Numerical Study of Entropy Generation in a Vertical Square Channel Packed with Saturated Porous Media

Dr. Abdulhassan A. Karamallah *, Dr. Wahid S. Mohammad* & Wissam Hashim Khalil*

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Abstract

Entropy generation in a vertical square channel packed with saturated porous media, and subjected to differentially heated isothermal walls has been numerically investigated.

The effect of Darcy, Reynolds, and Eckert numbers on Entropy generation was studied. The entropy generation was found to be inversely proportional to both Reynolds and Darcy number, while it was directly proportional with the Eckert number.

It was shown that as Darcy and Reynolds numbers were increasing, the Bejan number decreases, i.e., the irreversibility due fluid friction is dominated, while as the Eckert increases, their irreversibility due to heat transfer increases.

Keywords: entropy generation, porous media, Bejan number

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Cross sectional area of test section</td>
<td>m²</td>
</tr>
<tr>
<td>C_p</td>
<td>Specific heat at constant pressure</td>
<td>J/kg . K</td>
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</table>
### Symbol Description Dimension

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient</td>
<td>W/m².°C</td>
</tr>
<tr>
<td>$K$</td>
<td>Porous media Permeability</td>
<td>m²</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
<td>W/m.°C</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>$S$</td>
<td>Entropy</td>
<td>W/m.°C</td>
</tr>
<tr>
<td>$S_{gen.}$</td>
<td>Entropy generation per unit volume</td>
<td>W/m³.°C</td>
</tr>
<tr>
<td>$T$</td>
<td>Boundary condition of constant wall Temperature</td>
<td>-</td>
</tr>
<tr>
<td>$U,V,W$</td>
<td>Non-dimensional velocity components</td>
<td>-</td>
</tr>
<tr>
<td>$u,v,w$</td>
<td>Velocity components</td>
<td>m/s</td>
</tr>
<tr>
<td>$W$</td>
<td>Rate of work done</td>
<td>W</td>
</tr>
<tr>
<td>$X,Y,Z$</td>
<td>Non-dimensional axis</td>
<td>-</td>
</tr>
<tr>
<td>$x,y,z$</td>
<td>Coordinate axis</td>
<td>M</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Dimensionless temperature, $\frac{T - T_w}{T_m - T_w}$</td>
<td>-</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>N.s/m²</td>
</tr>
<tr>
<td>$\phi$</td>
<td>General dependent variable</td>
<td>-</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Dimensional Temperature different</td>
<td>$\frac{\Delta T}{T_o}$</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
<td>$Re = \frac{\rho \cdot u_{in} \cdot D_h}{\mu_l}$</td>
</tr>
<tr>
<td>Da</td>
<td>Darcy number</td>
<td>$Da = \frac{K}{W^2}$</td>
</tr>
</tbody>
</table>
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pe</td>
<td>Peclet number</td>
<td>( Pe = u D_h / \alpha_e )</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
<td>( Pr = \frac{\nu}{\alpha_e} )</td>
</tr>
<tr>
<td>Br</td>
<td>Brinkman number</td>
<td>( Ec Pr = \frac{\mu u^2}{k \Delta T} )</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>Temperature difference</td>
<td>°C</td>
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1- Introduction

The placement of porous materials in passages can enhance the transfer of heat to a flowing fluid. Porous passages are useful devices for cooling of engineering systems. There has been a current interest in utilization of porous passages for electronic cooling applications.

Studies related to a laminar flow in a channel filled with saturated porous media have significantly increased during recent years. This type of geometry and flow configuration are commonly observed in the field of electronics cooling system, solid matrix heat exchanger, geothermal system, nuclear waste disposal, microelectronic heat transfer equipment, coal and grain storage, petroleum industries, and catalytic converters. Meanwhile, the improvement in the thermal systems as well as the energy utilization during the convection in any fluid is one of the fundamental problems of the engineering processes, since the improved thermal systems will provide better material processing, energy conservation, and environmental effects [1].

