 v_2 \quad (u_1 = e^{x} \,,\, u_2 = e^{2x})$

to find v_1, v_2

$$D = \begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix} = e^{3x} \,, \quad D_1 = \begin{vmatrix} 0 & e^{2x} \\ xe^{3x}+1 & 2e^{2x} \end{vmatrix} = -xe^{5x} - e^{2x}$$

$$D_2 = \begin{vmatrix} e^{x} & 0 \\ e^{x} & xe^{3x}+1 \end{vmatrix} = xe^{4x} + e^{x}$$

$$v_1 = \int \frac{D_1}{D} = \int \frac{-xe^{5x} - e^{2x}}{e^{3x}} = -\int (xe^{2x} + e^{-x})\,dx = e^{-x} - \tfrac{1}{2}xe^{2x} - \frac{e^{2x}}{4}$$

$$v_2 = \int \frac{D_2}{D} = \int \frac{xe^{4x} + e^{x}}{e^{3x}} = \int (xe^{x} + e^{-2x})\,dx = xe^{x} - e^{x} - \frac{e^{-2x}}{2}$$

$$\Rightarrow y_p = e^{x}\left(e^{-x} - \frac{x}{2}e^{2x} - \frac{e^{2x}}{4}\right) + e^{2x}\left(xe^{x} - e^{x} - \frac{e^{-2x}}{2}\right)$$

$$= \tfrac{1}{2} + \tfrac{1}{2}xe^{3x} - \tfrac{3}{4}e^{3x}$$

$\Rightarrow y_g = y_h + y_p$

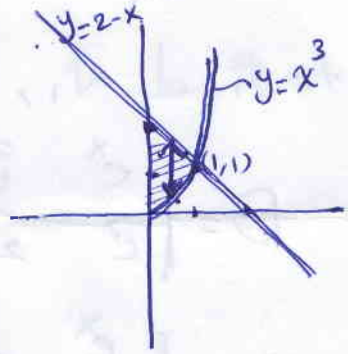
Q1/ b $\sum_{n=1}^{\infty} \frac{(n+5)!}{5!\, n!\, 5^n}$

By using Ratio Test $\rho = \frac{a_{n+1}}{a_n}$

$\Rightarrow \rho = \frac{(n+6)!}{5!\,(n+1)!\,5^{n+1}} \cdot \frac{5!\,n!\,5^n}{(n+5)!} = \frac{n+6}{(n+1).5}$

$\rightsquigarrow \lim_{n\to\infty} \frac{n+6}{(n+1)5} = \frac{1}{5} < 1 \rightsquigarrow$ Conv.

Q2/ a $I = \iint_R (x-y).dA$ $\quad y = x^3$, $y = 2-x$



$I = \int_0^1 \int_{x^3}^{2-x} (x-y)\,dy\,dx = \int_0^1 \left(xy - \frac{y^2}{2}\right]_{x^3}^{2-x} dx$

$= \int_0^1 \left[\left(2x - x^2 - \frac{(2-x)^2}{2}\right) - \left(x^4 - \frac{x^6}{2}\right)\right] dx$

$= \int_0^1 \left[2x - x^2 - \frac{4-4x+x^2}{2} - x^4 + \frac{x^6}{2}\right] dx$

$= x^2 - \frac{x^3}{3} - 2x + x^2 - \frac{x^3}{6} - \frac{x^5}{6} + \frac{x^7}{14}\Big]_0^1$

or $I = \int_0^1 \int_0^{\sqrt[3]{y}} (x-y)\,dx\,dy + \int_1^2 \int_0^{2-y} (x-y)\,dx\,dy$

Q2/(b) $\vec{V_1} = 3i - 6j - 2K$ } normal vectors
$\vec{V_2} = 2i + j - 2K$ } from P_1, P_2

$$\vec{V_1} \times \vec{V_2} = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 14i + 2j + 15K$$

Now we need a point, let $Z = 0 \implies 3x - 6y = 3$
$2x + y = 2$

$x = 1$, $y = 0$ وبحل المعادلات انيا نجد

$\implies$ The point $(1, 0, 0)$ lie in the line intersection

$\implies$ The par. equation of line is : $x = 1 + 14t$
$y = 2t$
$z = 15t$
$-\infty < t < \infty$

Q3/ $z'' + y' = \cos t$

$y'' - z = \sin t$

$z(0) = -1,\ z'(0) = -1$

$y(0) = 1,\ y'(0) = 0$

from one equation

$$s^2 Z(s) - sZ(0) - Z'(0) + sY(s) - y(0) = \frac{s}{s^2+1}$$

$$s^2 Z(s) + s \cancel{+1} + sY(s) \cancel{-1} = \frac{s}{s^2+1}$$

$$s^2 Z(s) + sY(s) = \frac{s}{s^2+1} - s = \frac{-s^3}{s^2+1} \quad ---(1)$$

from two equation:

$$s^2 Y(s) - sY(0) - Y'(0) - Z(s) = \frac{1}{s^2+1}$$

$$s^2 Y(s) - s - Z(s) = \frac{1}{s^2+1}$$

$$\Rightarrow s^2 Y(s) - Z(s) = \frac{1}{s^2+1} + s = \frac{1+s^3+s}{s^2+1} \quad ---(2)$$