Thermodynamics are broadly viewed as the science of energy, and the thermal engineering is concerned with making the best use of available energy resources. The science of thermodynamics is primarily built on two fundamental natural laws, known as the first and second laws. The first law is simply an expression of the conversion of energy principle, while the second law related with the concept of "entropy" which is the first distinction between reversible and irreversible processes in thermodynamics.

As the entropy generation takes place, the quality of the energy (i.e., the exergy) decreases [2]. The broader interpretation of the SLT (Second Law of Thermodynamics) suggests that the real "energy conservation" should include the practice of thermodynamic economy. Each energy transfer or conversion, all else being equal, should be arranged so that the total change in the entropy (entropy generation) is a minimum. This requires that energy...
It is easy to show that the rate of destruction of a useful work in an engineering system, $W_{\text{lost}}$, is directly proportional to the rate of entropy generation [2]:-

$$W_{\text{lost}} = T_0 S_{\text{gen}}$$  \hspace{1cm} (1)

Where $T_0$ is the absolute temperature of the surrounding temperature reservoir ($T_0$=constant). Equation (1) stresses the engineering importance of estimating the irreversibility or entropy generation rate of the convective heat transfer processes: If not used wisely, these processes contribute to the waste of the precious fuel resources.

The rate of the entropy generation per unit time and per unit volume $S_{\text{gen}}$ is given below:-

$$S_{\text{gen}} = \frac{k}{T^2} (\nabla T)^2 + \mu \frac{\Phi}{T}$$  \hspace{1cm} (2)

Where $k$ and $\mu$ are assumed constants.

In the last equation, $T$ represents the absolute temperature of the point where $S_{\text{gen}}$ is being evaluated. The two expressions in equation (2) illustrate the competition between the viscous dissipation and imperfect thermal contact (finite-temperature gradients) in the generation of entropy via a convective heat transfer.

The goal of optimization must first be established: it could be for a size reduction and/or to reduce the operating costs. The operational variables that could be optimized are the heat transfer rate, pumping power (pressure drop), flow rate, and fluid velocity. When considering the optimization by reducing the size (and hence saving the amount of material used), the increase in the manufacturing cost should be taken into account. The main objective is the improvement of the heat transfer. But it is counter balanced by the increase in the pumping power.

It is possible to find a design that is the balance between these two effects. One such optimization method is called the Entropy Generation Minimization (EGM)—the entropy produced is calculated and the parameters for a minimum entropy are found. A short article on the Entropy Generation Minimization was published in 1978, [4] gave a general criterion to rate the performance. It described the two types of losses a heat exchanger can have, losses due to the fluid-to fluid temperature difference ($\Delta T$) and losses due to the frictional pressure drop ($\Delta P$) losses. A decrease in the $\Delta T$ losses would result in an increase of $\Delta P$ losses. Both losses contribute to the irreversibility. The entropy was used to quantify this irreversibility, and minimizing the amount of the entropy produced will thus deliver an optimum design. The Entropy Generation Minimization method combined the principles of the heat transfer, fluid mechanics and thermodynamics and was used in the optimization of real, irreversible devices [5].

In conclusion, the Entropy Generation Minimization method is a well-established optimization method that is used on several heat transfer applications. It combines the fields of thermodynamics, heat transfer, and fluid mechanics. The method can optimize the real systems and can be adapted to suit a specific application.

Bejan (1979) [6] was the first author that studied the entropy generation
in a convective heat transfer in a pipe flow. In all cases considered in his research he found that the pipe wall region acted as a strong concentrator of irreversibility. Considerable research work was carried out to investigate the importance of entropy generation in the thermal systems of different cross sections and different thermal boundary conditions without taken into account the existing of porous media [7-18]. In the last mentioned papers they indicated all the parameters that affect the entropy generation minimization. The entropy generation rate in a laminar flow through a channel filled with saturated porous media was investigated by [19] and [20] for different thermal boundary conditions. The Brinkman model was employed. The result showed that the heat transfer irreversibility dominated over the fluid friction irreversibility (i.e. \( 0 \leq \phi \leq 1 \)) and the viscous dissipation had no effect on the entropy generation rate at the centerline of the channel. The effect of the temperature – dependent viscosity on the fully developed forced convection in a duct of a rectangular cross section occupied by a fluid saturated porous medium was analytically investigated by [21]. Having found the velocity and the temperature distribution, the second law of thermodynamics was invoked to find the local and the average entropy generation rate. The Bejan number and the heat transfer irreversibility were presented in terms of the Brinkman number, the Peclet number, the viscosity variation number, and the aspect ratio (width to height ratio). It was observed that the entropy generated due to the flow in a duct of a square cross section was more than those of rectangular counter – parts while increasing the aspect ratio decreased the entropy generation rate. Hooman et al. (2008) [22] A numerical study is reported to investigate the entropy generation due to forced convection in a parallel plate channel filled by a saturated porous medium. Two different thermal boundary conditions were considered being isoflux and isothermal walls. Increasing the porous media shape factor and the Brinkman number, and decreasing the dimensionless degree of irreversibility of the problem, as reflected in (Ns). Moreover, one concludes that different arrangement of the parameters will lead to completely different behavior for both (Ns) and (Be) as described. The present work is carried out to fulfill this gap in the literature. Its very important subject for it is engineering and industrial application such as regenerative heat exchanger used in energy conservation or waste heat recovery devices which are very important nowadays in saving man power and many.

2- Mathematical Model

A mathematical analysis of the partial differential equations that describe the flow of the fluid and the heat transfer in the duct is presented. These equations are based on the conservation of mass, momentum, and energy. The effect of introducing porous media in a duct packed bed using Darcy model equation is introduced. The entropy generation equation is also presented to study the Entropy Generation Minimization (EGM).
The most appropriate coordinate system to express the physical problem is the Cartesian Coordinates System (x, y, z). The point of the origin of these coordinates is the duct edge which lies at the intersection of the left wall of the duct with the bottom wall of the duct. In order to solve the governing equations, the following assumptions are used:
1- The flow is assumed to be steady and incompressible.
2- Body forces are neglected.
3- Constant property flow.
4- The flow is laminar.
5- Natural convection effects are negligible. This assumption based on the fact that the bed is dominated by the forced convection.
6- Physical properties of fluid and solid particle are constant.
7- Liquid and solid phases are in thermal equilibrium.
8- Heat transfer to the surrounding is negligible.

The governing equations of the fluid motion in a complete form that includes the conservation of mass, momentum, energy, are referred as Navier-Stokes equations. All theoretical and computational fluid dynamics are based on the Navier-Stokes equations, and the derivation of these equations are available in all textbooks, so only the last form related to the present will be presented. In the governing equation, the primitive variable will be used. This will give the solution of momentum equations similar that for scalar transport, which provided pressure field available. To study the effect of porous media through the duct, the Brinkman model is introduced. The momentum equation model includes the addition of the Darcy terms ($\frac{\mu}{k} \overline{V}$) to the classical momentum equation, where ($\overline{V}$) represent the local velocity vector. The using of the last model rise the term “Brinkman-Brinkman” model which proposed by [23], refers to a problem involving a saturated porous medium in which Brinkman momentum equation is used, and the thermal energy includes a viscous dissipation term involving Brinkman number:

Non-dimensionalization of Governing Equations

The governing equations may be non-dimensionalized to achieve the work requirements. First, it would provide the conditions upon which the dynamics and energetic similarity may be obtained for the geometrically similar conditions. Second, the solution of such equations would usually provide values within limits between zero and one. The following non-dimensional variables are introduced to non-dimensionalize the governing equations (continuity, momentum, and energy equations):

$$
X = \frac{x}{a}, \quad Y = \frac{y}{a}, \quad Z = \frac{z}{a}, \\
U = \frac{u}{u_o}, \quad V = \frac{v}{u_o}, \quad W = \frac{w}{u_o}, \\
P = \frac{\rho}{\rho u_o^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \\
Re = \frac{\rho h u_o^2}{\mu}, \quad Da = \frac{K}{a^2}, \\
Ec = \frac{u_o^2}{C_f \cdot \Delta T}, \quad Pr = \frac{\mu \cdot C_p}{k}
$$