$$\Rightarrow \begin{bmatrix} s^2 & s \\ -1 & s^2 \end{bmatrix} \begin{bmatrix} Z(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} \frac{-s^3}{s^2+1} \\ \frac{s^3+s+1}{s^2+1} \end{bmatrix}$$

$$\Rightarrow Z(s) = \frac{\begin{vmatrix} \frac{-s^3}{s^2+1} & s \\ \frac{s^3+s+1}{s^2+1} & s^2 \end{vmatrix}}{\begin{vmatrix} s^2 & s \\ -1 & s^2 \end{vmatrix}} =$$

$$Y(s) = \frac{\begin{vmatrix} s^2 & \frac{-s^3}{s^2+1} \\ -1 & \frac{s^3+s+1}{s^2+1} \end{vmatrix}}{\begin{vmatrix} s^2 & s \\ -1 & s^2 \end{vmatrix}} =$$

Then ~~xxx~~ Find $\mathcal{L}^{-1}\{Z(s)\} = -\cos t - \sin t$

$$\mathcal{L}^{-1}\{Y(s)\} = \cos t$$

Q4/ 9 $\quad f(x) = \frac{1}{x} \qquad x = 2$

$f(x) = x^{-1}$ $\quad \cancel{= \frac{-1}{x^2}}$

$f'(x) = -x^{-2} = \frac{-1}{x^2} \Longrightarrow f'(2) = \frac{-1}{2^2}$

$f''(x) = 2x^{-3} = \frac{2!}{x^3} \Longrightarrow f''(2) = \frac{2!}{2^3}$

$f'''(x) = -6x^{-4} = \frac{-3!}{x^4} \Longrightarrow f'''(2) = \frac{-3!}{2^4}$

$f^{(4)}(x) = 24x^{-5} = \frac{4!}{x^5} \Longrightarrow f^{(4)}(2) = \frac{4!}{2^5}$

for five terms:

$f(x) = f(a) + f'(a)(x-a) + f''(a).\frac{(x-a)^2}{2!} + f'''(a).\frac{(x-a)^3}{3!} + f^{(4)}(a)\frac{(x-a)^4}{4!}$

$$\Rightarrow f(x) = \frac{1}{2} - \frac{x-2}{2^2} + \frac{(x-2)^2}{2^3} - \frac{(x-2)^3}{2^4} + \cdots + (-1)^n \frac{(x-2)^n}{2^{n+1}}$$

To find the interval conv

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{2^{n+2}} \cdot \frac{2^{n+1}}{(x-2)^n}\right| < 1$$

$$\Rightarrow \lim_{n\to\infty}\left|\frac{x-2}{2}\right| < 1 \Rightarrow \boxed{0 < x < 4}$$

Q4/ⓑ $\mathcal{L}^{-1}\left\{\dfrac{3s-137}{s^2+2s+401}\right\}$

$$\frac{3s-137}{s^2+2s+401} = \frac{3s-137}{s^2+2s+1+401} = \frac{3s+3-3-137}{(s+1)^2+400}$$

$$= \frac{3(s+1)-140}{(s+1)^2+400} = \left\{\frac{3(s+1)}{(s+1)^2+400} - \frac{140}{(s+1)^2+400}\right\}$$

$$= 3e^{-t}\cos 20t - \frac{140}{20} e^{-t}\sin 20t$$

Q5/ⓐ $\vec{R}(t) = 2\cos t\, i + 3\sin t\, j \quad , t = \frac{\pi}{4}$

$$K = \frac{|\ddot{y}\dot{x} - \ddot{x}\dot{y}|}{\left((\dot{x})^2 + (\dot{y})^2\right)^{3/2}}$$

$\dot{y}(t) = -2\sin t \quad , \ddot{y}(t) = -2\cos t$

$\dot{x}(t) = 3\cos t \quad , \ddot{x}(t) = -3\sin t$

$$\Rightarrow K = \frac{\left|\left(-2\cos\frac{\pi}{4} * 3\cos\frac{\pi}{4}\right) - \left(-3\sin\frac{\pi}{4} * -2\sin\frac{\pi}{4}\right)\right|}{\left((3\cos t)^2 + (-2\sin t)^2\right)^{3/2}}$$

or $\quad K = \dfrac{|v \times a|}{|v|^3} \quad$, where $R(t) = 2\cos t\, i + 3\sin t\, j + 0\, k$

Q 5/(b) prove $I = \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$

$$I \cdot I = \int_0^\infty \int_0^\infty e^{-x^2} \cdot e^{-y^2} dx\,dy = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$$

Use polar Coord.

$0 \le r \le \infty$

$0 \le \theta \le \frac{\pi}{2}$

$$\Rightarrow I^2 = \int_0^\infty \int_0^{\pi/2} e^{-r^2} r\,dr\,d\theta$$

$$= \frac{\pi}{4}$$

$$\Rightarrow I = \sqrt{\frac{\pi}{4}} = \frac{\sqrt{\pi}}{2}$$