The following non-dimensional variables are introduced to non-dimensionalize the entropy generation equation: [24]
\[ Ns = \frac{S_{gen}}{S_o} \]
\[ S_o = \frac{k_f(\Delta T)^2}{h^2T_o^2} \]
\[ Br = Ec.Pt \]
\[ Br^* = \frac{Br}{\sqrt{Da}} \]
\[ \Omega = \frac{\Delta T}{T_o} \]

Substitution of the non-dimensional quantities into the governing equations yields:
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0
\]

\[
\left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} \right) = -\frac{\partial P}{\partial X}
\]
\[
+ \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) - \frac{1}{ReDa} U
\]

\[
\left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} \right) = \frac{\partial P}{\partial Y}
\]
\[
+ \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) - \frac{1}{ReDa} V
\]

\[
\left( U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} \right) = \frac{\partial P}{\partial Z}
\]
\[
+ \frac{1}{Re} \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) - \frac{1}{ReDa} W
\]

\[
\left( U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} \right) = \frac{1}{RePr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right)
\]
\[
+ \frac{Ec}{ReDa} \left( U^2 + V^2 + W^2 \right)
\]

\[ Ns = \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 + \left( \frac{\partial \theta}{\partial Z} \right)^2 + \frac{Br}{\Omega}
\]
\[
\left[ 2 \left( \frac{\partial U}{\partial X} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 + 2 \left( \frac{\partial W}{\partial Z} \right)^2 \right] + \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} \right) \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} \right)
\]
\[
+ 2 \left( \frac{\partial U}{\partial X} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 + 2 \left( \frac{\partial W}{\partial Z} \right)^2
\]
\[
\left[ \frac{2}{3} \left( \frac{\partial U}{\partial X} \right)^2 + \frac{2}{3} \left( \frac{\partial V}{\partial Y} \right)^2 + \frac{2}{3} \left( \frac{\partial W}{\partial Z} \right)^2 \right]
\]

3- Computational Implementation

The computational fluid dynamic is today an equal partner with a pure theory and a pure experiment in the analysis and solution of the fluid dynamic problems. Computational technique for the solution of the continuity, momentum (with considering Darcy model to treat the packed bed), energy, and entropy generation equations, was performed by using a finite volume method (FVM) to obtain the discretization form for these equations. These discretized equations were solved by using a SIMPLE algorithm with a hybrid scheme. A computer program based on this algorithm and using Fortran 90 language was built to meet the requirements of the problem. The SIMPLE algorithm was based on the staggered grid in which the velocities staggered midway between the grids. non-linear, it is sometimes necessary to relax the solution to avoid the divergence, and to ensure the
stability of the iterative processes. The value of relaxation factor in the procedure of solution was taken (0.7). The convergence criteria of the solution was \(10^{-4}\).

4- Results

As previously mentioned, the governing equations are used to provide a useful parameter to understand various thermal and hydrodynamic effects on the heat and flow through porous media, with the study of their effects on the entropy generation through the packed channel.

4.1 Validity of Numerical Code

For the sake of validation of the present code, a comparison between the present work and previous work studies from literature are presented in Figure (3). This figure shows a comparison for the velocity profile for different ranges of Darcy numbers with different works from the literature. From the Figure, it is shown that there is a fairly good agreement between the present result and the literature. Figure (4) shows a comparison for the temperature profile for different ranges of Darcy numbers, it is shown that the trend of the temperature distribution is similar for different Darcy numbers but the higher centerline temperature is less than that of the compared one, this is due to the double height used by [25].

4.2 Entropy Generation

In the case of an irreversibility analysis, there are a large number of parameters to vary, and the calculations are time consuming. The effect of the Reynolds, Darcy, and Eckert numbers on the entropy generation and Bejan number fields with different combinations of the above parameters are considered.

Having obtained the velocity and temperature distributions, the local entropy distribution can be calculated by inserting the velocity and temperature values in the entropy equation after being discretized. The entropy equation is discretized using the same finite difference approximations used for the continuity, axial momentum and energy equations. In Figures (5), (6) for entropy generation number, the profile is symmetric about the centerline of the channel due to the symmetric velocity and temperature distribution. For all parameters, each wall acts as a strong concentrator of irreversibility because of the high near-wall gradients of velocity and temperature. The maximum entropy generation occurs near the cold wall for all group parameters.

4.3 Effect of (Da, Re, Ec) Numbers on Entropy Generation

Figures (5), (6) show the effects of (Da, Re) numbers on entropy generation. At a constant Reynolds number, it's evident that as the permeability decreases (decreasing Darcy number), the entropy generation number increased. This is due, firstly to high restrictive medium which leads to more disorder in the fluid particles, and secondly to the high velocity and temperature gradient.

The values of the entropy generation number decreases as Reynolds number increases since the Reynolds number depends only on the flow geometry i.e., for higher values of Reynolds the characterized dimension will be greater and this means less velocity and temperature gradient in the axial direction, leading to decrease in the entropy generation.
As the Eckert number is decreased, the fluid friction entropy generation decreases exponentially, this is obvious from the entropy equation (10) as the Eckert number term is multiplied by the fluid friction term. The effect of the Eckert number on the entropy generation number is presented in Figure (7), from this Figure, for a constant Darcy number one can see that higher entropy generation attained for higher Eckert.

4.4 Bejan Number
The production of the Entropy generation may be contributed from two ways fluid friction entropy generation and heat transfer entropy generation. Bejan number measures the relative effect of the heat transfer and fluid friction entropy generation on the total entropy generation.

\[ Be = \frac{HTI}{HTI + FFI} \]

where:
HTI : Heat transfer Irreversibility.
FFI : Fluid friction Irreversibility.
The denominator of the above equation represent the total entropy generation through the system, While the numerator represent the entropy generation due to heat transfer irreversibility. Bejan Number takes the following values:

\[ Be > \frac{1}{2} \]: Irreversibility due to heat transfer dominates

\[ Be < \frac{1}{2} \]: Irreversibility due to fluid friction dominates

\[ Be = \frac{1}{2} \]: Irreversibility due to Heat transfer and Fluid Friction are equal

Figures (8) and (9) show the effect of the Darcy, Reynolds and Eckert numbers on the Bejan number. Figure (8) takes into account the variation of Reynolds number and its effect on the Bejan number, the curves show that as Reynolds number increases, Bejan number decreases and this means that the irreversibility due to heat transfer is decreased.

As Darcy number increase Figure (9), this will strengthen the effect of the fluid friction irreversibility on heat transfer irreversibility, therefore, resulting in a lower magnitude Bejan number at a given Reynolds and Eckert numbers.

Figure (9) presented also the effect of the Eckert number on the Bejan number. For a higher Eckert (Ec=10) where the heat transfer irreversibility is dominated over the fluid friction irreversibility, hence higher Bejan number attained. This is may be attributed to the large kinetic energy available to be converted to heat through the viscous dissipation.

5- Conclusions
1. The value of the entropy generation number decreases as the Reynolds number, Darcy number increases and Eckert number decreases.
2. The irreversibility due to fluid friction dominates for higher Darcy numbers, while as Darcy decreases, the irreversibility dominates due to the heat transfer.
3. The irreversibility due to the fluid friction is increased with increasing Eckert number for a given Reynolds number.

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Figure (1) the geometry of the duct and the coordinate axis
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Figure (2) Two dimensions section through the duct

Figure (3) Velocity distribution for different Darcy numbers at (Re=100), A: present work, B:[25], C: [26]
Figure (4) Comparison of non-dimensional temperature for present work with A: present work, B: [25]

Figure (5) Entropy generation at different Re and Da
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Figure (6) Entropy generation at Da=0.001, 0.01 for different Re

Figure (7) Entropy generation at Ec=0.1, 1, Da= (0.001, 1000) for different Re
Figure (8) Bejan number at Ec=0.1, 1, 10, Da=(0.001, 1000) for different Re
Figure (9) Bejan number at $Ec=0.1, 1, Re=100, 200, 500$ for different $Da$. 